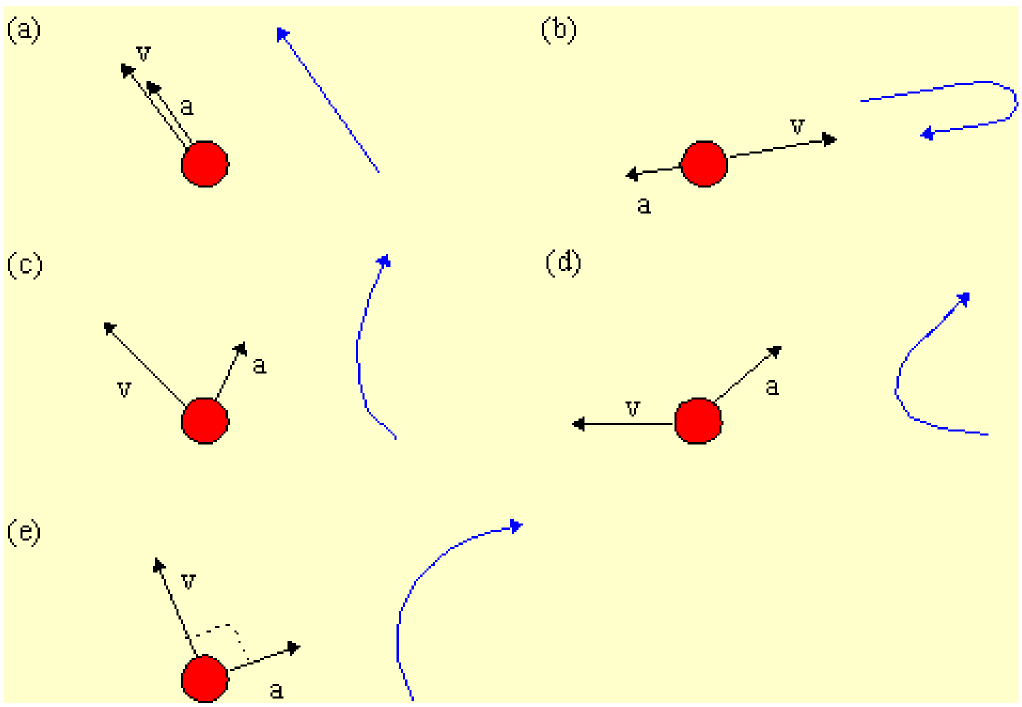
Physics 1120: 2D Kinematics Solutions

1. In the diagrams below, a ball is on a flat horizontal surface. The inital velocity and the constant acceleration of the ball is indicated. Describe qualitatively how motion the motion of the ball will change.

   As time passes, the velocity will tend to point in the direction of **a**.



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2. A particle has initial position $\mathbf{r}_0$ = <5 m, -3 m, 0 m> and initial velocity $\mathbf{v}_0$ = <0.4 m/s, -4 m/s, 2 m/s>. The acceleration is constant, **a** = <3 m/s^2, 4 m/s^2, -2m/s^2>. Find the position of the particle after t = 2.5 seconds. What was the magnitude of the particle's displacement during this time?

   The ***i***, ***j***, and ***k*** components are completely independent of one another. The final x position of the particle depends only on the x components, similarly for the y and z components. Thus the problem reduces to handling three 1D kinematics problems. For the given information, we use our kinematics equation to find x, y, and z.

$$\begin{aligned}\Delta x &= v_{0x}t + \tfrac{1}{2}a_x t^2 \\ &= (0.4\,m/s)(2.5s) + \tfrac{1}{2}(3m/s^2)(2.5s)^2 \\ &= 10.375m\end{aligned}$$

$$\begin{aligned}\Delta y &= v_{0y}t + \tfrac{1}{2}a_y t^2 \\ &= (-4\,m/s)(2.5s) + \tfrac{1}{2}(4\,m/s^2)(2.5s)^2 \\ &= 2.5m\end{aligned}$$

$$\begin{aligned}\Delta z &= v_{0z}t + \tfrac{1}{2}a_z t^2 \\ &= (2\,m/s)(2.5s) + \tfrac{1}{2}(-2\,m/s^2)(2.5s)^2 \\ &= -1.25m\end{aligned}$$

Thus the final displacement is thus **r** = <x, y, z> = <10.375 m, 2.5 m, -1.25 m>.

The magnitude of the displacement is found by using the 3D version of Pythagoras' Theorem:

$$\Delta r = |\Delta\vec{r}| = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} = \sqrt{(10.375)^2 + (2.5)^2 + (-1.25)^2} = 10.74m.$$

The straight-line distance between the starting and end points is 10.7 m.

To find the final position of the particle, one must examine the definition of displacement, $\mathbf{r} = \mathbf{r}_f - \mathbf{r}_0$, which can be rearranged into $\mathbf{r}_f = \mathbf{r}_0 + \mathbf{r}$. Thus we find that $\mathbf{r}_f = \mathbf{r}_0 + \mathbf{r}$ = <5 m, -3 m, 0 m> + <10.375 m, 2.5 m, -1.25 m> = <15.375 m, -0.5 m, -1.25 m>.

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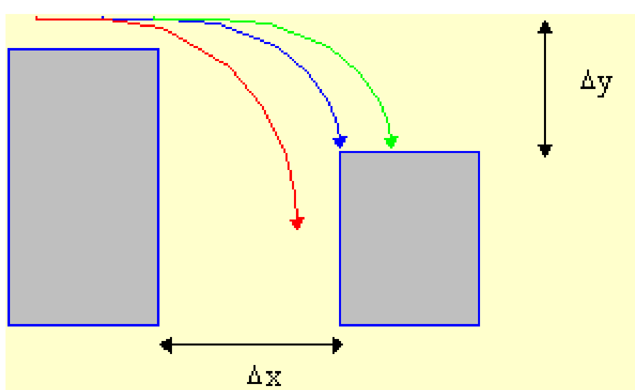
3. You are trapped on the top of a burning building. Death is imminent and help is nowhere in sight. There is a safe building 6.50 m away and 3.00 m lower. You decide to try and make it across. You run horizontally off your building at 8.10 m/s. Do you make it across? If you don't, how much faster must you be going?

First we sketch the situation and possible outcomes.



While you are jumping, you are a projectile. We solve projectile motion problems by considering the x and y components separately, keeping in mind that the time in air is common. We write out the ***i*** and ***j*** information in separate columns including the information that we can infer or that we are supposed to know like the fact that running horizontally implies that $v_{oy} = 0$..

i		*j*	
$\Delta x_{safe} = 6.50$ m		$\Delta y_{safe} = -3.00$ m	minus indicates down
$a_x = 0$	No x component for projectiles	$a_y = -g = -9.81$ m/s^2	gravity acts down
$v_{0x} = 8.10$ m/s		$v_{0y} = 0$ m/s	horizontal takeoff means no vertical component
$t_{air} = ?$	common	$t_{air} = ?$	

Looking at the x information, we see that we have enough data to find t_{air}. The kinematics equation that has all four quantities is $\Delta x = v_{0x}t + ½a_xt^2$. Since $a_x = 0$ for a projectile, this equation become $\Delta x = v_{0x}t$. Solving for t, we get

$$t = \Delta x / v_{0x} = (6.50 \text{ m}) / (8.10 \text{ m/s}) = 0.8025 \text{ s} .$$

This is the time it would take you to cross a horizontal distance of 6.50 m. You must be in the air for at least this long if you are to safely make it across to the next building.

On the other hand, looking at the y information, we see that we also have enough data to find t_{air}. The kinematics equation that has all four quantities is $\Delta y = v_{0y}t + ½a_yt^2$. We know $v_{0y} = 0$ since you ran off the roof horizontally and that $a_y = -g$, thus this equation becomes $\Delta y = -½gt^2$. Solving for t, we get

$$t = \{ -2\Delta y / g \}^{½} = \{(-2\times-3.00 \text{ m}) / (-9.81 \text{ m/s}) \}^{½} = 0.7821 \text{ s} .$$

This is the time it takes you to fall a vertical distance of 3.00 m. If you do reach the other building, then this is how long you were in the air.

Since the time it takes to cross the horizontal distance is less than the time you have, you have don't make it across.

To make it across safely you of course would need to run off the roof faster. Since $a_x = 0$ for projectiles, the kinematics equation become $\Delta x = v_{0x}t$ where t is now the 0.7821 s. Solving for v_{0x}, we get ,

$$v_{0x} = \Delta x / t = (6.50 \text{ m}) / (0.7821 \text{ s}) = 8.31 \text{ m/s} .$$

If you were able to run at 8.31 m/s you would safely make it to the other building.

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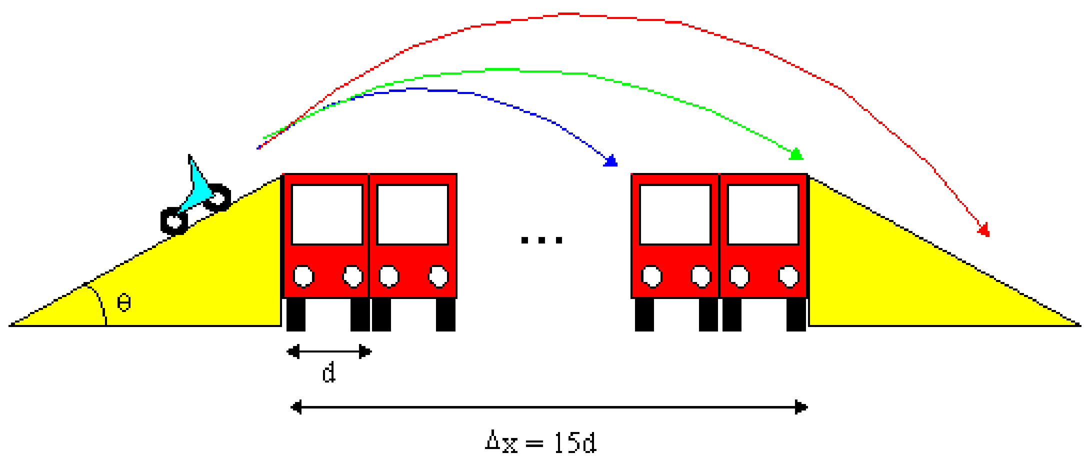
4. A stunt motorcyclist is trying to jump over fifteen buses set side to side. Each bus is 2.50 m wide and a 30.0° ramp has been installed on either side of the line of buses. What is the minimum speed at which she must travel to safely reach the other side. How long will she be in the air?

First we sketch the situation and possible outcomes.



While the motorcyclist is jumping, she is a projectile. We solve projectile motion problems by considering the x and y components separately, keeping in mind that the time in air is common. We write out the ***i*** and ***j*** information in separate columns including the information that we can infer or that we are supposed to know. We see from the sketch that if the motorcyclist is successful, then this is an example of level-to-level flight and $\Delta y = 0$. Note that the initial velocity is broken into components.

i		***j***	
$\Delta x_{safe} = 15 \times 2.50$ m $= 37.5$ m		$\Delta y_{safe} = 0$	level to level flight
$a_x = 0$	No x component for projectiles	$a_y = -g = -9.81$ m/s^2	gravity acts down
$v_{0x} = v_0 \cos\theta$	v_0 is unknown	$v_{0y} = v_0 \sin\theta$	
$t_{air} = ?$	common	$t_{air} = ?$	common

Looking at the x any y information, we see that we have two unknowns, v_0 and t, for both. While we cannot solve any equation for x or y since there are two unknowns, both can be solved together. The appropriate kinematics equations that has all four quantities for x and for y is:

$$\Delta x = v_{0x}t + \tfrac{1}{2}a_x t^2 \text{ , and } \Delta y = v_{0y}t + \tfrac{1}{2}a_y t^2.$$

We substitute in known quantities to get

$$\Delta x = v_0 \cos\theta \ t \text{ , and } 0 = v_0 \sin\theta \ t - \tfrac{1}{2}gt^2.$$

We can divide the second equation by t and we get

$$\Delta x = v_0 \cos\theta \ t, \text{ and } v_0 \sin\theta = \tfrac{1}{2}gt.$$

We rewrite the first equation as $t = \Delta x / v_0 \cos$, which we substitute into the second equation to get $v_0 \sin = \tfrac{1}{2}g[\Delta x / v_0 \cos\theta]$. Getting v_0 by itself we have $v_0 = \{g/(2\sin\cos\theta)\}^{1/2}$. Plugging in the appropriate numbers, we get $v_0 = 20.61$ m/s $= 74.2$ km/h. Since is the speed that the motorcyclist must have on liftoff to successfully reach the other ramp.

We can substitute this value into $t = \Delta x / v_0 \cos\theta$ to find the time in air to be 2.10 s.

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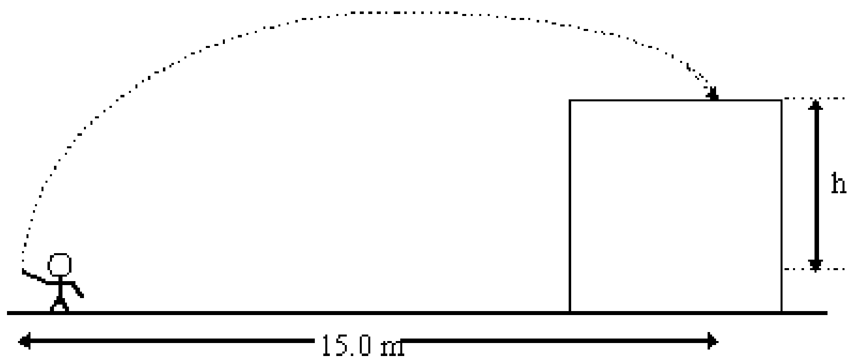
5. A boy throws a rock with speed v = 18.3 m/s at an angle of θ = 57.0° over a building. The rock lands on the roof 15.0 m in the x direction from the boy. How long was the rock in the air? How much taller, height h, is the building than the boy? Ignore air resistance.



The rock is a projectile. We solve projectile motion problems by considering the x and y components separately, keeping in mind that the time in air is common. We write out the ***i*** and ***j*** information in separate columns including the information that we can infer or that we are supposed to know. We see from the sketch that h = Δy, the vertical displacement. Note that the initial velocity is broken into components.

i		***j***	
$\Delta x = 15.0$ m		$\Delta y = h$	
$a_x = 0$	No x component for projectiles	$a_y = -g = -9.81$ m/s^2	gravity acts down
$v_{0x} = v_0 \cos\theta$ $= 18.3 \times \cos(57°)$ $= 9.9669$ m/s		$v_{0y} = v_0 \sin\theta$ $= 18.3 \times \sin(57°)$ $= 15.3477$ m/s	
$t_{air} = ?$	common	$t_{air} = ?$	common

Looking at the x information, we see that we have enough data to find t_{air}. The kinematics equation that has all four quantities is $\Delta x = v_{0x}t + ½a_xt^2$. Since $a_x = 0$ for a projectile, this equation become $\Delta x = v_{0x}t$. Solving for t, we get

$$t = \Delta x / v_{0x} = (15.0 \text{ m}) / (9.9669 \text{ m/s}) = 1.505 \text{ s} .$$

Looking at the y information, we see that we now have enough data to find h. The kinematics equation that has all four quantities is $\Delta y = v_{0y}t + ½a_yt^2$. Since $\Delta y = h$ and $a_y = -g$, this equation become $h = v_{0y}t - ½gt^2$. Substituting in the appropriate numbers reveals that h = 12.0 m. The building is 12.0 m taller that the boy where we have assumed that the ball left the boy's hand at head height which is a reasonable assumption.

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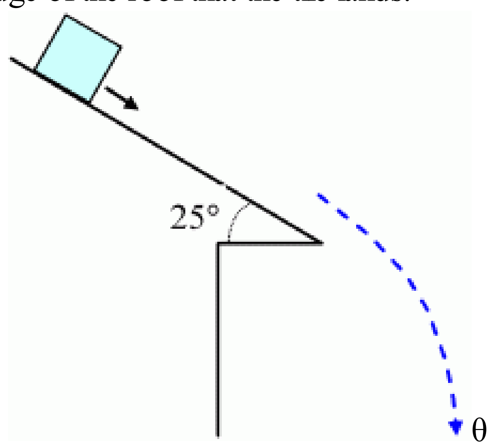
6. A tile, initially at rest, slides down a roof for a distance of 3.75m before falling off the roof. The height of the building from ground to eave is 8.40 m. The acceleration of the tile as it slides is 2.10 m/s^2.
(a) Determine the speed of the tile just as it leaves the roof.
(b) Determine the vertical component of the velocity just before it leaves the roof.
(c) Determine the horizontal component of the velocity just before it leaves the roof.
(d) Determine how long it takes to hit the ground after leaving the roof.
(e) Determine how far from the edge of the roof that the tile lands.



When the tile slides down the roof, it travels in a straight line. That is a 1D kinematics problem. When it leaves the roof, it becomes a projectile problem.

(a) We solve the 1D problem first. We write down all the given data and unknowns:

$\Delta x = 3.75$ m	
$v_0 = 0$	starts from rest
$a = 2.10$ m/s^2	
v_f	need this for the second part

Inspecting the data, we see that we can use the kinematics equation $2a\Delta x = (v_f)^2 - (v_0)^2$. Solving for v_f, we find

$$v_f = \{2a\Delta x + (v_0)^2\}^{½} = \{2(3.75 \text{ m/s}^2)(3.75 \text{ m}) + (0)^2\}^{½} = 3.9686 \text{ m/s} .$$

This is the speed that the tile leaves the roof and is the initial velocity for the second part of the problem.

(b) The vertical component of the velocity of the tile as it leaves the roof is $v_{0x} = -v_f\sin(25°) = -1.6772$ m/s. Note that the minus sign indicates that the tile is moving downwards.

(c) The horizontal component of the velocity of the tile as it leaves the roof is $v_{0x} = v_f\cos(25°) = 3.5968$ m/s.

(d) The tile is now a projectile. We solve projectile motion problems by considering the x and y components separately, keeping in mind that the time in air is common. We write out the ***i*** and ***j*** information in separate columns including the information that we can infer or that we are supposed to know. We see from the sketch that Δy is the vertical distance that the tile falls. Note that the initial velocity is broken into components.

i		***j***	
$\Delta x = ?$		$\Delta y = -8.40$ m	minus means down
$a_x = 0$	No x component for projectiles	$a_y = -g = -9.81$ m/s^2	gravity acts down
$v_{0x} = 3.5968$ m/s		$v_{0y} = -1.6772$ m/s	
$t_{air} = ?$	common	$t_{air} = ?$	common

Looking at the y information, we see that we have enough data to find t_{air}. The kinematics equation that has all four quantities is $\Delta y = v_{0y}t + ½a_yt^2$. Substituting in the appropriate numbers reveals that we have a quadratic in t:

$$-8.40 = -1.6772t + ½(-9.81)t^2 .$$

The two solution to this quadratic are t = 1.149 s and t = -1.491 s. We take the positive solution as that is the solution for times after the tile left the roof.

(e) Looking at the x information, we see that we now have enough data to find Δx. The kinematics equation that has all four quantities is $\Delta x = v_{0x}t + ½a_xt^2$. Since $a_x = 0$ for a projectile, this equation become $\Delta x = v_{0x}t$. Substituting in the appropriate numbers, we get $\Delta x = v_{0x}t = 3.5968$ m/s × 1.149 s = 4.13 m. The tile land 4.13 m from the eave of the roof.

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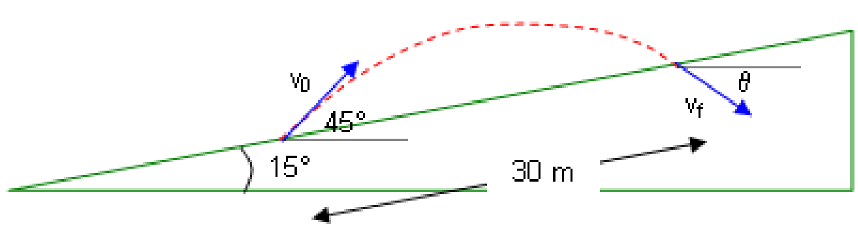
7. A boy is on the side of a hill. The hill makes a 15° incline with respect to horizontal. The boy throws a rock up the side of the hill. The boy throws the rock at 45° with respect to horizontal and the rock lands 30 m away up the hill. Find how fast the boy threw the rock. What angle does the rock make with horizontal before it lands? Ignore the boy's height.



We know

$$\mathbf{v}_0 = \boldsymbol{i}\ v_0\cos(45°) + \boldsymbol{j}\ v_0 \sin(45°)$$
$$\mathbf{v}_f = \boldsymbol{i}\ v_f \cos(\theta) - \boldsymbol{j}\ v_f \sin(\theta)$$
$$\mathbf{a} = -\boldsymbol{j}\ g = -\boldsymbol{j}\ 9.81 \text{ m/s}^2$$
$$\Delta\mathbf{r} = \boldsymbol{i}\ 30 \cos(15°) + \boldsymbol{j}\ 30 \sin(15°) \text{ m}$$
$$= \boldsymbol{i}\ 28.978 + \boldsymbol{j}\ 7.765 \text{ m}$$

Applying our kinematic equations to the situation:

$$v_f \cos(\theta) = v_0 \cos(45°) \qquad (1)$$
$$28.978 = v_0 \cos(45°)\ t \qquad (2)$$
$$7.765 = v_0 \sin(45°)t - ½gt^2 \qquad (3)$$
$$-v_f \sin(\theta) = v_0 \sin(45°) - gt \qquad (4)$$
$$2(9.81)(7.765) = [-v_f \sin(\theta)]^2 - [v_0 \sin(45°)]^2 \qquad (5)$$

Examining the equations, we find that we can eliminate v_0t from equation (3) using equation (2). This yields

$$7.765 = [28.978/\cos(45°)] \sin(45°) - ½gt^2 .$$

This can be simplified and solved for t. We find t = 2.0796 s. Knowing t, equation (2) can be solved to yield v_0 = 19.706 m/s. With v_0 and t, equations (1) and (4) become $v_f \cos(\theta) = 13.934$ and $v_f \sin(\theta) = 6.467$. The ratio of these two equations yields $\tan(\theta) = 0.4641$ or $\theta = 24.9°$. Also v_f = 15.36 m/s.

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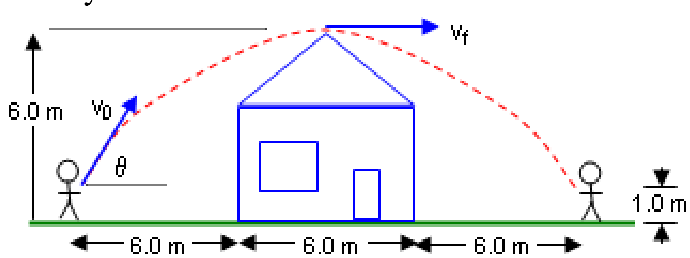
8. You are 6.0 m from the wall of a house. You want to toss a ball to your friend who is 6.0 m from the opposite wall. The throw and catch each occur 1.0 m above the ground.
(a) What minimum velocity, expressed in ***ij*** notation, will allow the ball to clear the roof?
(b) At what angle and speed do you throw the ball?



The main point of interest is the rooftop. To just clear the roof, requires that the roof be at the top of the parabola. We know

$$\mathbf{v}_0 = \boldsymbol{i}\ v_0 \cos(\theta) + \boldsymbol{j}\ v_0 \sin(\theta)$$
$$\mathbf{v}_f = \boldsymbol{i}\ v_f + \boldsymbol{j}\ 0$$
$$\mathbf{a} = -\boldsymbol{j}\ g = -\boldsymbol{j}\ 9.81 \text{ m/s}^2$$

$\Delta\mathbf{r} = \boldsymbol{i}\ 9 + \boldsymbol{j}\ 5$ m

Applying our kinematic equations to the situation:

$v_f = v_0 \cos(\theta)$	(1)
$9 = v_0 \cos(\theta)\ t$	(2)
$5 = v_0 \sin(\theta)t - \frac{1}{2}gt^2$	(3)
$0 = v_0 \sin(\theta) - gt$	(4)
$2(-9.81)(5) = -[v_0 \sin(\theta)]^2$	(5)

Equation (5) allows us to find the y component of the initial velocity $v_{0y} = v_0 \sin(\theta) = 9.905$ m/s. We can use this result to find t from equation 4, t = 1.0096 s. Knowing t we can use equation (2) to find the x component of the initial velocity $v_{0x} = v_0 \cos(\theta) = 8.914$ m/s. Thus the initial velocity is

$$\mathbf{v}_0 = \boldsymbol{i}\ 8.914 + \boldsymbol{j}\ 9.905 \text{ m/s} .$$

The angle θ can be found from the ratio $[v_0 \sin(\theta)] / [v_0 \cos(\theta)] = 9.905/8.914$ or, more simply, $\tan(\theta) = 1.111$ which yields $\theta = 48.0°$.

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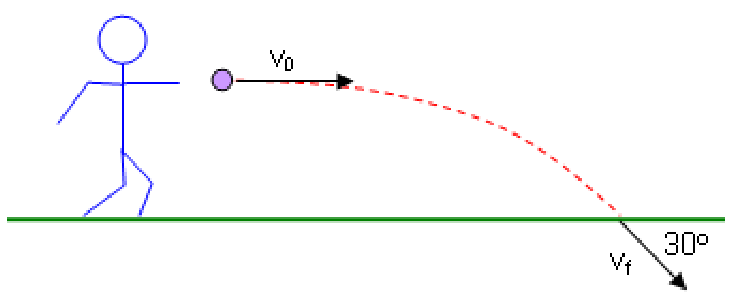
9. A boy throws a ball horizontally from a shoulder height of 1.10 m. Just before the ball touches down on the level ground it makes an angle of 30° with the ground. Determine the initial velocity of the ball as it left the boy's hand.



We know

$$\mathbf{v}_0 = \boldsymbol{i}\ v_0 + \boldsymbol{j}\ 0$$
$$\mathbf{v}_f = \boldsymbol{i}\ v_f \cos(30°) - \boldsymbol{j}\ v_f \sin(30°)$$
$$\mathbf{a} = -\boldsymbol{j}\ g = -\boldsymbol{j}\ 9.81 \text{ m/s}^2$$
$$\Delta\mathbf{r} = \boldsymbol{i}\ \Delta x - \boldsymbol{j}\ 1.10 \text{ m}$$

Applying our kinematic equations to the situation:

$$v_0 = v_f \cos(30°) \qquad (1)$$

$$\Delta x = v_0\, t \qquad (2)$$

$$-1.10 = -½gt^2 \qquad (3)$$

$$-\, v_f \sin(30°) = -gt \qquad (4)$$

$$2(-9.81)(-1.10) = [-v_f \sin(30°)]^2 \qquad (5)$$

To determine $v_{0,}$ we need to know v_f or Δx in equation (1) or (2). We can use equation (5) to find v_f = 9.291 m/s. Thus, from equation (1), v_0 = 8.046 m/s.

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10. Since the earth is rotating at constant speed, there is a slight centripetal acceleration. For a person on the equator, calculate this acceleration. The earth has a radius of 6380 km and recall that it takes one day to make a complete rotation.

The definition of centripetal acceleration is $a_c = v^2/R$. If we assume uniform circular motion, then $v = 2\pi R/T$, where T is the period of motion - one day in this case. Substituting the equation for v into the equation for a_c yields,

$$a_c = 4\pi^2 R/T^2 = 4\pi^2(6380 \times 10^3\text{ m})/(1\text{ d} \times 24\text{ h/d} \times 3600\text{ s/h})^2 = 3.37 \times 10^{-2}\text{ m/s}^2\ .$$

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11. The orbit of the earth about the Sun takes one year. The orbit is approximate circular with a radius of 1.50×10^{11} m. What centripetal acceleration does this imply?.

The definition of centripetal acceleration is $a_c = v^2/R$. If we assume uniform circular motion, then $v = 2\pi R/T$, where T is the period of motion - one year in this case. Substituting the equation for v into the equation for a_c yields,

$$a_c = 4\pi^2 R/T^2 = 4\pi^2(1.50 \times 10^{-11}\text{ m})/(365\text{ d} \times 24\text{ h/d} \times 3600\text{ s/h})^2 = 5.95 \times 10^{-3}\text{ m/s}^2\ .$$

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Questions? mike.coombes@kwantlen.ca