Physics 1100: DC Circuits Solutions

1. In the diagram below, $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, and $R_3 = 15\ \Omega$. The battery supplies an emf of $\varepsilon = 0.30$ V. What is the equivalent resistance, R_S? What is the current through each resistor? What is the voltage drop across each resistor? What is the power expended in each resistor?
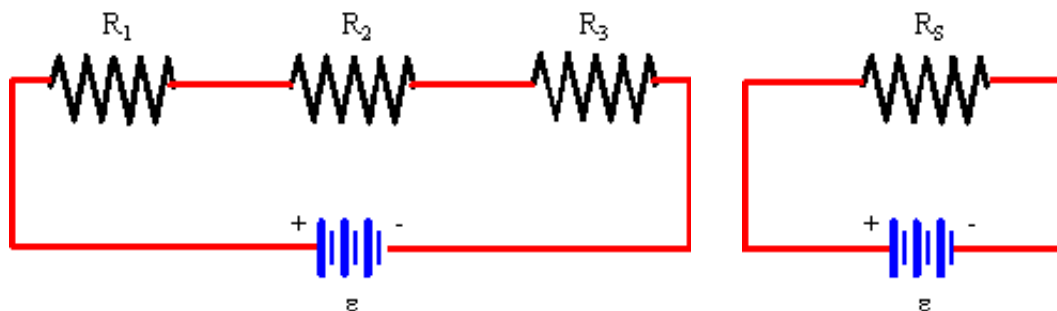
(i) Since there are no nodes or branches between the three resistors, the three resistors are in series. For series resistors, the equivalent resistance is

$$R_S = R_1 + R_2 + R_3 = 5\ \Omega + 10\ \Omega + 15\ \Omega = 30\ \Omega\ .$$

(ii) Resistors in series each carry the same current as their equivalent resistance, R_S. The definition of equivalent means that the three resistors could be replaced by R_S without affecting any other aspect of the circuit.

Using Ohm's Law the current through R_S is

$$I = \varepsilon/R_S = 0.30\ \text{V} / 30 = 0.010\ \text{A}\ .$$

Thus R_1, R_2, and R_3 each carry 0.010 A.

(iii) Again using Ohm's Law, the voltage across each resistor is given by $V = IR$. The results are given below.

(iv) The power dissipated by each resistor is given by $P = I^2R = V^2/R$ as shown below.

Resistor (Ω)	*I* (Amps)	*V=IR* (Volts)	*P=I²R* (Watts)
5	0.010	0.050	0.50×10^{-3}

10	0.010	0.100	1.00×10^{-3}
15	0.010	0.150	1.50×10^{-3}
30	0.010	0.300	3.00×10^{-3}

2. In the diagram below, $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, and $R_3 = 15\ \Omega$. The battery supplies an emf of $\varepsilon = 0.30$ V. What is the equivalent resistance, R_P? What is the voltage drop across each resistor? What is the current through each resistor? What is the power expended in each resistor?
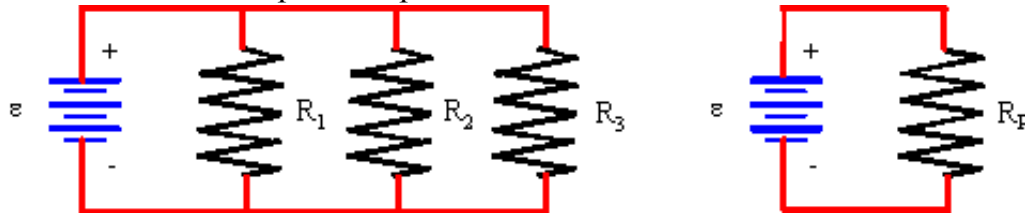
(i) Since the three resistors share two common points or nodes, the three resistors are in parallel. For parallel resistors, the equivalent resistance is

$$1/R_P = 1/R_1 + 1/R_2 + 1/R_3 = 1/(5\ \Omega) + 1/(10\ \Omega) + 1/(15\ \Omega) = 11/30\ \Omega^{-1}.$$

So the equivalent resistance is $R_P = 30/11 = 2.727\ \Omega$.

(ii) Resistors in parallel each have the same voltage drop as their equivalent resistance, R_P. The definition of equivalent means that the three resistors could be replaced by R_P without affecting any other aspect of the circuit. So the voltage drop across R_P, and thus across R_1, R_2, and R_3, is $\varepsilon = 0.30$ V.

(iii) Using Ohm's Law, the current through each resistor is given by $I = V/R$. The results are given below.

(iv) The power dissipated by each resistor is given by $P = I^2R = V^2/R$ as shown below.

Resistor (Ω)	*V* (Volts)	*I=V/R* (Amps)	*P=V²/R* (Watts)
5	0.30	0.060	18.0×10^{-3}
10	0.30	0.030	9.0×10^{-3}
15	0.30	0.020	6.0×10^{-3}
30	0.30	0.110	33.0×10^{-3}

3. For the circuits shown below, identify all nodes and branches. Are any nodes at the same potential? Which resistors, if any, are in series? Which resistors, if any, are in parallel?

(a)
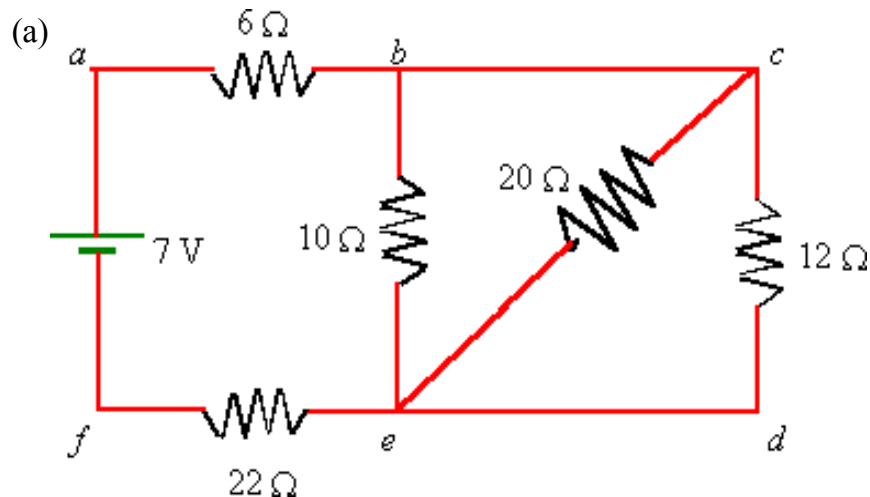
Nodes are where three or more wires come together. Thus points b, c, and e are nodes. Branches are the paths between nodes. Therefore the branches are $bafe$, be,bc,ce, and cde. Any two nodes connected by just a straight wire (no resistors or batteries in between) are at the same potential. Hence nodes b and c are at the same potential. Resistors in the branch carry the same current and are said to be in series. Here the 6-Ω and 22-Ω resistors are in series even though there is a battery between them. Resistors which share the same two nodes are said to be in parallel. The 20-Ω and 12-Ω resistors are in parallel because they have nodes c and e in common. The 10-Ω resistor is also in parallel with the 20-Ω and 12-Ω resistors because nodes b and c are equivalent.

(b)
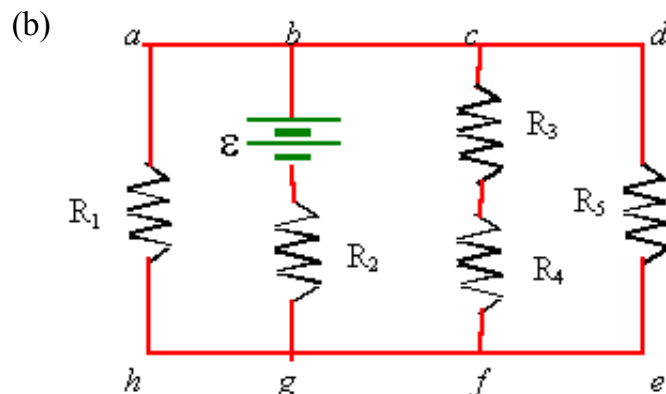
Nodes are where three or more wires come together. Thus points b, c, f, and g are nodes. Branches are the paths between nodes. Therefore the branches are $bahg,bg,bc,bg,cf,cdef$, and fg. Any two nodes connected by just a straight wire (no resistors or batteries in between) are at the same potential. Hence nodes b and c are at the same potential as are nodes f and g. Resistors in the same branch carry the same current and are said to be in series. Here the resistors R_3 and R_4 are in series. Resistors which share the same two nodes are said to be in parallel. R_1 and R_5 are in parallel because have nodes b and g are equivalent to nodes c and f.

4. Find the equivalent resistance of the circuit shown below. Find the voltage drop over, current through, and power dissipated by each resistor. Put your results in a table.
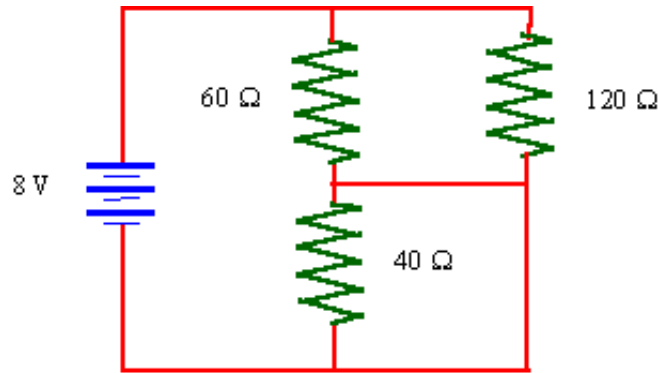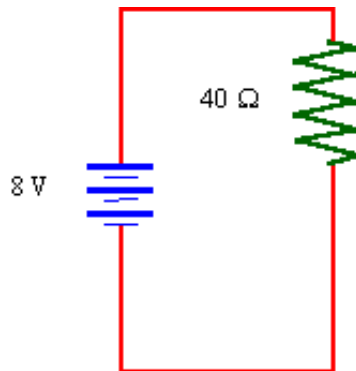

The 40 Ω resistor is in parallel with just a wire of zero resistance - that is it is being short-circuited. The two arms may be replaced by a single wire since

$$R_P = [1/40 + 1/0]^{-1} = 0 .$$

The 60 Ω resistor and the 120 Ω resistor are connected in parallel; they can be replaced by a single resistor with value

$$R_P = [1/60 + 1/120]^{-1} = 40 .$$

Thus the equivalent circuit looks like

Thus the equivalent resistance of the circuit is 40 Ω. The current from the battery is I = V/R = (8 V)/(40 Ω) = 0.20 A. The equivalent resistor is actually the 60 Ω resistor and the 120 Ω resistor connected in parallel, so they all have the same potential difference of 8 V. The 60 Ω being twice as small as the 120 Ω resistor gets $^2/_3$P of the current, 0.133 A, while the 120 Ω resistor the rest, 0.067 A.

| Resistor	Potential Difference (V)	Current (A)	Power (W)

|			
60 Ω	8	0.133	1.067
120 Ω	8	0.067	0.533
40 Ω	0	0	0

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5. Find the equivalent resistance of the circuit shown below. Find the voltage drop over, current through, and power dissipated by each resistor. Put your results in a table.
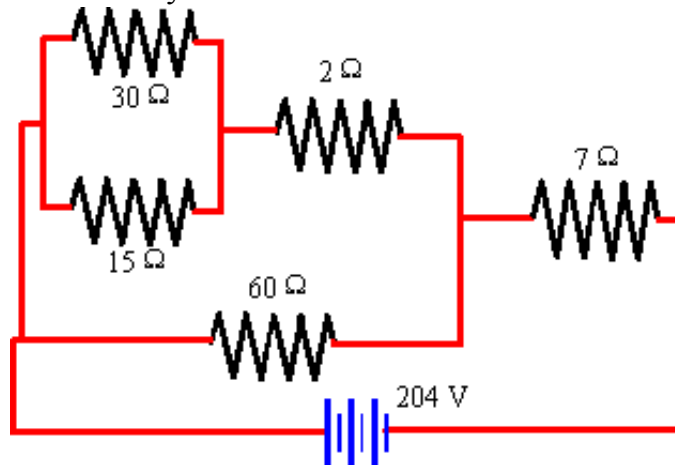

We reduce circuits which are a combination of series and parallel resistors piece by piece. Examining the circuit we see that the 30 Ω and 15 Ω resistor are in parallel. Since $1/30 + 1/15 = 1/10$, these resistors may be replace by a single 10 Ω resistor, as shown below.
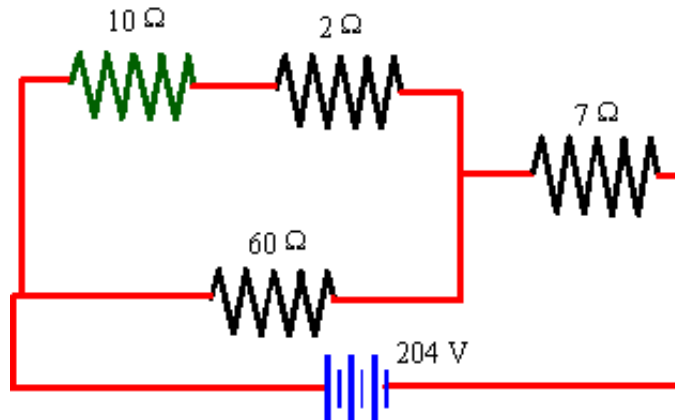

Next the 10 Ω and 2 Ω resistor are in series. Since $10 \, \Omega + 2 \, \Omega = 12 \, \Omega$, we replace these two resistors by a single 12 Ω resistor.

12 Ω

7 Ω

60 Ω

204 V
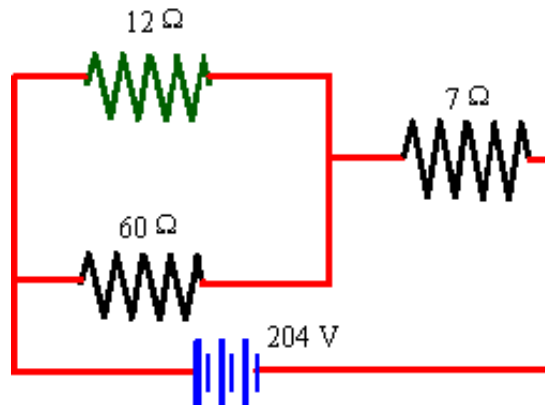
Next we observe that the 12 Ω and 60 Ω resistors are in parallel. Since 1/12 + 1/60 = 6/60, these resistors may be replace by a single 10 Ωresistor, as shown below.

10 Ω   7 Ω
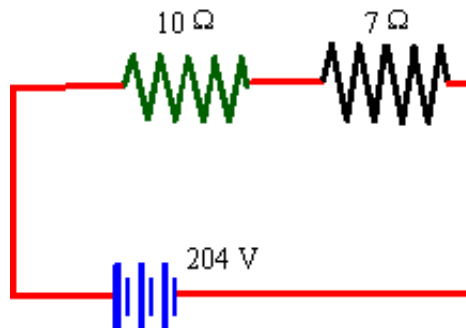
204 V

Finally, we note that the 10 Ω and 7 Ω resistor are in series and may be replaced by a single 17 Ω resistor as shown below.

17 Ω
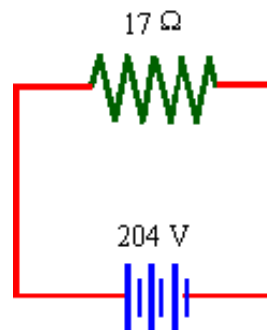
204 V

The equivalent resistance is thus 17 Ω. It has a voltage drop of 204 V. Using Ohm's Law, it carries a current $I_{17} = V/R = 204$ V / 17 Ω = 12 A. It dissipates a power of $P_{17} = V^2/R = 2448$ W.

To find V, I, and P for the other resistors we work our way back up, one level at a time. First, the 17 Ω resistor was as 10 Ω and a 7 Ω resistor in series. Recall that all three must thus carry the same current, $I_{10} = I_7 = I_{17} = 12$ A. The voltage drop is found from Ohm's Law, V = IR. For the 10 Ω resistor this is $V_{10} = I_{10}R_{10} = (12$ A$)(10$ Ω$) = 120$ V. For the 7 Ω resistor this is $V_7 = I_7 R_7 = (12$ A$)(7$ Ω$) = 84$ V. Note that the total is 204 V, the same as across the 17 Ω resistor. We can now calculate the power dissipated by each using $P = I^2 R$. For the 10 Ω resistor this is $P_{10} = (12$ A$)^2(10$ Ω$) = 1440$ W. For the 7 Ω resistor this is $P_7 = (12$ A$)^2(7$Ω$) = 1008$ W. Note that the total is 2448 W, the same as through the equivalent resistor.

Next recall that the 10 Ω resistor was actually a pair of 12 Ω and 60 Ω resistors in parallel. We know that resistors in parallel each have the same voltage drop as the equivalent resistor, so $V_{12} = V_{60} = V_{10} = 120$ V. The current through each may be found from Ohm's Law, $I = V/R$. Hence the 12 Ω resistor carries a current $I_{12} = V_{12}/R_{12} = 120$ V $/ 12$ Ω $= 10$ A. Similarly, the 60 Ω resistor carries a current $I_{60} = V_{60}/R_{60} = 120$ V $/ 60$ Ω $= 2$ A. Notice that the total current is 12 A, the same as through the equivalent resistor. We can find the power through each using $P = V^2/R$. The power dissipated by the 12 Ω resistor is $P_{12} = (120$ V$)^2/(12$ Ω$) = 1200$ W. The power dissipated by the 60 Ω resistor is $P_{60} = (120$ V$)^2/(60$ Ω$) = 240$ W. The total power dissipated by the two resistors is 1440 W, the same as through their equivalent resistor.

Next, the 12 Ω resistor was as 10 Ω and a 2 Ω resistor in series. Recall that all three must thus carry the same current, $I_{10} = I_2 = I_{12} = 10$ A. The voltage drop is found from Ohm's Law, $V = IR$. For the 10 Ω resistor this is $V_{10} = I_{10}R_{10} = 100$ V. For the 2 resistor this is $V_2 = I_2R_2 = 20$ V. Note that the total is 120 V, the same as across the equivalent 12 Ω resistor. We can now calculate the power dissipated by each using $P = I^2R$. For the 10 Ω resistor this is $P_{10} = (10$ A$)^2(10$Ω$) = 1000$ W. For the 2 Ω resistor this is $P_2 = (10$ A$)^2(2$ Ω$) = 200$ W. Note that the total is 1200 W, the same as through the equivalent resistor.

The last step is to observe that the 10 Ω resistor was actually a pair of 15 Ω and 30 Ω resistors in parallel. We know that resistors in parallel each have the same voltage drop as the equivalent resistor, so $V_{15} = V_{30} = V_{10} = 100$ V. The current through each may be found from Ohm's Law, $I = V/R$. Hence the 15 Ω resistor carries a current $I_{15} = V_{15}/R_{15} = 100$ V $/ 15$ Ω $= 6.67$ A. Similarly, the 30 Ω resistor carries a current $I_{30} = V_{30}/R_{30} = 100$ V $/ 30$ Ω $= 3.33$ A. Notice that the total current is 10 A, the same as through the equivalent resistor. We can find the power through each using $P = V^2/R$. The power dissipated by the 15 Ω resistor is $P_{15} = (100$ V$)^2/(15$ Ω$) = 666.7$ W. The power dissipated by the 30 Ω resistor is $P_{30} = (100$ V$)^2/(30$ Ω$) = 333.3$ W. The total power dissipated by the two resistors is 1000 W, the same as through their equivalent resistor.

Summarizing the results, in a table we have

| *Resistor* (Ω)	*V* (Volts)	*I* (Amps)	*P* (Watts)
30	100	3.33	333.3
15	100	6.67	666.7
2	20	10	200
60	120	2	240
7	84	12	1008
17	204	12	2448

6. Find the equivalent resistance of the circuit shown below. Find the voltage drop over, current through, and power dissipated by each resistor. Put your results in a table.
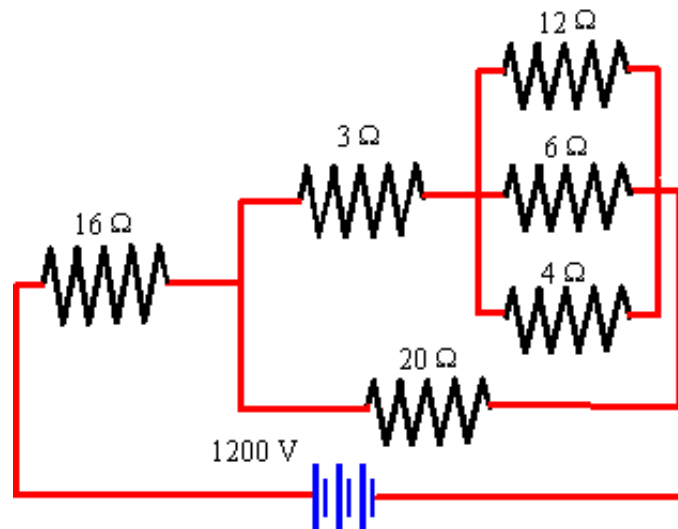

We reduce circuits which are a combination of series and parallel resistors piece by piece. Examining the circuit we see that the 12 Ω , 6 Ω , and 4 Ω resistor are in parallel. Since $1/12 + 1/6 + 1/4 = 6/12$, these resistors may be replace by a single 2 Ω resistor, as shown below.
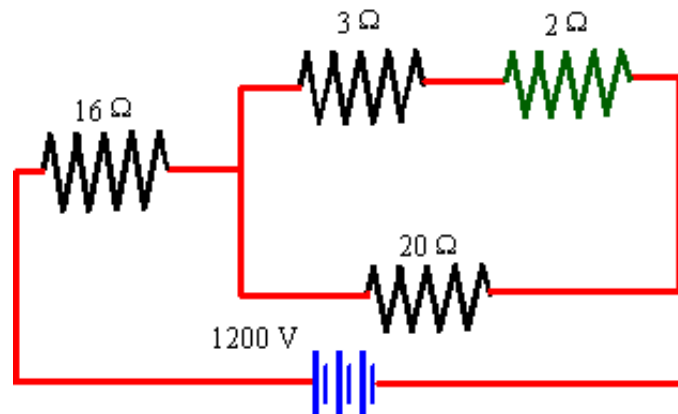

Next the 2 Ω and 3 Ω resistor are in series. Since $2 \, \Omega + 3 \, \Omega = 5 \, \Omega$, we replace these two resistors by a single 5 Ω resistor.
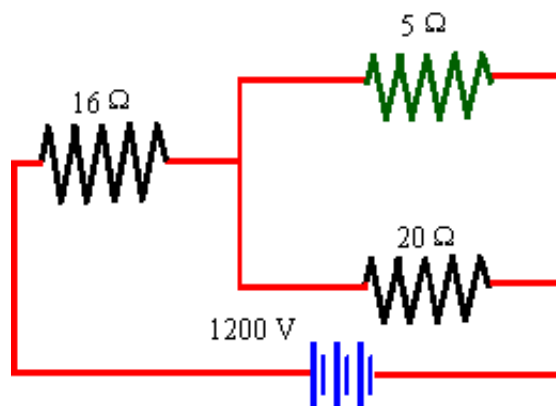
Next we observe that the 5 Ω and 20 Ω resistors are in parallel. Since $1/5 + 1/20 = 5/20$, these resistors may be replace by a single 4 Ω resistor, as shown below.
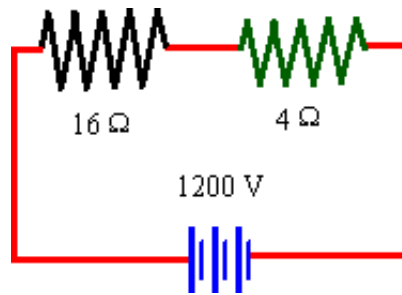
Finally, we note that the 4 Ω and 16 Ω resistor are in series and may be replaced by a single 20 Ω resistor as shown below.
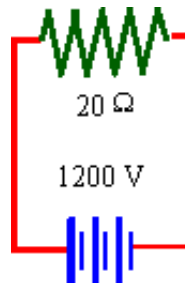
The equivalent resistance is thus 20 Ω . It has a voltage drop of 1200 V. Using Ohm's Law, it carries a current I_{20} = V/R = 1200 V / 20 Ω = 60 A. It dissipates a power of P_{20} = V^2/R = 72 × 10^3 W.

To find V, I, and P for the other resistors we work our way back up, one level at a time. First, the 20 Ω resistor was as 16 Ω and a 4 Ω resistor in series. Recall that all three must thus carry the same current, I_{16} = I_4 = I_{20} = 60 A. The voltage drop is found from Ohm's Law, V = IR. For the 16 Ω resistor this is V_{16} = $I_{16}R_{16}$ = (60 A)(16 Ω) = 960 V. For the 4 Ω resistor this is V_4 = I_4R_4 = (60 A)(4 Ω) = 240 V. Note that the total is 1200 V, the same as across the 20 Ω resistor. We can now calculate the power dissipated by each using P = I^2R. For the 16 Ω resistor this is P_{16} = $(60 A)^2$(16 Ω) = 57.6 kW. For the 4 Ω resistor this is P_4 = $(60 A)^2$(4 Ω) = 14.4 kW. Note that the total is 72 kW, the same as through the equivalent resistor.

Next recall that the 4 Ω resistor was actually a pair of 5 Ω and 20 Ω resistors in parallel. We know that resistors in parallel each have the same voltage drop as the equivalent resistor, so V_5 = V_{20} = V_4 = 240 V. The current through each may be found from Ohm's Law, I = V/R. Hence the 5 Ω resistor carries a current I_5 = V_5/R_5 = 240 V / 5 Ω = 48 A. Similarly, the 20 Ω resistor carries a current I_{20} = V_{20}/R_{20} = 240 V / 20 Ω = 12 A. Notice that the total current is 60 A, the same as through the equivalent resistor. We can find the power through each using P = V^2/R. The power dissipated by the 5 Ω resistor is P_5 = $(240 V)^2$/(5 Ω) = 11.52 kW. The power dissipated by the 20 Ω resistor is P_{20} = $(240 V)^2$/(20 Ω) = 2.88 kW. The total power dissipated by the two resistors is 14.4 kW, the same as through their equivalent resistor.

Next, the 5 Ω resistor was as 3 Ω and a 2 Ω resistor in series. Recall that all three must thus carry the same current, I_3 = I_2 = I_5 = 48 A. The voltage drop is found from Ohm's Law, V = IR. For the 3 Ω resistor this is V_3 = I_3R_3 = 144 V. For the 2 Ω resistor this is V_2 = I_2R_2 = 96 V. Note that the total is 240 V, the same as across the 5 Ω resistor. We can now calculate the power dissipated by each using P

$= I^2R$. For the 3 Ω resistor this is $P_{10} = (48 \text{ A})^2(3\Omega) = 6912$ W. For the 2 Ω resistor this is $P_2 = (48 \text{ A})^2(2 \text{ Ω}) = 4608$ W. Note that the total is 11520 W, the same as through the equivalent resistor.

The last step is to observe that the 2 Ω resistor was actually a trio of 12 Ω , 6 Ω , and 4 Ω resistors in parallel. We know that resistors in parallel each have the same voltage drop as the equivalent resistor, so $V_{12} = V_6 = V_4 = V_2 = 96$ V. The current through each may be found from Ohm's Law, $I = V/R$. Hence the 12 Ω resistor carries a current $I_{12} = V_{12}/R_{12} = 96$ V / 12 Ω = 8 A. Similarly, the 6 Ω resistor carries a current $I_6 = V_6/R_6 = 96$ V / 6 Ω = 16 A. As well, the 4 Ω resistor carries a current $I_4 = V_4/R_4 = 96$ V / 4 Ω = 24 A. Notice that the total current is 48 A, the same as through the equivalent resistor. We can find the power through each using $P = V^2/R$. The power dissipated by the 12 Ω resistor is $P_{12} = (96 \text{ V})^2/(12 \text{ Ω}) = 768$ W. The power dissipated by the 6 Ω resistor is $P_6 = (96 \text{ V})^2/(6 \text{ Ω}) = 1536$ W. The power dissipated by the 4 Ω resistor is $P_4 = (96 \text{ V})^2/(4 \text{ Ω}) = 2304$ W. The total power dissipated by the two resistors is 4608 W, the same as through their equivalent resistor.

Summarizing the results, in a table we have

Resistor (Ω)	V (Volts)	I (Amps)	P (Watts)
16	960	60	57600
20	240	12	2880
3	144	48	6912
12	96	8	768
6	96	16	1536
4	96	24	2304
20	1200	60	72000

Top

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7. A student is working with the circuit shown in the diagram below. She uses an ammeter and finds that the current through the branch with the 300-Ω resistor to be I = 0.050 A.
(a) What is the voltage drop across the 300-Ω resistor?
(b) What is the voltage drop across the 150-Ω resistor?
(c) What is the current through the 150-Ω resistor?
(d) What is the current through the 100-Ω resistor?
(e) What is the voltage drop across the 100-Ω resistor?
(f) What is the emf of the battery?
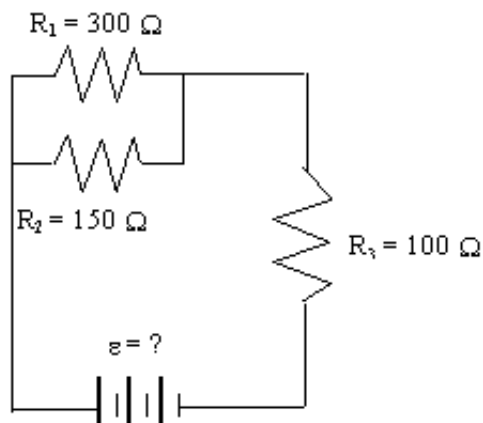
(a) The voltage drop across the 300-Ω resistor is given by Ohm's Law; $V = IR = 300\ \Omega \times 0.050\ A = 15.0\ Volts$.

(b) The 150-W resistor is in parallel with the 300-Ω resistor are in parallel, so there is also a 15.0 Volt drop across it.

(c) Using Ohm's Law again, the current through the 150-Ω resistor is given by $I = V/R = (15.0\ V)/(150\ \Omega) = 0.100\ A$.

(d) The current through the 300-Ω resistor and current through the 150-Ω resistor join at the node to become the current through the 100-Ω resistor. The current through the 100-Ω resistor is thus $0.050\ A + 0.100\ A = 0.150\ A$.

(e) The voltage drop across the 100-Ω resistor is given by Ohm's Law; $V = IR = 100\ \Omega \times 0.150\ A = 15.0\ Volts$.

(f) The emf of the battery must equal the sum of the voltage drops as you go around the circuit. Passing through either the 150-Ω resistor or 300-Ω resistor, there is drop of 15.0 Volts. Next there is a drop of 15.0 Volts over the 100-Ω resistor. In total there is a drop of 30.0 Volts. So the emf must be 30.0 Volts.

Top

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8. A galvanometer has a coil resistance of 250 Ω and requires a current of 1.5 mA for full-scale deflection. This device is to be used in an ammeter that has a full-scale current of 25.0 mA. What is the value of the shunt resistance? What is the equivalent resistance of the ammeter?

The shunt resistance is the resistor connected in parallel with the coil as shown in the diagram below. The current entering and leaving the branch is I = 25.0 mA. We want the current through the coil to be $I_G = $ 1.5 mA.
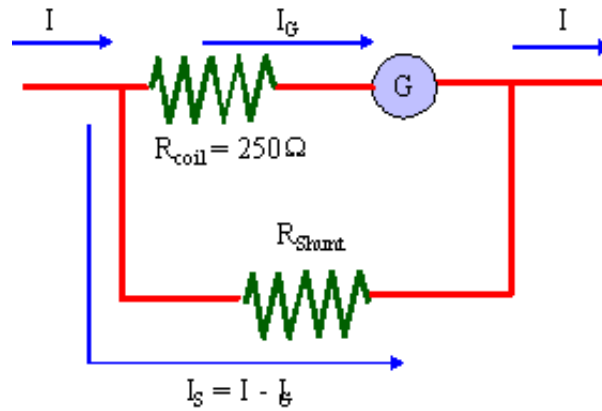
Since the voltage drop across each arm is the same

$$I_G R_C = (I - I_G) R_S .$$

Solving for R_S, we find

$$R_S = I_G R_C / (I - I_G) = (1.5 \text{ mA})(250 \ \Omega)/(25 \text{ mA} - 1.5 \text{ mA}) = 16 \ \Omega .$$

We would need the shunt to have a resistance of 16 Ω.

Since the coil and shunt are is parallel, the equivalent resistance - the ammeter resistance - is

$$R_A = (1/R_C + 1/R_S)^{-1} = (1/(250 \ \Omega) + 1/(15.96 \ \Omega))^{-1} = 15 \ \Omega.$$

9. Consider the circuit in diagram (a) below. What is the current in this circuit? Now consider the circuit in diagram (b) below. The only difference is that an ammeter has been added so that the current could be measured. The ammeter is the same one as the previous problem. What is the current reading on the ammeter? Why is it different from the theoretical value that you found for diagram (a)?

(a)   (b)
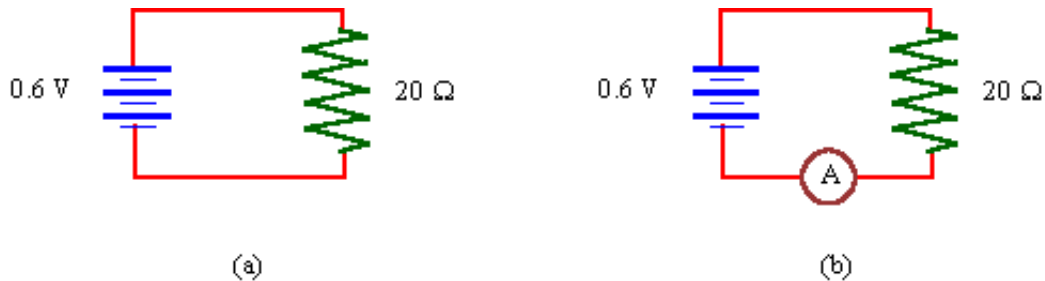
For diagram (a) we just use Ohm's law to determine the current

$$I = V/R = (0.6 \text{ V})/(20 \ \Omega) = 0.030 \text{ A} = 30 \text{ mA}.$$

For diagram (b) we must remember that this ammeter has a resistance $R_A = 15 \ \Omega$ which is not much smaller that the resistance already there 20 Ω. The equivalent resistance of the circuit in (b) is therefore

$$R_{eq} = 15\ \Omega + 20\ \Omega = 35\ \&Omega.$$

Now using Ohm's law to determine the current

$$I_A = V/R_{eq} = (0.6\ \text{V})/(35\ \Omega) = 0.017\ \text{A} = 17\ \text{mA}\ .$$

10. The equivalent resistance of a voltmeter is 140,000 Ω. The voltmeter uses a galvanometer that has a full-scale deflection of 180 μA. What is the maximum voltage that can be measured by the voltmeter?

A voltmeter is connected is parallel with the resistor of interest, as shown in the diagram below

$I_G = 180\mu A$

$R_V = 140,000\Omega$

R

I

I

$I_R = I - I_G$
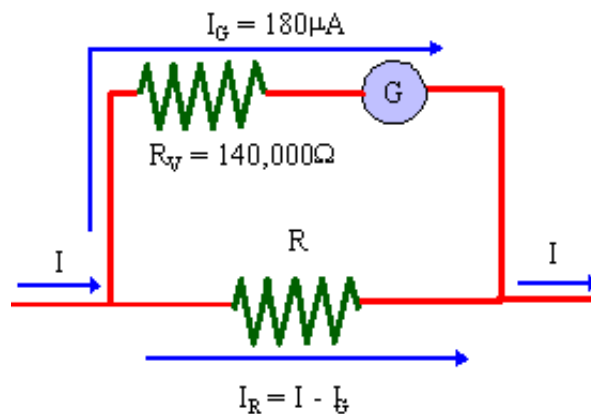
Since the voltmeter is parallel with the resistor, the voltage drop across both is the same. The voltage drop across the coil is

$$V_{coil} = R_V I_G = 140,000\ \Omega \times 180\ \text{A} = 25.2\ \text{V}\ .$$

The voltage drop over the resistor is 25.2 Volts.

11. Consider the circuit in diagram (a) below. What is the potential difference over the 140 kΩ resistor in this circuit? Now consider the circuit in diagram (b) below. The only difference is that a voltmeter has been added so that the potential difference could be measured. The voltmeter is the same one as the previous problem. What is the voltage reading on the voltmeter? Why is it different from the theoretical value that you found for diagram (a)?

(a)   (b)
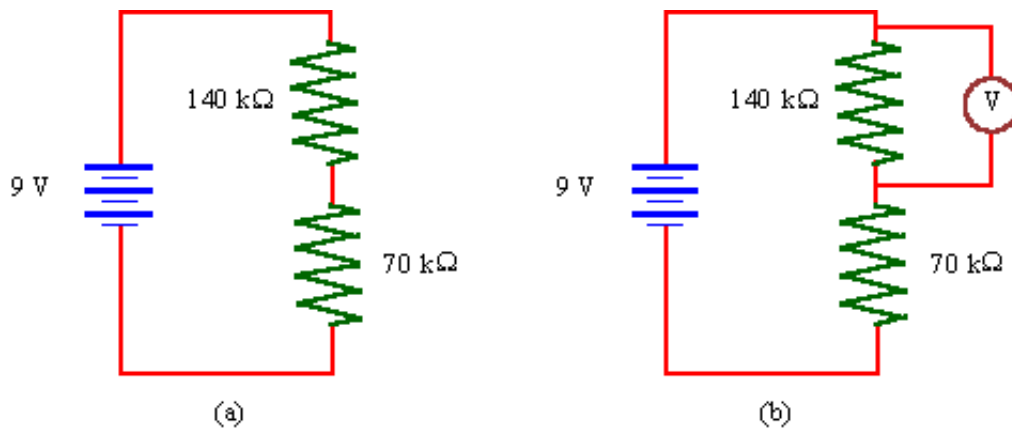
For diagram (a) we can determine the potential difference over the 140 kΩ resistor quite simply. It is twice as big as the 70 kΩ resistor. Hence it uses twice as much energy and so has twice the potential difference. That is it gets 2/3 × 9 Volts = 6 Volts.

For diagram (b) we must remember that this voltmeter has a resistance R_V = 140 kΩ which is <u>not</u> much greater that the resistance it is connected in parallel with, 140 k Ω. The equivalent resistance of the voltmeter and the 140 kΩ resistor in (b) is therefore

$$R_{eq} = (1/(140 \text{ k}\Omega) + 1/(140 \text{ k}\Omega))^{-1} = 70 \text{ k } \Omega \text{ .}$$

This equivalent resistance is the same size as the other resistor in circuit so each shares the half the potential difference or ½ × 9 Volts = 4.5 Volts. Now recall that the equivalent resistance and the two parallel resistors must each have the same potential difference. So the 140 kΩ resistor has 4.5 Volts over it and that is what the voltmeter displays.

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