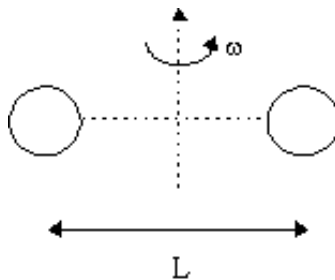


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Physics 1120: Moment of Inertia Solutions

1. The moment of inertia of an oxygen molecule about an axis through the centre of mass and perpendicular to the line joining the atoms is $1.95 \times 10^{-46} \text{ kg}\cdot\text{m}^2$. The mass of an oxygen atom is $2.66 \times 10^{-26} \text{ kg}$. What is the distance between the atoms? Treat the atoms as particles.

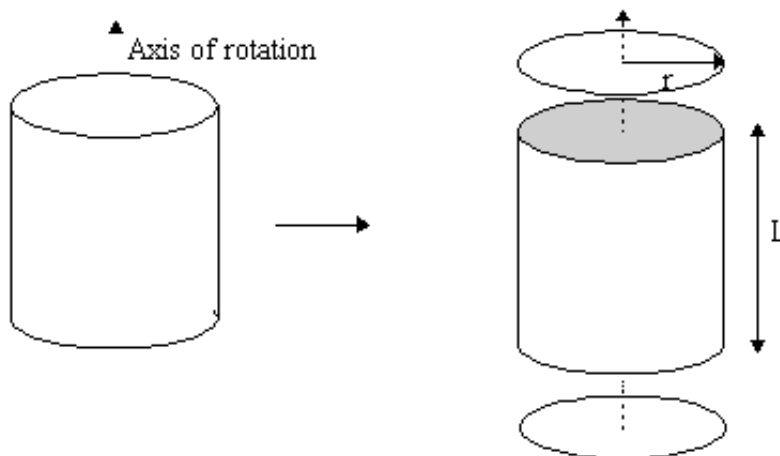


The moment of inertia for point particles is given by $I = \sum m_i r_i^2 = M(\frac{L}{2})^2 + M(\frac{L}{2})^2 = \frac{1}{2} ML^2$.

Rewriting this for L yields $L = \sqrt{\frac{2I}{M}} = \sqrt{\frac{2 \times (1.95 \times 10^{-46})}{2.66 \times 10^{-26}}} = 121 \times 10^{-10} \text{ m}$.

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2. An empty beer can has a mass of 50 g, a length of 12 cm, and a radius of 3.3 cm. Assume that the shell of the can is a perfect cylinder of uniform density and thickness.
- What is the mass of the lid/bottom?
 - What is the mass of the shell?
 - Find the moment of inertia of the can about the cylinder's axis of symmetry.



(a) The mass has three components, $M = M_{\text{lid}} + M_{\text{shell}} + M_{\text{bottom}} = 50 \text{ g}$, where the lid and the bottom are identical. Mass is proportional to the surface area for uniform objects. The area of the lid and bottom is that of a circle, $A_{\text{circle}} = \pi r^2$. The surface area of a cylindrical shell is $A_{\text{shell}} = 2\pi rL$. So the total area of can is $A_{\text{total}} = 2A_{\text{circle}} + A_{\text{shell}} = 2\pi rL + 2\pi r^2 = 2\pi r(r + L)$. So the mass of the lid or bottom is given by

$$M_{\text{lid}} = \frac{A_{\text{circle}}}{A_{\text{total}}} M = \frac{r}{2(r + L)} M. \text{ Thus } M_{\text{lid}} = 5.392 \text{ g.}$$

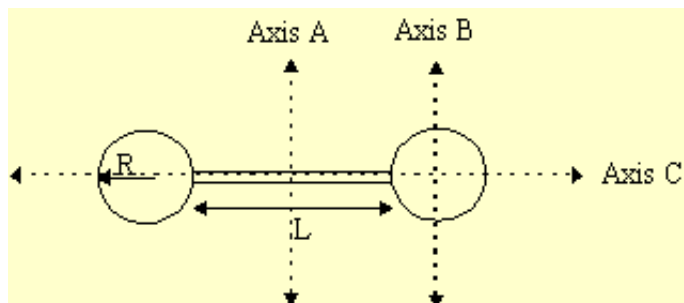
(b) Similarly, the mass of the shell is given by $M_{\text{shell}} = \frac{A_{\text{shell}}}{A_{\text{total}}} M = \frac{L}{r + L} M$. Thus $M_{\text{shell}} = 39.216 \text{ g}$.

(c) The total moment of inertia of the beer can is given by the sum of the individual pieces, $I_{\text{total}} = I_{\text{lid}} + I_{\text{shell}} + I_{\text{bottom}}$. Since each piece is revolving about its centre of mass, we do not need the parallel axis theorem. Looking up the moments of inertia of a flat solid disk and a thin cylindrical shell, we have $I_{\text{total}} = 2 \times \frac{1}{2} M_{\text{lid}} r^2 + M_{\text{shell}} r^2 = 4.86 \times 10^{-5} \text{ kg}\cdot\text{m}^2$.

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3. A dumbbell consists of two uniform spheres of mass M and radius R joined by a thin rod of mass m , length L , and radius r (see diagram).

- What is the moment of inertia about the centre of mass and perpendicular to the rod (Axis A)?
- About an axis through one sphere and perpendicular to the rod (Axis B)?
- About an axis along the rod (Axis C)?



(a) The moment of inertia of a composite body is equal to the sum of the moments of its individual pieces, $I_{\text{total}} = I_{\text{sphere1}} + I_{\text{rod}} + I_{\text{sphere2}}$. For axis A, the rod is rotating about its centre of mass. Each sphere is a distance $R + L/2$ from the axis of rotation, so we must use the parallel axis theorem. Recall that the moment of inertia of a rod about its centre is $\frac{1}{12} M L^2$ and that the moment of inertia of a solid sphere about its centre is $\frac{2}{5} M R^2$. Thus we have

$$\begin{aligned} I_A &= \frac{1}{12} M_{\text{rod}} L^2 + 2 \times \left(\frac{2}{5} M_{\text{sphere}} R^2 + M_{\text{sphere}} (R + L/2)^2 \right) \\ &= \frac{1}{12} M_{\text{rod}} L^2 + 2 M_{\text{sphere}} \left(\frac{2}{5} R^2 + (R + L/2)^2 \right) \end{aligned}$$

(b) For axis B, the rod's centre is $R + L/2$ away from the axis of rotation. One sphere's centre is $L + 2R$

from the axis of rotation. The last sphere is rotating about axis B. Thus r

$$I_A = \left(\frac{2}{5} M_{\text{sphere}} R^2 + M_{\text{sphere}} (2R + L)^2 \right) + \left(\frac{1}{12} M_{\text{bar}} L^2 + M_{\text{bar}} \left(R + \frac{L}{2} \right)^2 \right) + \frac{2}{5} M_{\text{sphere}} R^2$$

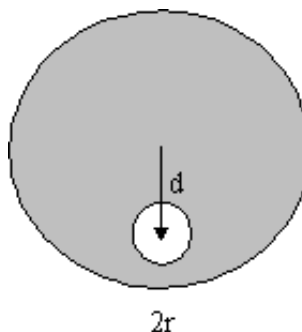
$$= M_{\text{bar}} \left(\frac{1}{12} L^2 + \left(R + \frac{L}{2} \right)^2 \right) + M_{\text{sphere}} \left(\frac{4}{5} R^2 + (2R + L)^2 \right)$$

(c) The bar and both spheres are rotating about their own centres when rotating about axis c. However, note that the bar is a cylinder of radius r in this configuration.

$$I_A = \frac{1}{2} M_{\text{bar}} r^2 + 2 \times \left(\frac{2}{5} M_{\text{sphere}} R^2 \right) = \frac{1}{2} M_{\text{bar}} r^2 + \frac{4}{5} M_{\text{sphere}} R^2$$

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4. A hole of radius r has been drilled in a flat, circular plate of radius R. The centre of the hole is at a distance d from the centre of the circle. The mass of the complete body was M. Find the moment of inertia for rotation about an axis through the centre of the plate.



We must treat the hole as an object of negative mass. The inertia of the object is then just

$I_{\text{total}} = I_{\text{plate}} - I_{\text{hole}}$. The plate and the hole are just disks and the inertia of a disk is $\frac{1}{2} MR^2$. The hole is not rotating about its own centre of mass, so we must use the parallel axis theorem,

$$I_{\text{total}} = \frac{1}{2} M_{\text{plate}} R^2 - \left(\frac{1}{2} M_{\text{hole}} r^2 + M_{\text{hole}} d^2 \right) = \frac{1}{2} M_{\text{plate}} R^2 - M_{\text{hole}} \left(\frac{1}{2} M r^2 + d^2 \right).$$

We are not yet finished; we do not know the mass of the hole. However since the plate is uniform, mass is proportional to area, so

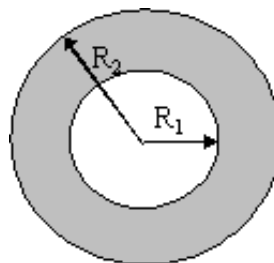
$$M_{\text{hole}} = \frac{A_{\text{hole}}}{A_{\text{plate}}} M_{\text{plate}} = \frac{\pi r^2}{\pi R^2} M_{\text{plate}} = \frac{r^2}{R^2} M_{\text{plate}}$$

Substituting this into our equation for the moment of inertia yields

$$I_{\text{total}} = \frac{1}{2} M_{\text{plate}} R^2 - \frac{r^2}{R^2} M_{\text{plate}} \left(\frac{1}{2} r^2 + d^2 \right) = \frac{1}{2} M_{\text{plate}} \left(R^2 - \frac{r^2}{R^2} (r^2 + 2d^2) \right).$$

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5. A thick spherical shell has an inner radius R_1 , an outer radius R_2 , and a mass M . The material that the spherical shell is made of is uniform. Find the moment of inertia of the thick shell about an axis through the centre of the sphere. The volume of a sphere is $4\pi r^3/3$.



We must treat the hole at the centre as a sphere of negative mass. Since the moment of inertia of a sphere about its centre is $\frac{2}{5}MR^2$, the moment of inertia of this object is

$$I_{total} = I_{sphere} - I_{hole} = \frac{2}{5} M_{sphere} R_2^2 - \frac{2}{5} M_{hole} R_1^2 .$$

We are not yet finished since we do not know the mass of either the sphere or of the hole. We must find a way to relate these masses to the mass of the spherical shell, M . Fortunately the object is uniform so the mass of each is related to its volume. The volume of the spherical shell is simply the difference of its outer and inner volumes,

$$V_{total} = \frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3 .$$

Thus the mass of the sphere is given by

$$M_{sphere} = \frac{A_{sphere}}{A_{total}} M = \frac{\frac{4}{3}\pi R_2^3}{\frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3} M = \frac{R_2^3}{R_2^3 - R_1^3} M .$$

Similarly, the mass of the hole is given by

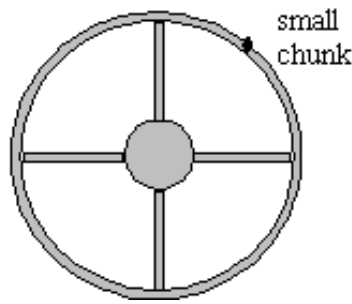
$$M_{hole} = \frac{A_{hole}}{A_{total}} M = \frac{R_1^3}{R_2^3 - R_1^3} M .$$

Thus the moment of inertia of a thick spherical shell is

$$I_{total} = \frac{2}{5} \frac{R_2^3}{R_2^3 - R_1^3} M R_2^2 - \frac{2}{5} \frac{R_1^3}{R_2^3 - R_1^3} M R_1^2 = \frac{2}{5} M \frac{R_2^5 - R_1^5}{R_2^3 - R_1^3} .$$

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6. The object shown in the diagram below consists of a 100-kg, 25.0 cm radius cylinder connected by four 5.00-kg, 0.75-m long thin rods to a thin-shelled outer cylinder of mass 20.0 kg. A small chunk of metal of mass 1.00 kg has been welded to the outer cylinder. What is the moment of inertia of the object about the centre of the inner cylinder? Treat the metal chunk as a point mass.

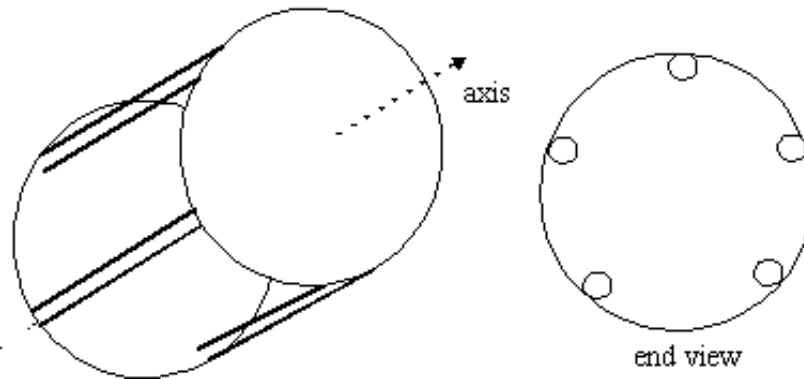


The moment of inertia of a composite body is equal to the sum of the moments of its individual pieces, $I_{total} = I_{shell} + I_{disk} + 4I_{rods} + I_{chunk}$. The rods are not rotating about their centre of mass, so we must use the parallel axis theorem. The centre of each rod is $R_{cyl} + L/2$ from the axis of rotation at the centre of the object. The moments of inertia for a cylindrical shell, a disk, and a rod are MR^2 , $\frac{1}{2}MR^2$, and $\frac{1}{12}ML^2$ respectively. The moment of inertia of a point mass is $M_{point}R^2$. Thus the total moment of inertia is:

$$I = M_{shell}R_{shell}^2 + \frac{1}{2}M_{cyl}R_{cyl}^2 + 4\left(\frac{1}{12}M_{rod}L^2 + M_{rod}(R_{cyl} + L/2)^2\right) + M_{chunk}R_{shell}^2 = 32.9 \text{ kg} \cdot \text{m}^2$$

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7. The object in the diagram below consists of five thin cylinders arranged in a circle. A thin disk has been welded to the tops and the bottoms of the cylinders. The cylinders each have a mass of 250 g, a length of 15.0 cm, and a radius of 1.00 cm. The disks are each 125 g and have a radius of 5.00 cm. Find the moment of inertia of the whole object about an axis through the centres of the disks.



The moment of inertia of a composite body is equal to the sum of the moments of its individual pieces, $I_{total} = 2I_{disk} + 5I_{rods}$. The cylinders are not rotating about their centre of mass, so we must use the parallel axis theorem. The centre of each rod is $R_{disk} - R_{cyl}$ from the axis of rotation at the centre of the object. The moments of inertia for a disk or a cylindrical rod are $\frac{1}{2}MR^2$. The moment of inertia of a point mass is $M_{point}R^2$. Thus the total moment of inertia is:

$$I = 2 \times \frac{1}{2}M_{disk}R_{disk}^2 + 5\left(\frac{1}{2}M_{rod}r^2 + M_{rod}(R_{disk} - R_{cyl})^2\right) = 2.375 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

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Physics

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