Physics 1120 Uncertainty Propagation

1. Round the following to the correct number of significant figures:
   (a) 71.85234 ± 0.02672 (b) 13.6 ± 0.210 (c) 0.0044667 ± 0.000081

   Recall that we keep two digits if the first non-zero digit of the uncertainty is a 1 or a 2.

   (a) 71.852 ± 0.027

   (b) 13.60 ± 0.21

   (c) 0.00447 ± 0.00008

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2. Apply error propagation rules to the following:
   a. Let $A = 79.5 \pm 0.6$, $B = 27.8 \pm 0.4$, and $C = 54.6 \pm 0.3$. Evaluate $F = A - B + C$.

      First we determine the principle value,

      $$P(F) = P(A) - P(B) + P(C) = 79.5 - 27.8 + 54.6 = 106.3 .$$

      Using the rule for addition and subtraction, the uncertainty is given by

      $$\begin{aligned}\delta(F) &= \delta(A - B + C) \\ &= [(\delta A)^2 + (\delta B)^2 + (\delta C)^2]^{1/2} \\ &= [(0.6)^2 + (0.4)^2 + (0.3)^2]^{1/2} \\ &= 0.78\end{aligned}$$

      Thus $F = 106.3 \pm 0.78 = 106.3 \pm 0.8$.

   b. Let $A = -12.1 \pm 0.2$ and $B = 3.45 \pm 0.06$. Evaluate $F = A/B$.

      First we determine the principle value,

      $$P(F) = P(A)/P(B) = -12.1/3.45 = -3.5072 .$$

      Using the rule for multiplication and division, the uncertainty is given by

$$\delta(F) = \delta(A/B)$$
$$= P(A/B)[(\delta(A)/P(A))^2 + (\delta(B)/P(B))^2]^{1/2}$$
$$= P(F)[(\delta(A)/P(A))^2 + (\delta(B)/P(B))^2]^{1/2}$$
$$= |-3.5072| \times [(0.02 / -12.1)^2 + (0.06 / 3.45)^2]^{1/2}$$
$$= 0.0841$$

Thus $F = -3.5072 \pm 0.0841 = -3.51 \pm 0.08$.

c. Let $A = 15.4 \pm 0.2$, $B = 7.85 \pm 0.03$, and $C = 6.24 \pm 0.08$. Evaluate $F = A/(BC)$.

First we determine the principle value,

$$P(F) = P(A/BC) = 15.4/(7.85 \times 6.24) = 0.3144 \text{ .}$$

Using the rule for multiplication and division, the uncertainty is given by

$$\delta(F) = \delta(A/BC)$$
$$= P(F)[(\delta(A)/P(A))^2 + (\delta(B)/P(B))^2 + (\delta(B)/P(B))^2]^{1/2}$$
$$= 0.3144 \times [(0.2 /15.4)^2 + (0.03/7.85)^2 + (0.08/6.24)^2]^{1/2}$$
$$= 0.00586$$

Thus $F = 0.3144 \pm 0.0058 = 0.314 \pm 0.006$.

d. Let $R = 0.151 \pm 0.005$. Evaluate $V = (4/3)R^3$.

First we determine the principle value,

$$P(V) = (4/3)P(R^3) = (4/3)(0.151)^3 = 0.01442 \text{ .}$$

Using the rule for roots and powers, the uncertainty is given by

$$\delta(V) = (4\pi/3)\delta(R^3)$$
$$= (4\pi/3)(3)[P(R)]^2\delta(R)$$
$$= 4\pi(0.151)^2(0.005)$$
$$= 0.00143$$

Thus $V = 0.01442 \pm 0.00143 = 0.014 \pm 0.001$.

e. Let $X = 14.75 \pm 0.09$. Evaluate $Y = 3X^{1/2}$.

First we determine the principle value,

$$P(Y) = 3P(X^{½}) = 3(14.75)^{½} = 11.522 .$$

Using the rule for roots and powers, the uncertainty is given by

$$\begin{aligned} \delta(Y) &= 3\delta(X^{½}) \\ &= 3(½)[P(X)]^{-½}\delta(X) \\ &= (3/2)(0.09)/(14.75)^{½} \\ &= 0.035 \end{aligned}$$

Thus Y = 11.522 ± 0.035 = 11.52 ± 0.04 .

f. Let θ = 27.5 ± 0.5°. Evaluate Y = *sin*(θ).

First we determine the principle value,

$$P(Y) = P\{sin(\theta)\} = sin(27.5°) = 0.46175 .$$

Using the rule for functions, the uncertainty is given by

$$\begin{aligned} \delta(Y) &= \delta\{sin(\theta)\} \\ &= \delta\theta \; cos(P(\theta)) \\ &= (0.5° \times \pi/180°)cos(27.5) \\ &= 0.00774 \end{aligned}$$

Thus Y = 0.46175 ± 0.00774 = 0.462 ± 0.008 .

g. Let λ = 3.51 ± 0.06. Evaluate $N = e^{-\lambda}$

First we determine the principle value,

$$P(N) = P(e^{-\lambda}) = e^{-3.51} = 0.029897.$$

Using the rule for functions, the uncertainty is given by

$$\begin{aligned} \delta(N) &= \delta(e^{-\lambda}) \\ &= \delta\lambda P(e^{-\lambda}) \\ &= (0.06)(0.029897) \\ &= 0.00179 \end{aligned}$$

Thus N = 0.029897 ± 0.00179 = 0.030 ± 0.002 .

h. Let x = 0.75 ± 0.03. Evaluate φ = *arctan*(x).

First we determine the principle value,

$$P(\varphi) = \arctan(0.75) = 36.870° \text{ or } 0.64350 \text{ rad}.$$

Since we are dealing with a function

$$\delta\varphi = \delta x / (1 + x^2) = 0.03 / (1 + 0.75^2) = 0.019 \text{ radians or } \delta\varphi = 1.1°.$$

Thus, $\varphi = 36.9 \pm 1.1°$ or $\varphi = 0.644 \pm 0.019$ rad.

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3. Find algebraic expressions for the absolute error in the following:

(a) $V = \frac{4}{3}\pi R^3$ $R = 3.22 \pm 0.04$

First let $A = R^3$. Using the power rule, $\delta A/A = 3\delta R/R$.

Thus $V = (4\pi/3)A$, where $4\pi/3$ is a constant. So

$$\begin{aligned} \delta V &= (4\pi/3)\delta A \\ &= (4\pi/3)(R^3)(3\delta R/R) \\ &= 4\pi R^2 \delta R \end{aligned}$$

Evaluating we find $V = 139.85 \pm 5.21 = 140 \pm 5$.

(b) $z = \sqrt{x^2 + y^2}$ $x = 2.25 \pm 0.04$, $y = 3.72 \pm 0.04$

First let $A = x^2$. Using the power rule, $\delta A = 2x\delta x$. Evaluating $A = 5.0625$ and $\delta A = 0.18$.

Similarly, let $B = x^2$. Evaluating this gives $B = 13.8384$. The uncertainty formula is $\delta B = 2y\delta y$ which has the value $\delta B = 0.2976$.

Next let $C = A + B$. This is $C = 18.9009$. Using the addition rule, we find

$$\begin{aligned} \delta C &= [(\delta A)^2 + (\delta B)^2]^{1/2} \\ &= [(0.18)^2 + (0.2976)^2]^{1/2} \\ &= 0.3478 \end{aligned}$$

Now we have $z = C^{1/2}$ or $z = 4.3475$. Using the power rule,

$$\begin{aligned} \delta z &= \tfrac{1}{2}\delta C / C^{1/2} \\ &= \tfrac{1}{2}(0.3478)/4.3475 \end{aligned}$$

$$= 0.04$$

Evaluating we find $z = 4.35 \pm 0.04$.

(c) $R = A\textit{cos}(\theta)$ $\qquad A = 4.27 \pm 0.07$ $\theta = 35.0 \pm 0.9°$

First let $X = \textit{cos}(\theta)$. The pricipal value is $X = 0.81915$. Using the rule for functions $\delta X = \delta\theta \textit{sin}(\theta)$ where $\delta\theta$ must be given in *radians*. Since $\delta\theta = 0.9° \times \pi/180° = 0.01571$, $\delta X = 0.009010$.

Now we have $R = AX$. Evaluating this gives $R = (4.27)(0.81915) = 3.49777$. Using the rule for multiplication to find the uncertainty yields

$$\begin{aligned}\delta R/R &= [(\delta A/A)^2 + (\delta X/X)^2]^{½} \\ &= [(0.07/4.27)^2 + (0.009010/0.81915)^2]^{½} \\ &= 0.01980\end{aligned}$$

Cross-multiplying to find δR, we get $\delta R = 3.49777(0.01980) = 0.06927$.

The result is $R = 3.50 \pm 0.07$.

(d) $d = v_0 + at$ $\qquad v_0 = 12.4 \pm 0.2$, $a = -3.51 \pm 0.11$, $t = 2.52 \pm 0.08$

Let $Y = at$. The principle value is $Y = (-3.51)(2.52) = -8.8452$. To find the uncertainty we use the multiplication rule.

$$\begin{aligned}\delta Y &= Y[(\delta a/a)^2 + (\delta t/t)^2]^{½} \\ &= (-8.8452)[(0.11/-3.51)^2 + (.08/2.52)^2]^{½} \\ &= -0.39457 \\ &= 0.39457\end{aligned}$$

Note that uncertainties are always positive.

Now we are left with a simple sum, $d = v_0 + Y$. The principle value is $d = 12.4 + -8.8452 = 3.5548$. Using the addition rule for the uncertainty we find

$$\begin{aligned}\delta d &= [(\delta v_0)^2 + (\delta Y)^2]^{½} \\ &= [\ (0.2)^2 + (0.39457)^2]^{½} \\ &= 0.4424\end{aligned}$$

Thus we find $v = 3.5548 \pm 0.4424 = 3.6 \pm 0.4$.

(e) $X = R\ \textit{tan}^2(\theta)$ $\qquad R = 6.85 \pm 0.12$ $\theta = 33.0 \pm 0.8°$

Let $A = \textit{tan}(\theta)$. The principle value is $A = 0.64941$. Using the rule for the uncertainty in this function, δA

$= \delta\theta / cos^2(\theta)$. Recall that $\delta\theta$ must be used in radians. So $\delta\theta = 0.8° \times \pi/180° = 0.013963$. The uncertainty is $\delta A = (0.013963)/\cos^2(33°) = 0.019851$.

We now have $X = RA^2$, so let $B = A^2$. This has principle value $B = 0.42173$. The rule for this power function yields the uncertainty $\delta B = 2A\delta A$ with value $\delta B = 2(0.64941)(0.019851) = 0.025783$.

So we are left with $X = RB$. This has principle value $X = (6.85)(0.42173) = 2.88885$. Using the rule for multiplication to find the uncertainty

$$\delta X = X\,[(\delta R/R)^2 + (\delta B/B)^2]^{1/2}$$
$$= (2.88885)[(0.12/6.85)^2 + (0.025783/0.42173)^2]^{1/2}$$
$$= 0.18372$$

Thus we have $X = 2.89 \pm 0.18$.

(f) $v = \sqrt{Rg\tan(\theta)}$ $\qquad R = 6.85 \pm 0.12,\ g = 9.81 \pm 0.01,\ \theta = 43.0 \pm 0.8°$

Let $A = tan(\theta)$ which has principle value $A = 0.93252$. Using the rule for this function, $\delta A = \delta\theta / cos^2(\theta)$ where $\delta\theta$ must be in radians. Evaluating $\delta A = (0.8° \times \pi/180°)/\cos^2(43°) = 0.026104$.

We now have $v = [RgA]^{1/2}$, so let $B = RgA$. The principle value is $B = (6.85)(9.81)(0.93252) = 66.6639$. The uncertainty propagation rule for this multiplication yields

$$\delta B = B\,[(\delta R/R)^2 + (\delta g/g)^2 + (\delta A/A)^2]^{1/2}$$
$$= (66.6639)[(0.12/6.85)^2 + (0.01/9.81)^2 + (0.026104/0.93252)^2]^{1/2}$$
$$= 2.2025$$

So now $v = B^{1/2}$ which, when evaluated, yields $v = (66.6639)^{1/2} = 8.16480$. The rule for the uncertainty in this function is

$$\delta v = \tfrac{1}{2}\delta B / B^{1/2}$$
$$= \tfrac{1}{2}(2.2025)/(8.16480)$$
$$= 0.13488$$

Thus we have $v = 8.16 \pm 0.13$.

(g) $d = v_0 t + \tfrac{1}{2}at^2$ $\qquad v_0 = 12.4 \pm 0.2,\ a = -3.51 \pm 0.11,\ t = 2.52 \pm 0.08$

This is not a formula that we can work with. Suppose we let $A = v_0 t$ and $B = \tfrac{1}{2}at^2$ so that $d = A + B$. We do not have a rule to find δd for this simple addition since δA and δB are not independent of one another since they both will include a δt dependence. We will have to wait until we can handle multivariate calculus to do this problem correctly.

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