

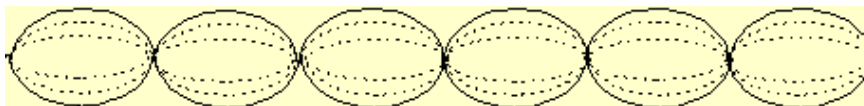
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Physics 1120: Standing Waves and Sound Level Solutions

Sound Level

1. A wire of length 4.35 m and mass 137 g is under a tension of 125 N. A standing wave has formed which has seven nodes including the endpoints. What is the frequency of this wave? Which harmonic is it? What is the fundamental frequency? The maximum amplitude at the antinodes is 0.0075 m, write an equation for this standing wave.

First we sketch the standing wave.



The equations for a string fixed at both ends are $f_n = n \frac{v}{2L}$ and $\lambda_n = \frac{2L}{n}$. Examining the sketch, we see that $n = \# \text{node} - 1 = 6$, so that this is the **sixth** harmonic. We are given L , so we need the speed of the wave v to determine f_n . The

speed of the wave can be found from the formula $v = \sqrt{\frac{F_{\text{tension}}}{\mu}}$, where μ is the linear density given by $\mu = \frac{M}{L}$. Using the given data, the speed may be computed

$$v = \sqrt{\frac{125 \text{ N}}{(0.137 \text{ kg} / 4.35 \text{ m})}} = 63.0 \text{ m/s}$$

Hence, $f_6 = 6 \frac{(63.0 \text{ m/s})}{2(4.35 \text{ m})} = 6 \times 7.24 \text{ Hz} = 43.4 \text{ Hz}$.

The fundamental, or $n = 1$, frequency is $f_1 = 7.24 \text{ Hz}$.

The equation of a standing wave is given by $y_n = A_n \sin\left(\frac{2\pi}{\lambda_n} x\right) \cos(2\pi f_n t)$. In this case, we have

$$y_6 = 0.0075 \sin(4.33x) \cos(273t),$$

where $\lambda_n = \frac{2L}{n}$ has been used.

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2. A string fixed at one end only is vibrating in its third harmonic. The wave function is $y(x,t) = 0.02 \sin(3.13x) \cos(512t)$,

where y and x are in metres and t is in seconds. (a) What is the wavelength of the wave? (b) What is the length of the string. (c) What is the speed of the transverse wave in the string?

The equation of a standing wave is given by $y_n = A_n \sin\left(\frac{2\pi}{\lambda_n} x\right) \cos(2\pi f_n t)$. We are told that this is the third harmonic, so $n = 3$.

(a) Since $\lambda_n = \frac{4L}{n}$ for strings with one free end, we have $\lambda_3 = \frac{4L}{3} = \frac{2\pi}{3.13}$. From this we determine that the wavelength, $\lambda_3 = 2.01$ m.

(b) We also find the length of the string to be $L = 1.51$ m.

(c) The speed of the wave is given by $v = \lambda_n f_n$. Comparing the equation of the standing wave with the particular given wave, we see that $2\pi f_n = 512$. Thus

$$v = \frac{2\pi}{3.13} \times \frac{512}{2\pi} = 164 \text{ m/s}$$

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3. Three successive resonance frequencies for a certain string are 175, 245, and 315 Hz. (a) Find the ratio of these three modes. (b) How can you tell that this string has an antinode at one end? (c) What is the fundamental frequency? (d) Which harmonics are these resonance frequencies? (e) If the speed of transverse waves on this string is 125 m/s, find the length of the string?

(a) The ratio of these three frequencies is 175:245:315, or 535:735:935, or 5:7:9.

(b) For a string fixed at both ends, the resonant frequencies are given by $f_n = n \frac{v}{2L}$, where $n = 1, 2, 3, 4, \dots$ For a string fixed at only one end, the resonant frequencies are given by $f_n = n \frac{v}{4L}$, where $n = 1, 3, 5, 7, \dots$ Since the given sequence has only odd numbers, we may conclude that the string is fixed at only one end.

(c) The fundamental frequency is the greatest common factor of the sequence, so $f_1 = 35.0$ Hz.

(d) Since $f_1 = \frac{v}{4L}$, we can rearrange this equation to find the length,

$$L = \frac{v}{4f_1} = \frac{125 \text{ m/s}}{4 \times 35 \text{ Hz}} = 0.893 \text{ m}$$

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4. Having no ruler at hand, you decide to determine the length of a hollow open-ended tube by holding one end next to a

sound source and varying the frequency of the sound. The lowest frequency that causes resonance in the tube is $f = 420$ Hz. Assuming that the speed of sound is 343 m/s, what is the length of the tube? Sketch the standing wave.

For a tube open at one end, the standing waves frequencies obey the formula $f_n = n \frac{v}{4L}$ where $n = 1, 3, 5, \dots$. The lowest frequency corresponds to $n = 1$. Since we are given v and L , we have

$$f_1 = 420 \text{ Hz} = 1(343 \text{ m/s}) / 4L.$$

Solving for L yields,

$$L = (343 \text{ m/s}) / 4(420 \text{ Hz}) = 0.408 \text{ m}.$$

The tube is 41 cm long.

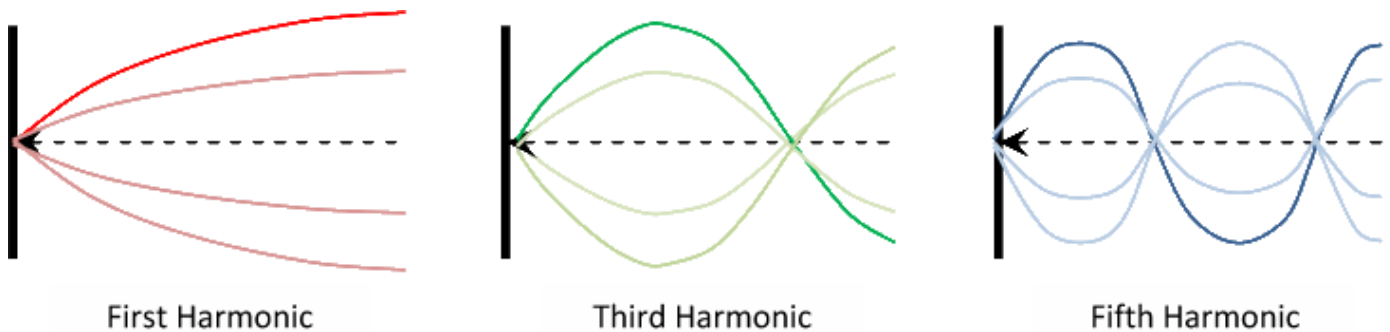
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5. In an archery contest, a 1.15-m arrow sticks in the target and vibrates. If the speed of the waves in that arrow is 150 m/s, what are the three lowest resonance frequencies for the that vibration? Sketch the standing waves.

The patterns will be of a standing wave with one fixed and one free end.

Such patterns obey the formula $f_n = n \frac{v}{4L}$, where $n = 1, 3, 5, 7, \dots$. Solving $f_n = n(31.61 \text{ Hz})$. so the lowest three frequencies are $f_1 = 31.61$ Hz, $f_3 = 97.83$ Hz, and $f_5 = 163.04$ Hz.

The first three patterns are shown below.

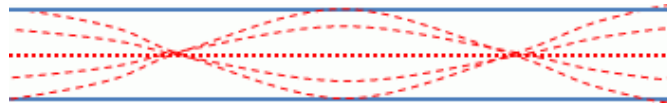


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6. A narrow hollow tube of length $L = 3.50$ m is open at both ends and sits on two sawhorses. A wind is blowing and the tube is "moaning". What is the lowest possible frequency for the sound produced? Sketch the standing wave. Take the speed of sound to be 340 m/s.



The open ends will be antinodes and the location of the sawhorses must be nodes, so the lowest frequency standing wave in the tube looks like

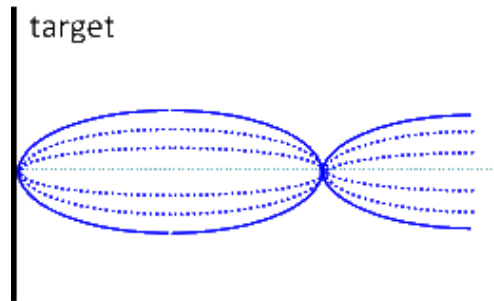


which is the $n = 2$ harmonic. The frequency of the standing wave is therefore

$$f = nv / 2L = (2)(340 \text{ m/s}) / (2)(3.50 \text{ m}) = 97.1 \text{ Hz}$$

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7. While watching an archery show you taped earlier, you notice that when the 1.05 m arrows hit a target they vibrate with the following pattern. Using frame advance on your VCR you deduce that the arrow vibrates 110 times per second. What is the speed of the waves travelling through the arrow?



The patterns will be of a standing wave with one fixed and one free end. This is the third harmonic.

Such patterns obey the formula $f_n = n \frac{v}{4L}$, where $n = 1, 3, 5, 7, \dots$. Here $n = 3$. Solving for the speed of the wave $v = 4Lf_n/n = (4)(1.05 \text{ m})(110 \text{ Hz})/3 = 154 \text{ m/s}$.

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Sound Level

8. A typical speaker diaphragm vibrates with a maximum displacement of 2.00 mm. Assuming that this is also the maximum displacement of the nearby air molecules, find the maximum pressure amplitude. Take the frequency of the speaker to be $f = 3000 \text{ Hz}$, the density of the air to be $\rho = 1.29 \text{ kg/m}^3$, and the speed of sound to be 340 m/s.

maximum pressure amplitude is given by the formula

$$p_0 = \rho \omega v \delta_0 = (1.29 \text{ kg/m}^3)(2\pi \times 3000 \text{ Hz})(340 \text{ m/s})(0.002 \text{ m}) = 16.5 \times 10^3 \text{ Pascal}$$

Since ordinary atmosphere pressure is approximately 1×10^5 , this is about a 16% change in pressure.

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9. When one student is doing an exam in an otherwise very quiet room, the sound level is 45 dB. What is the intensity of the noise produced by the student? If there are 30 equally noisy students in the room, and assuming that you are the same distance from all the students, what would the new sound level be?

Sound intensity is related to sound level by the formula, $I = I_0 \times 10^{(\beta/10)}$. So the intensity of one student is

$$I_{\text{student}} = (1 \times 10^{-12} \text{ W/m}^2)10^{(45/10)} = 3.162 \times 10^{-8} \text{ W/m}^2.$$

For incoherent sound sources, intensity adds. So with the 30 students the sound intensity is

$$I_{\text{group}} = 30 I_{\text{student}} = 9.487 \times 10^{-7} \text{ W/m}^2.$$

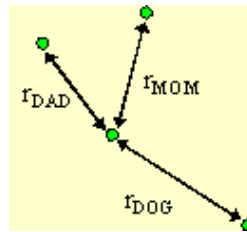
Using the formula for sound level,

$$\beta = 10\log(I/I_0) = 10\log(9.487 \times 10^{-7} / 1 \times 10^{-12}) = 59.8 \text{ dB}.$$

The sound level in the classroom reaches 60 dB.

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10. You've been out very late and when you come home your parents are very angry and start shouting at you. This upsets the family dog who starts howling. The diagram below shows their positions with you in the middle. The distances are $r_{\text{DAD}} = 1.20 \text{ m}$, $r_{\text{MOM}} = 1.35 \text{ m}$, and $r_{\text{DOG}} = 2.00 \text{ m}$. The power in their voices are respectively, $P_{\text{DAD}} = 1.25 \text{ mW}$, $P_{\text{MOM}} = 0.85 \text{ mW}$, and $P_{\text{DOG}} = 1.00 \text{ mW}$. Find the intensity of sound from each source at your position. Treat the sources as incoherent (in the physical sense) and find the sound level. Be sure to include the effects of the normal background sound level of 40 dB. $I_0 = 10^{-12} \text{ W/m}^2$.



These are all incoherent sound sources, so the intensities add

$$I_{\text{total}} = I_{\text{DAD}} + I_{\text{MOM}} + I_{\text{DOG}} + I_{\text{background}}.$$

Assuming that we are dealing with spherical sound sources, $I = P / 4\pi r^2$, so

$$I_{\text{DAD}} = (1.25 \times 10^{-3} \text{ W}) / 4\pi(1.2 \text{ m})^2 = 6.908 \times 10^{-5} \text{ W/m}^2,$$

$$I_{\text{MOM}} = (0.85 \times 10^{-3} \text{ W}) / 4\pi(1.35 \text{ m})^2 = 3.711 \times 10^{-5} \text{ W/m}^2, \text{ and}$$

$$I_{\text{DOG}} = (1.00 \times 10^{-3} \text{ W}) / 4\pi(2 \text{ m})^2 = 1.989 \times 10^{-5} \text{ W/m}^2.$$

The background intensity is found from the background sound level using

$$I_{\text{background}} = I_0 \times 10^{\beta/10} = (1 \times 10^{-12} \text{ W/m}^2) \times 10^6 = 1 \times 10^{-6} \text{ W/m}^2.$$

Thus the total intensity is $I_{\text{total}} = 12.71 \times 10^{-5} \text{ W/m}^2$ and the sound level at your position is,

$$\beta = 10\log(I_{\text{total}}/I_0) = 10\log[(12.71 \times 10^{-5})/(1 \times 10^{-12})] = 81.0 \text{ dB}.$$

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Physics

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