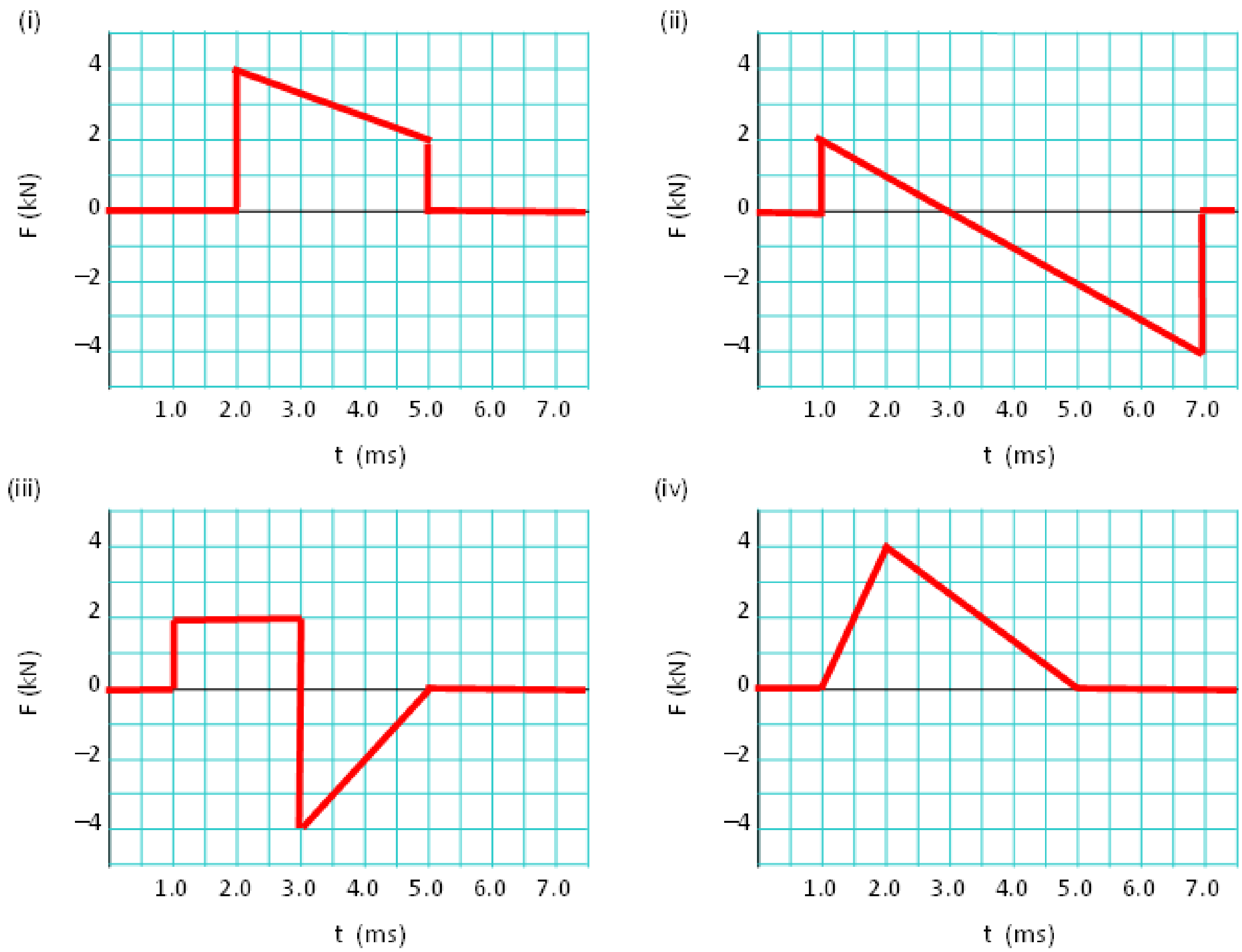
Physics 1120: Momentum and Impulse Solutions

1. The diagrams below are graphs of Force in kiloNewtons versus time in milliseconds for the motion of a 5-kg block moving to the right at 4.0 m/s.
   (a) What is the magnitude and direction of the impulse acting on the block in each case?
   (b) What is the magnitude and direction of the average force acting on the block in each case?
   (c) What is the magnitude and direction of the final velocity of the block in each case?



   a. Impulse is given by the area under the F-t curves. Since we have simple shapes, it is easy to find the area. For rectangles area is height × base and for triangles area is half the height × base.

      i. I = 3 kN × 3 ms = 9 N-s

      ii. I = −1 kN × 6 ms = −6 N-s

      iii. I = 2 kN × 2 ms + ½(−4 kN) × 2 ms = 0 N-s

      iv. I = ½(4 kN) × 4 ms = 8 N-s

If the impulse is positive, the net area was above the curve and it is directed to the right, if negative to the left.

b. We know $\mathbf{I} = \mathbf{F}_{ave}\Delta t$ where Δt is how long the collision lasts. We read Δt from the graphs, so $\mathbf{F}_{ave} = \mathbf{I}/\Delta t$.

   i. F_{ave} = (9 N-s)/(3 ms) = 3000 N

   ii. F_{ave} = (−6 N-s)/(6 ms) = -1000 N

   iii. F_{ave} = (0 N-s)/(4 ms) = 0 N

   iv. F_{ave} = (8 N-s)/(4 ms) = 2000 N

   If the average force is positive it is directed to the right, if negative to the left. The impulse and force have the same direction.

c. Impulse is also equal to the difference in momentum, $\mathbf{I} = m\mathbf{v}_f - m\mathbf{v}_i$. We can rearrange our equation for $\mathbf{v}_f$, $\mathbf{v}_f = \mathbf{I}/m + \mathbf{v}_i$.

   i. v_f = (9 N-s)/(5.0 kg) + 4 m/s = 5.8 m/s

   ii. v_f = (?6 N-s)/(5.0 kg) + 4 m/s = 2.8 m/s

   iii. v_f = (0 N-s)/(5.0 kg) + 4 m/s = 4 m/s

   iv. v_f = (8 N-s)/(5.0 kg) + 4 m/s = 5.6 m/s

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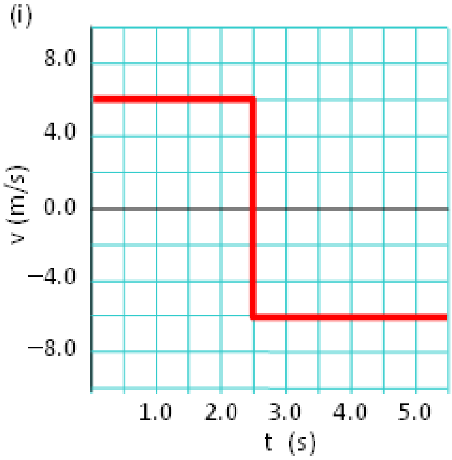
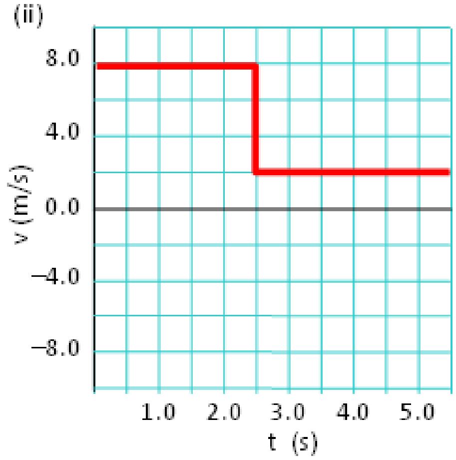
2. The diagrams below are the velocity versus time graphs for the collision of motion of a 4-kg block with a wall. The collision lasts for 20 milliseconds in each case.
   (a) What is the magnitude and direction of the impulse acting on the block in each case?
   (b) What is the magnitude and direction of the average force acting on the block in each case?

(i)



(ii)



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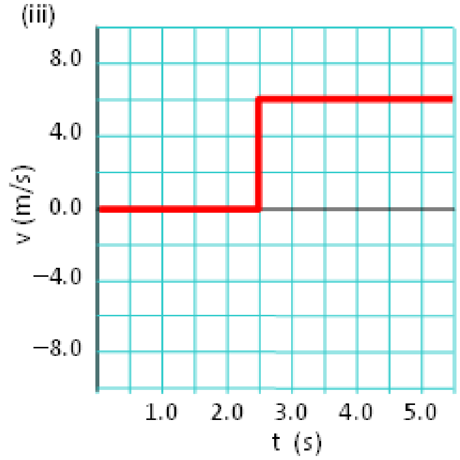

a. Impulse is also equal to the difference in momentum, $\mathbf{I} = m\mathbf{v}_f - m\mathbf{v}_i$. We have the mass, m = 4 kg.

i. $I = (4\ \text{kg}) \times (-6\ \text{m/s} - 6\ \text{m/s}) = -48$ N-s

ii. $I = (4\ \text{kg}) \times (2\ \text{m/s} - 8\ \text{m/s}) = -24$ N-s

iii. $I = (4\ \text{kg}) \times (6\ \text{m/s} - 0\ \text{m/s}) = +24$ N-s

If the impulse is positive it is directed to the right, if negative to the left.

b. We know $\mathbf{I} = \mathbf{F}_{ave}\Delta t$ where Δt is how long the collision lasts. We have already calculated I and we are given Δt = 20 ms, so $\mathbf{F}_{ave} = \mathbf{I}/\Delta t$.

i. $F_{ave} = (-48\ \text{N-s})/(20\ \text{ms}) = -2400$ N

ii. $F_{ave} = (-24\ \text{N-s})/(20\ \text{ms}) = -1200$ N

iii. $F_{ave} = (+24\ \text{N-s})/(20\ \text{ms}) = +1200$ N

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3. You've been rowdy and obnoxious in a bar and are now in the process of being thrown out by the scruff of the neck by the bouncer. The bouncer has hold of you for 5.0 s and you are take from a seated position to a final speed of 2.75 m/s. If your mass is 70.0 kg, what was your final momentum? What impulse and average force did the bouncer exert on your person? Assume all motion is in a straight line.

Momentum is defined by $\boldsymbol{p} = m\boldsymbol{v}$. Taking the direction of motion as positive, your initial momentum was zero and your final momentum is

$$p = (70.0\ \text{kg})(2.75\ \text{m/s}) = 192.5\ \text{kg-m/s}\ .$$

Impulse is defined as the change in momentum

$$\boldsymbol{I} = \boldsymbol{p}_f - \boldsymbol{p}_i = 192.5\ \text{kg-m/s}\ .$$

Average force is related to impulse by $\boldsymbol{I} = \boldsymbol{F}_{average}\Delta t$, so

$$F_{average} = I / \Delta t = 192.5\ \text{kg-m/s} / 5\ \text{s} = 38.5\ \text{N}\ .$$

This is the average force exerted on you and is in the same direction as your motion.

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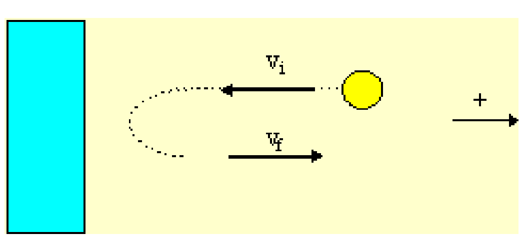
4. A ball of mass 0.500 kg with speed 15.0 m/s collides with a wall and bounces back with a speed of 10.5 m/s. If the motion is in a straight line, calculate the initial and final momenta and impulse. If the ball exerted an average force of 1000 N on the wall, how long did the collision last?



Momentum is defined by $\boldsymbol{p} = m\boldsymbol{v}$. Taking the right as positive, the initial momentum of the ball is

$$p_i = (0.5 \text{ kg})(-15 \text{ m/s}) = -7.5 \text{ kg-m/s} .$$

The final momentum is

$$p_f = (0.5 \text{ kg})(10.5 \text{ m/s}) = 5.25 \text{ kg-m/s} .$$

Impulse is defined as the change in momentum

$$\boldsymbol{I} = \boldsymbol{p}_f - \boldsymbol{p}_i = 12.75 \text{ kg-m/s} .$$

Average force is related to impulse by $\boldsymbol{I} = \boldsymbol{F}_{average}\Delta t$, and the wall would exert this force on the ball to the right. Therefore

$$\Delta t = I / F_{average} = 12.75 \text{ kg-m/s} / +1000 = 0.013 \text{ s}.$$

The ball is in contact with the wall for approximately 13 milliseconds.

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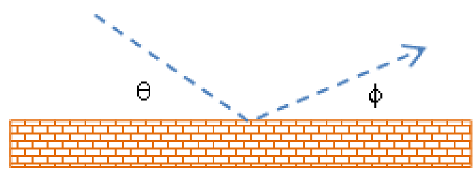
5. A ball of mass 0.25 kg glances of a wall as shown in the diagram. The ball approaches at 15 m/s at θ = 30° and leaves at 12 m/s at φ = 20°. The collision lasts for 15 milliseconds.
(a) What are the components of the impulse experienced by the ball?
(b) What are the components of the average force acting on the ball?



a. We know Impulse is equal to the difference in momentum, $\mathbf{I} = m\mathbf{v}_f - \mathbf{v}_i$. This is a vector equation and to get components we consider the x and y components separately.

   $I_x = mv_{fx} - mv_{ix} = (0.25) \times (12\cos 20° - 15\cos 30°) = -0.4285$ N-s

   $I_y = mv_{fy} - mv_{iy} = (0.25) \times (12\sin 20° - (-15\sin 30°)) = +2.9011$ N-s

   or $\mathbf{I} = -\boldsymbol{i}0.4285 + \boldsymbol{j}2.9011$ N-s.

b. We know $\mathbf{I} = \mathbf{F}_{ave}\Delta t$ where Δt is how long the collision lasts. We have already calculated I and we are given Δt = 15 ms, so $\mathbf{F}_{ave} = \mathbf{I}/\Delta t$

   so $\mathbf{F}_{ave} = (-\boldsymbol{i}0.4285 + \boldsymbol{j}2.9011 \text{ N-s}) / (15 \text{ ms}) = -\boldsymbol{i}28.6 + \boldsymbol{j}193.4$ N.

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6. Explain why a person wearing a seatbelt in a car accident is less likely to be seriously hurt than the person who isn't wearing a seatbelt.

Wearing a seatbelt would not effect the person's initial and final momentum. Since impulse is the change in momentum, the impulse experienced by the person would be the same in either case. However, it is not impulse which is dangerous but the magnitude of the forces acting on the person's body. Impulse and average force are related by $\boldsymbol{I} = \boldsymbol{F}_{average}\,\Delta t$. For the same impulse, the average force will be higher as the collision time Δt decreases. Cars are designed to crumple on impact. This crumpling is designed to make a collision last as long as possible. If a person is wearing a seatbelt, the time for the person's change in momentum is the same as that of the car thereby minimizing the force on the occupant. If the person is not wearing a seatbelt, the Law of Inertia dictates that the person will keep moving forward (Note some people describe this by saying that the person was thrown forward by the force of impact. Why is this wrong?). This means that the person will impact the steering wheel or windshield. The steering wheel and windshield can't collapse as nicely as the front of the car so Δt is much smaller and thus the average force is much higher.

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7. A lion of mass 120 kg leaps at a hunter with a horizontal velocity of 12m/s. The hunter has an automatic rifle firing bullets of mass 15 g with a muzzle speed of 630m/s and he attempts to stop the lion in midair. How many bullets would the hunter have to fire into the lion to stop its horizontal motion? Assume the bullets stick inside the lion.

   The total momentum of the system, the bullets and the lion, would be zero since there are no external forces to consider,

   $$\mathbf{P} = m_{lion}v_{lion} - nm_{bullet}v_{bullet} = 0\ ,$$

   where the lion is assumed to be moving in the positive direction. Rearranging the equation to find n,

   $$n = m_{lion}v_{lion} / m_{bullet}v_{bullet} = (120\text{ kg } 12\text{ m/s}) / (0.015\text{kg } 630\text{ m/s}) = 152.4\ .$$

   It would take 153 bullets to stop the lion dead, so to speak.

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8. On a frictionless surface, a 6.0-kg rock approaches from the left at 3.5 m/s. It collides elastically with a 9.0-kg rock which is approaching from the right at 1.7 m/s.
   (a) Without changing coordinate systems, find the final velocities of the rocks.
   (b) Use a coordinate system in which the 9.0-kg rock is not moving to find the final velocities of the rocks.
   (c) Use a coordinate system in which the 3.5-kg rock is not moving to find the final velocities of the rocks.

   (a) We are dealing with a collision, so we know we must conserve momentum,

   $$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}\ . \qquad (1)$$

   We are told that the collision is elastic which means that kinetic energy is conserved. For a 1D collision, this is the same as

   $$v_{2f} - v_{1f} = -(v_{2i} - v_{1i})\ . \qquad (2)$$

   Substituting in the give data, our equations become

   $$6v_{1f} + 9v_{2f} = 6\times3.5 + 9\times(-1.7) = 5.7\ ,$$

   $$v_{2f} - v_{1f} = -(-1.7 - 3.5) = 5.2\ .$$

   Solving the two equations in two unknowns, we find $v_{1f} = -2.74$ m/s and $v_{2f} = 2.46$ m/s.

(b) To switch to a coordinate system in which the second stone is not moving, we subtract v_{2i} to each,

$$u_{1i} = v_{1i} - v_{2i} = 3.5 \text{ m/s} - (-1.7 \text{ m/s}) = 5.2 \text{ m/s} ,$$

$$u_{2i} = v_{2i} - v_{2i} = 0 .$$

In all frames of reference, this is still an elastic collision, so the following is true,

$$m_1u_{1f} + m_2u_{2f} = m_1u_{1i} + m_2u_{2i} , \qquad (3)$$

$$u_{2f} - u_{1f} = -(u_{2i} - u_{1i}) . \qquad (4)$$

These equations become

$$6u_{1f} + 9u_{2f} = 6\times5.2 + 9\times0 = 31.2 ,$$

$$u_{2f} - u_{1f} = -(0 - 5.2) = 5.2 .$$

Solving, we find $u_{1f} = -1.04$ m/s and $u_{2f} = 4.16$ m/s.

However, we want the results in our frame of reference. To undo our change we add v_{2i} to each

$$v_{1f} = u_{1f} + v_{2i} = -1.04 \text{ m/s} + (-1.7 \text{ m/s}) = -2.74 \text{ m/s} ,$$

$$v_{2f} = u_{2f} + v_{2i} = 4.16 \text{ m/s} + (-1.7 \text{ m/s}) = 2.46 \text{ m/s} .$$

Notice that we get exactly the same result.

(c) We first must determine the velocity of the centre of mass

$$V_{CM} = (\Sigma\, m_iv_i)/M_{total} = [6\times3.5 \text{ m/s} + 9\times(-1.7 \text{ m/s})] / 15 \text{ kg} = 0.38 \text{ m/s} .$$

As in part (b), we find the velocity of each stone in the CM frame of reference by subtracting V_{CM} from each:

$$u_{1i} = v_{1i} - V_{CM} = 3.5 \text{ m/s} - 0.38 \text{ m/s} = 3.12 \text{ m/s} ,$$

$$u_{2i} = v_{2i} - V_{CM} = -1.7 \text{ m/s} - 0.38 \text{ m/s} = -2.08 \text{ m/s} .$$

In a zero momentum frame of reference, an elastic collision only changes the direction of the velocities of each rock, $u_{1f} = -3.12$ m/s and $u_{2f} = +2.08$ m/s.

However, we want the results in our frame of reference. To undo our change we add V_{CM} to each

$$v_{1f} = u_{1f} + V_{CM} = -3.12 \text{ m/s} + 0.38 \text{ m/s} = -2.74 \text{ m/s} ,$$

$$v_{2f} = u_{2f} + V_{CM} = +2.08 \text{ m/s} + 0.38 \text{ m/s} = 2.46 \text{ m/s} .$$

Again, we get exactly the same result but with a little less calculation.

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9. If the collision in question #2 had been perfectly inelastic, what would have been the final velocity of the rocks? How

much kinetic energy would have been lost in the collision?

In any kind of collision, momentum is conserved so

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}\ .$$

We are told that the collision is perfectly elastic which means that kinetic energy is not conserved and that the rocks stick together, $v_{1f} = v_{2f} = v_f$. Our equation becomes

$$(m_1 + m_2)v_f = m_1v_{1i} + m_2v_{2i}\ .$$

This allows us to find v_f immediately

$$v_f = [m_1v_{1i} + m_2v_{2i}]/(m_1 + m_2) = [6\text{ kg}\times3.5\text{ m/s} + 9\text{ kg}\times(-1.7\text{ m/s})]/\ 15\text{ kg} = 0.38\text{ m/s}\ .$$

Notice that this is the same as the velocity of the Centre of Mass.

The initial kinetic energy is

$$K_i = ½m_1(v_{1i})^2 + ½m_2(v_{2i})^2 = ½(6)(3.5)^2 + ½(9)(-1.7)^2 = 49.755\text{ J}\ .$$

The final energy is

$$K_f = ½(m_1 + m_2)(v_f)^2 = ½(6+9)(0.38)^2 = 1.083\text{ J}\ .$$

The difference in energy is

$$\Delta K = K_f - K_i = -48.7\text{ J}\ .$$

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10. If the collision in question #2 had a coefficient of restitution e = 0.600, what would have been the final velocity of the rocks? How much kinetic energy would have been lost in the collision?

In any kind of collision, momentum is conserved so

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}\ .$$

We are told that the coefficient of restitution

$$v_{2f} - v_{1f} = -\mathbf{e}(v_{2i} - v_{1i})\ .$$

Substituting in the give data, our equations become

$$6v_{1f} + 9v_{2f} = 6\times3.5 + 9\times(-1.7) = 5.7\ ,$$

$$v_{2f} - v_{1f} = -(0.6)(-1.7 - 3.5) = 3.12\ .$$

Solving the two equations in two unknowns, we find $v_{1f} = -1.492$ m/s and $v_{2f} = 1.628$ m/s.

The initial kinetic energy is

$$K_i = ½m_1(v_{1i})^2 + ½m_2(v_{2i})^2 = ½(6)(3.5)^2 + ½(9)(-1.7)^2 = 49.755 \text{ J} .$$

The final energy is

$$K_f = ½m_1(v_{1f})^2 + ½m_2(v_{2f})^2 = ½(6)(-1.492)^2 + ½(9)(-1.628)^2 = 18.605 \text{ J} .$$

The difference in energy is

$$\Delta K = K_f - K_i = -31.1 \text{ J} .$$

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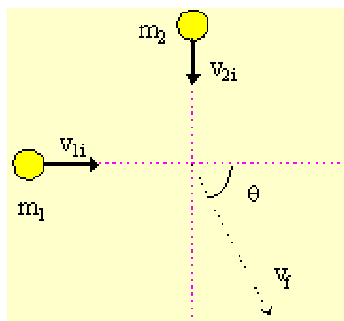
11. A 50.0-kg skater is traveling due east at 3.00 m/s. A 70.0-kg skater is moving due south at 7.00 m/s. They collide and hold on to one another after the collision. Determine the magnitude and direction of their velocity after the collision. Ignore the effects of friction.



In any kind of collision, momentum is conserved so

$$(m_1 + m_2)\mathbf{v}_f = m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} . \qquad (1)$$

Now momentum and velocity are vector quantities and the ***i*** and ***j*** components must be handled separately

$$(m_1 + m_2)v_{fx} = m_1v_{1ix} + m_2v_{2ix} , \qquad (1a)$$

$$(m_1 + m_2)v_{fy} = m_1v_{1iy} + m_2v_{2iy} . \qquad (1b)$$

So we can rearrange these equations to find the components of the final velocity

$$v_{fx} = (m_1v_{1ix} + m_2v_{2ix}) / (m_1 + m_2) , \qquad (2a)$$

$$v_{fy} = (m_1v_{1iy} + m_2v_{2iy}) / (m_1 + m_2) . \qquad (2b)$$

Using the given values, we find

$$v_{fx} = [(50 \text{ kg})(3 \text{ m/s}) + (70 \text{ kg})(0)] / (50 \text{ kg} + 70\text{kg}) = 1.25 \text{ m/s} , \qquad (2a)$$

$$v_{fy} = [(50 \text{ kg})(0) + (70 \text{ kg})(-7 \text{ m/s}) / (50 \text{ kg} + 70 \text{ kg}) = 4.083 \text{ m/s} . \qquad (2b)$$

To find the magnitude and direction of the final velocity, we use Pythagoras' Theorem and trigonometry,

$$v_f = [(v_{fx})^2 + (v_{fx})^2]^{½} = 4.27 \text{ m/s, and}$$

$$\theta = \tan^{-1}(|v_{fy}/v_{fx}|) = 72.98°.$$

The final velocity of the pair is 4.27 m/s at 73.0° south of east.

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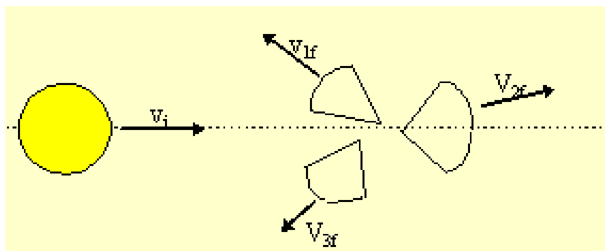
12. A curling rock is traveling down the ice when it mysteriously explodes into three parts. After the explosion, one piece having 27.0% of the total mass moves at a speed of V_{1f} = 14.2 m/s at an angle of 42.0° to the positive y axis. A second with 52.0% of the total mass move at a speed of V_{2f} = 18.9 m/s at an angle of 17.8° to the positive x axis. The third piece moves with speed V_{3f} = 35.9 m/s at 39.0° to the negative y axis. What was the speed of the stone before the explosion?



In any kind of collision, momentum is conserved so

$$m_1\mathbf{V}_{1f} + m_2\mathbf{V}_{2f} + m_3\mathbf{V}_{3f} = (m_1 + m_2 + m_3)\mathbf{V}_i . \qquad (1)$$

Now momentum and velocity are vector quantities and the ***i*** and ***j*** components must be handled separately

$$m_1V_{1fx} + m_2V_{2fx} + m_3V_{3fx} = (m_1 + m_2 + m_3)V_{ix} . \qquad (1a)$$

$$m_1V_{1fy} + m_2V_{2fy} + m_3V_{3fy} = (m_1 + m_2 + m_3)V_{iy} . \qquad (1b)$$

So we can rearrange these equations to find the components of the final velocity

$$V_{ix} = (m_1V_{1fx} + m_2V_{2fx} + m_3V_{3fx}) / (m_1 + m_2 + m_3) , \qquad (2a)$$

$$V_{iy} = (m_1V_{1fy} + m_2V_{2fy} + m_3V_{3fy}) / (m_1 + m_2 + m_3) . \qquad (2b)$$

Taking the masses of the piece to be m_1 = 0.27M, m_2 = 0.52M, and m_3 = (1-0.27-0.52)M = 0.21M where M is the total mass, and using the given values, we find

$$V_{ix} = [0.27(-14.2\sin(42°)) + 0.52(18.9\cos(17.8°)) + 0.21(35.9\sin(39°))] = 2.05 \text{ m/s} ,$$

$$V_{iy} = [0.27(14.2\cos(42°)) + 0.52(18.9\sin(17.8°)) + 0.21(-35.9\cos(39°))] = 0.0 \text{ m/s} .$$

The stone had an initial speed of 2.05 m/s to the right.

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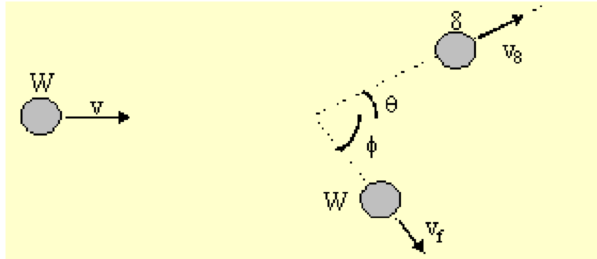
13. In a pool game, balls of the same mass undergo elastic collisions. Suppose the white ball collides with a stationary 8-ball off-centre. After the collision, both balls travel at an angle to the original velocity of the white ball as shown in the diagram. If the initial speed of the white ball is v and θ = 30.0°, determine φ and the speeds of the balls.



In any kind of collision, momentum is conserved so

$$m_W\mathbf{v}_f + m_8\mathbf{v}_8 = m_W\mathbf{v} \,. \qquad (1)$$

Now momentum and velocity are vector quantities and the ***i*** and ***j*** components must be handled separately

$$m_W v_{fx} + m_8 v_{8x} = m_W v_x \,, \qquad (1a)$$

$$m_W v_{fy} + m_8 v_{8y} = m_W v_y \,. \qquad (1b)$$

Since this is an elastic collision, kinetic energy is conserved and we have

$$\tfrac{1}{2}m_W(v_f)^2 + \tfrac{1}{2}m_8(v_8)^2 = \tfrac{1}{2}m_W v^2 \,. \qquad (2)$$

Since billiard balls have the same mass $m_W = m_8 = m$, can be eliminated from the equations above. The equations become

$$v_f\cos\varphi + v_8\cos\theta = v \,, \qquad (3a)$$

$$-v_f\sin\varphi + v_8\sin\theta = 0 \,, \qquad (3b)$$

$$(v_f)^2 + (v_8)^2 = v^2 \,. \qquad (3c)$$

Now $\cos(30°) = (\surd 3)/2$ and $\sin(30°) = \tfrac{1}{2}$. Thus equation (3b) yields

$$v_8 = 2v_f\sin\varphi \,. \qquad (4)$$

Using this result to eliminate v_8 from (3a) and (3c), we get

$$v = v_f[(\surd 3)\sin\varphi + \cos\varphi] \,. \qquad (5a)$$

$$v^2 = (v_f)^2[1 + 4\sin^2\varphi] \,. \qquad (5b)$$

We put the expression for v from (5a) into (5b) and get

$$[(\surd 3)\sin\varphi + \cos\varphi]^2 = [1 + 4\sin^2\varphi] \,.$$

Expanding the square on the left-hand side

$$3\sin^2\varphi + (2\surd 3)\sin\varphi\cos\varphi + \cos^2\varphi = 1 + 4\sin^2\varphi \,.$$

Rearranging, this becomes

$$(2\surd 3)\sin\varphi\cos\varphi = 1 + \sin^2\varphi - \cos^2\varphi \,.$$

Recall that $\cos^2\varphi = 1 - \sin^2\varphi$, so that the right-hand side simplifies and we get

$$(2\sqrt{3})\sin\varphi\cos\varphi = 2\sin^2\varphi .$$

Eliminating common terms, we get

$$(\sqrt{3})\cos\varphi = \sin\varphi .$$

Recall that $\tan\varphi = \sin\varphi/\cos\varphi$, so that the above is actually $\tan\varphi = \sqrt{3}$. This yields

$$\varphi = 60°.$$

With this result we can go back to equation (4) and find that

$$v_8 = (\sqrt{3})v_f .$$

Using the result for φ and equation (5a) we get

$$v = 2v_f .$$

Thus $v_8 = (\sqrt{3})v/2$.

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