Physics 1120: Waves Solutions

1. A wire of length 4.35 m and mass 137 g is under a tension of 125 N. What is the speed of a wave in this wire? If the tension is doubled, what is the speed? If the mass is doubled?

   The speed of a wave on a string is given by the formula $v = \sqrt{F_{tension}/\mu}$, where is the linear density given by $\mu = M/L$. Thus the speed is

   $$v = \sqrt{125N / (0.137kg/4.35m)} = 63.0\, m/s$$.

   If we double the tension, v = 89.1 m/s .

   If we double the mass, v = 44.5 m/s .

   Top

2. A wave on a string has the formula $y = 0.030\sin(0.55x - 62.8t + \pi/3)$. What is the wavelength, frequency, period, phase constant, and speed of the wave. The string has a linear density μ = 0.020 kg/m. What is the tension in the string? What is the rate of energy flow of the wave? Sketch the reference circle, the snapshot graph at $t = 0$, and the history graph at $x = 0$.

   First recall that the formula for a wave on a string is given by $y = A\sin(kx \pm \omega t + \varphi_0)$ Thus, by inspection, we have A = 0.030 m, $\frac{2\pi}{\lambda} = 0.55$, $\frac{2\pi}{T} = 62.8$, and $\varphi_0 = \pi/3$. Solving for λ and T yields λ = 11.4 m and T = 0.100 s.

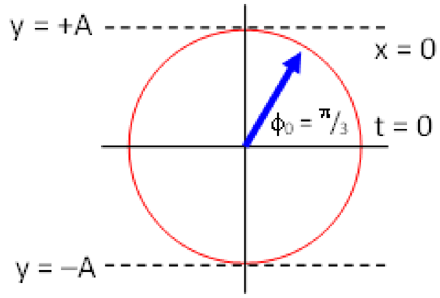
   We know that frequency is given by f = 1/T = 10.0 Hz.

   As well, the speed of the wave is given by v = λ/T = 114 m/s.

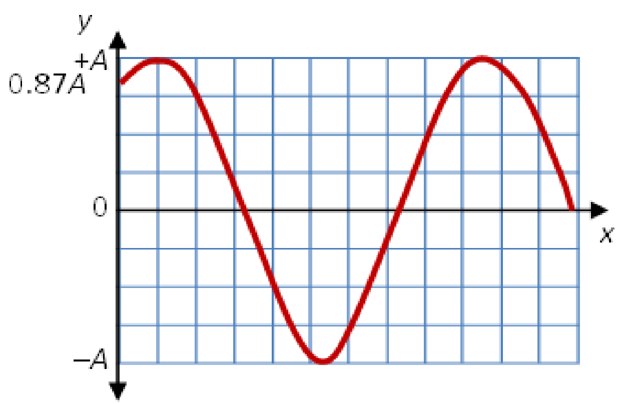
   To find the tension in the string, we take $v = \sqrt{F_{tension}/\mu}$ and rewrite it as $F_{tension} = \mu v^2$. For the given values, $F_{tension}$ = 260 N.

   The rate of energy flow, or energy per unit time, or power, is given by the formula $P = \frac{1}{2}\mu v \omega^2 A^2$, where ω = 2πf. For the given values, P = 4.05 Watts.

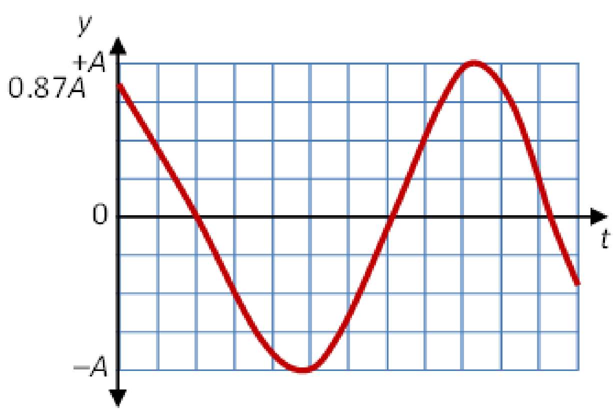
   The phase constant puts the vector in the first quadrant.



The snapshot graph, a plot of y versus x, for $t = 0$ can be sketched by referring to the phasor or reference circle diagram. The y-component of the amplitude is positive at $x = 0$, $y = A\sin(\pi/3) = 0.866A$. As x increases, the phase $0.55x - 62.8t + \pi/3$ also increases and the vector rotates counter clockwise to maximum y amplitude A. The snapshot graph therefore looks like:



The history graph, a plot of y versus t, for $x = 0$ can be sketched by referring to the phasor or reference circle diagram. The y-component of the amplitude is positive at $t = 0$, $y = A\sin(\pi/3) = 0.866A$. As t increases, the phase $0.55x - 62.8t + \pi/3$ decreases and the vector rotates clockwise to zero amplitude. The history graph therefore looks like:



Top

---

3. On a point on a string, a peak of a harmonic wave is observed to pass every 0.050 s. The distance between peaks is 0.75 m. The height of the peak is 0.025 m. Assume that the wave is moving to the left. What is the equation of this wave (take $t = 0$ at the first peak)? What is the speed of the wave? The string has a linear density $\mu = 0.020$ kg/m. What is the tension in the string? What is the rate of energy flow of the wave? Sketch the reference circle, the snapshot graph at $t = 0$, and the history graph at $x = 0$.

Since the peak of the harmonic wave is observed to pass every 0.050 s, T = 0.050 s. Since the distance between successive peaks is one wavelength, $\lambda = 0.75$ m. The amplitude of a wave is given by the height of the peak, so A = 0.025 m.

Next recall that the formula for a wave on a string is given by $y = A\sin(kx \pm \omega t + \varphi_0)$. For waves moving to the left, we need the + sign. Since we start the clock $t = 0$ at a peak, the phase constant φ_0 is π. So the required equation must be

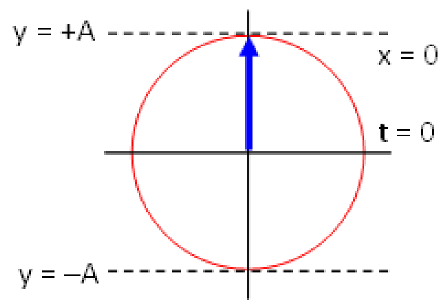
$$y = 0.025\sin(2\pi x/0.075 + 2\pi t/0.050 + \pi).$$

The speed of the wave is given by $v = \lambda/T = 15.0$ m/s.

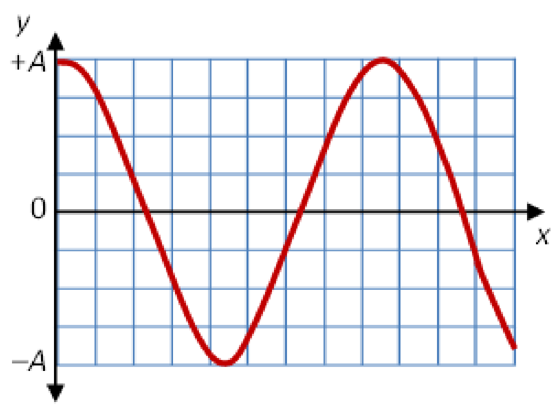
From the previous question, we saw that $v = \sqrt{F_{tension}/\mu}$ could be rearranged to give $F_{tension} = \mu v^2$. For the given values, $F_{tension} = 0.045$ N.

The rate of energy flow, or energy per unit time, or power, is given by the formula $P = \frac{1}{2}\mu v \omega^2 A^2$, where $\omega = 2\pi f$. For the given values, P = 0.296 Watts.

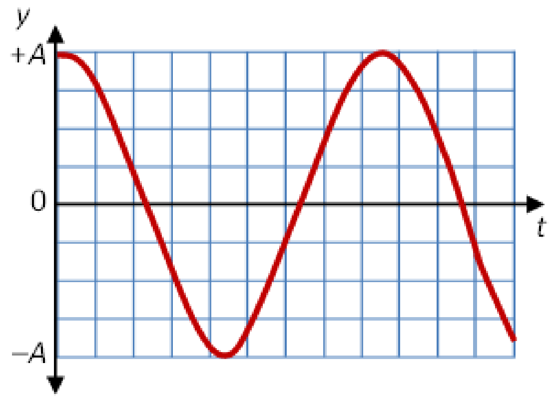
The phase constant puts the vector on the y-axis between the first and second quadrants.



The snapshot graph, a plot of y versus x, for $t = 0$ can be sketched by referring to the phasor or reference circle diagram. The y-component of the amplitude is maximum, A. As x increases, the phase $8.38x + 126t + \pi$ also increases and the vector rotates counter clockwise to zero amplitude. The snapshot graph therefore looks like:

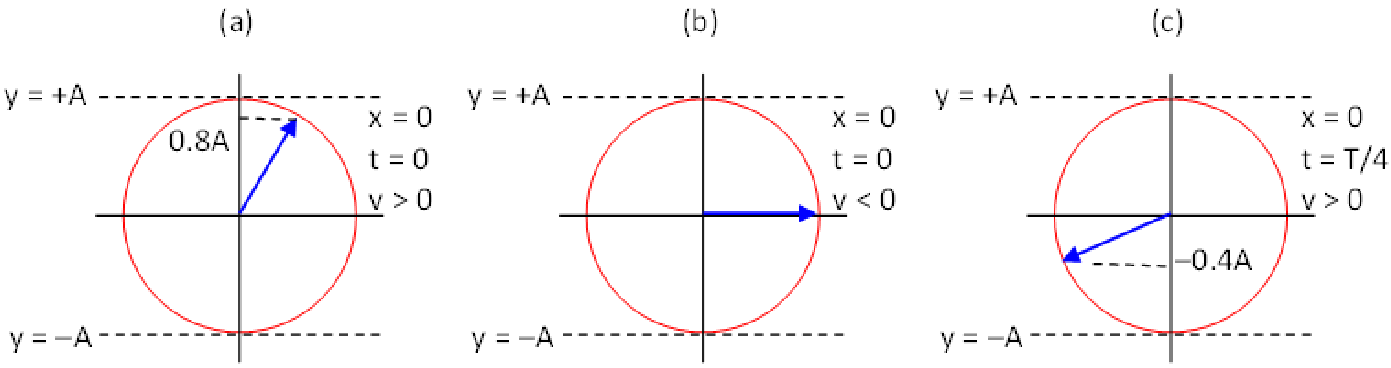


The history graph, a plot of y versus t, for $x = 0$ can be sketched by referring to the phasor or reference circle diagram. The y-component of the amplitude is maximum, A. As t increases, the phase $8.38x + 126t + \pi$ also increases and the vector rotates counter clockwise to zero amplitude. The history graph therefore looks like identical to the snapshot graph:



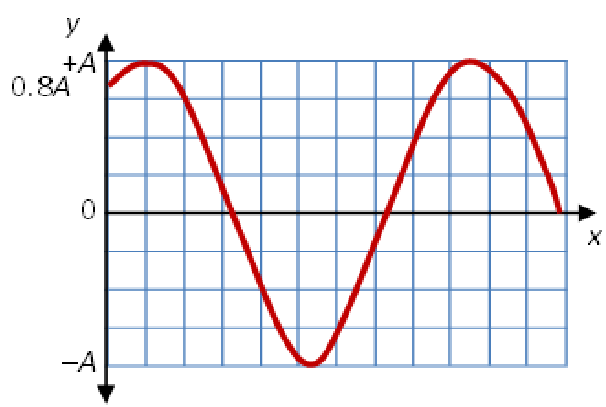
Top

4. Sketch the associated snapshot and history graphs for the following reference circle diagrams. What is the phase constant φ_0 in each case?

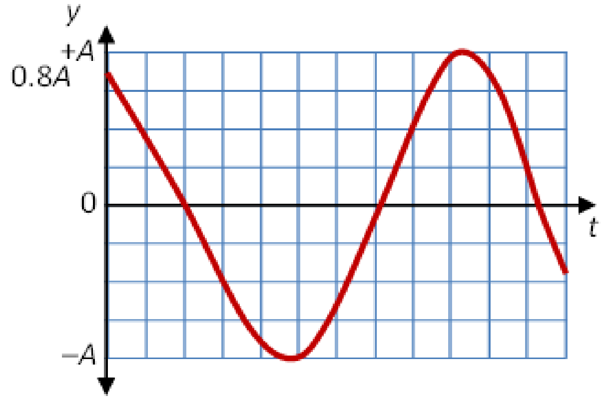


(a) The phase constant φ_0 is the angle measured counter clockwise from the positive x-axis to the vector when the diagram shows $x = 0$ and $t = 0$. Simple trigonometry indicates that $\varphi_0 = \arcsin(0.8) = 53.1°$ or 0.927 rad.

The vector in the phasor diagram rotates counter clockwise with increasing x in the y versus x, or snapshot graph. So here the y starts at $0.8A$ and rises to maximum amplitude A. The snapshot graph looks like.

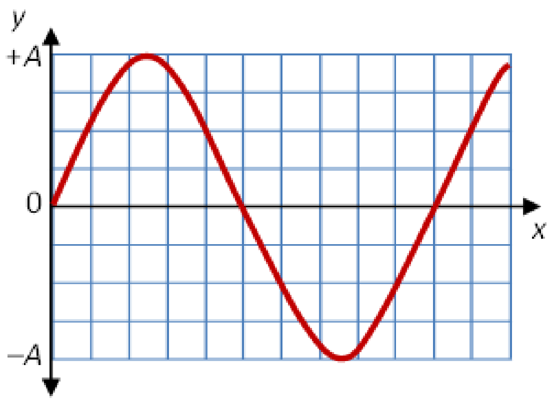


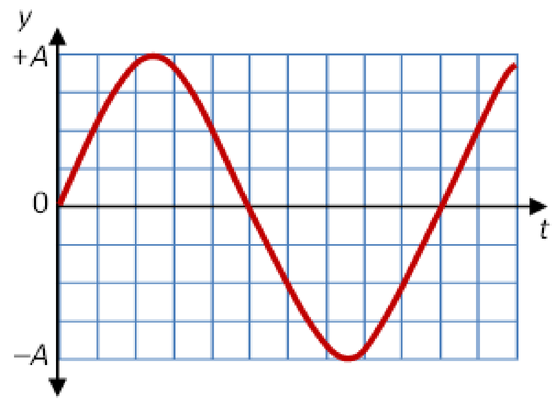
The vector in the phasor diagram rotates clockwise with increasing t in the y versus t, or history graph if the wave is moving to the right as it does here. So here the y starts at $0.8A$ and falls to equilibrium, zero amplitude. The history graph looks like.



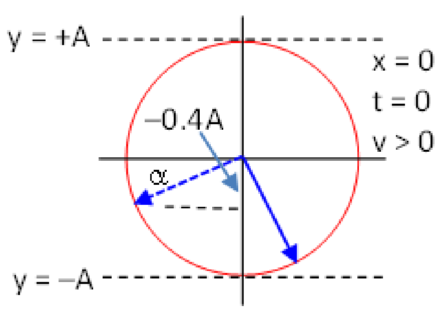
(b) The phase constant φ_0 is zero.

Since the wave is moving to the left, the snapshot and history graphs look identical. The vector in the phasor diagram rotates counter clockwise with increasing x or with increasing t. The initial amplitude is zero and rises to a maximum.

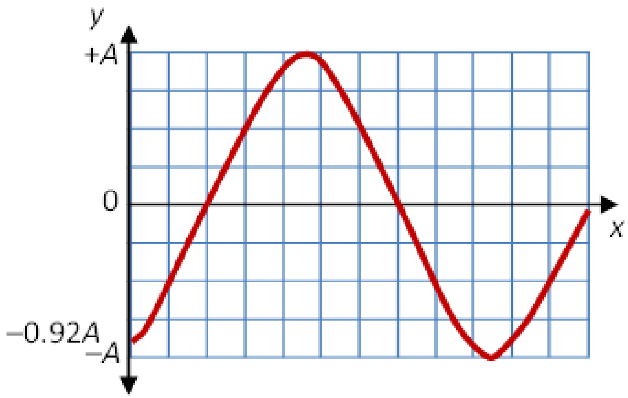


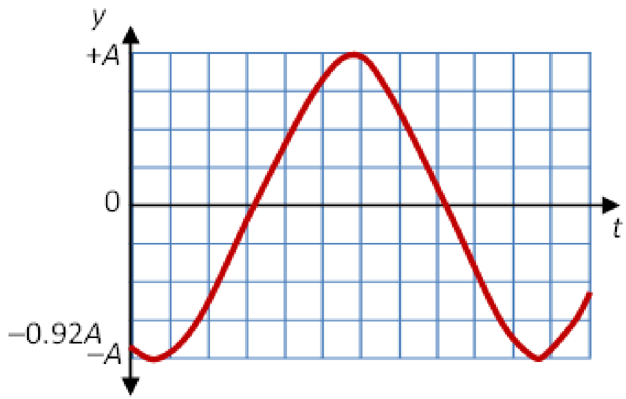


(c) Note that this phasor diagram is not given at $t = 0$ but at the later time $t = T/4$. The phase constant φ_0 is the angle measured counter clockwise from the positive x-axis to the vector when the diagram shows the vector at $x = 0$ and $t = 0$. Since this wave is moving to the right, the vector in the phasor diagram would have already moved one-quarter of the circle clockwise in this time ($T/4$ &rarrow; $2\pi/4$). We need to rotate the vector back one-quarter of the circle counter clockwise to see what it would look like at $t = 0$. This is shown below.



From the original information it is easy to determine the angle $\alpha = \arcsin(0.4) = 23.58° = 0.412$ *rad*. Thus the phase constant $\varphi_0 = 180° + \alpha + 90° = 293.58°$ or 5.124 *rad*. At $x = 0$ and $t = 0$, the amplitude is $y = A\sin(293.58°) = -0.917A$. The vector in the phasor diagram will rotate counter clockwise with increasing x in the snapshot graph heading to zero amplitude. As the wave is travelling to the right, the vector in the phasor diagram will rotate clockwise with increasing t in the history graph heading to the minimum amplitude, $-A$. Both graphs are shown below.

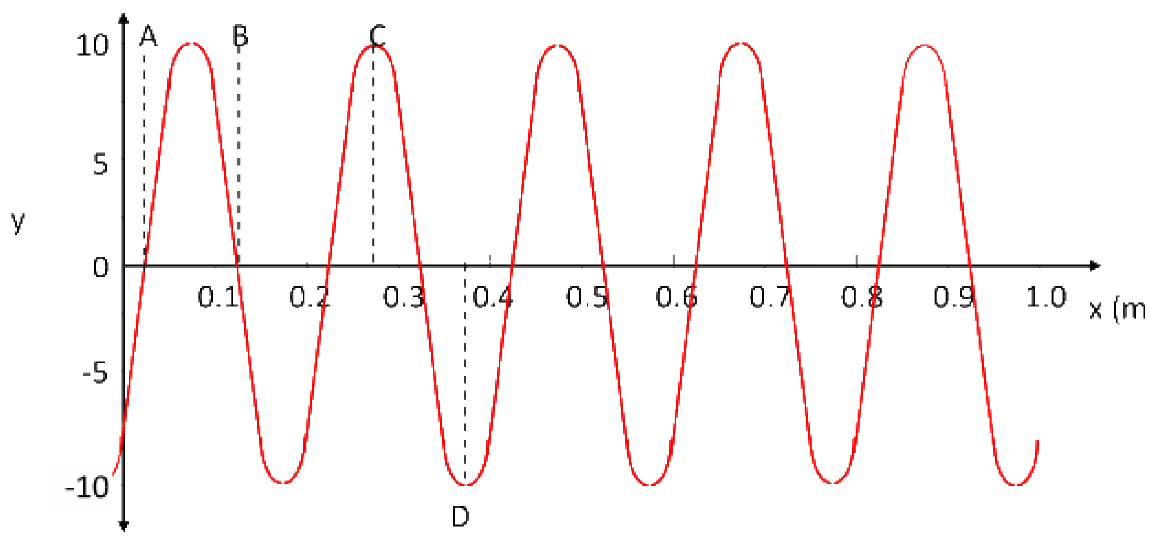




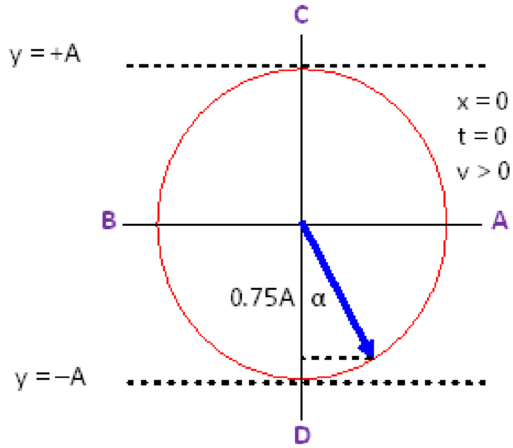
Top

---

5. Consider the snapshot graph below for a wave moving to the right at t = 0. What is the phase constant φ_0? Sketch the reference circle at the indicated points *A*, *B*, *C*, and *D*. Where exactly does point A occur? Point B?



The phase constant φ_0 is the angle measured counter clockwise from the positive x-axis to the vector when the phasor diagram shows $x = 0$ and $t = 0$. The snapshot graph above shows that the initial amplitude is -7.5 or $-0.75A$ where $A = 10$. The graph next has zero amplitude. Since the vector in a phasor diagram moves counter clockwise with increasing x, the vector must be in the fourth quadrant as shown in the diagram below.



We can find the angle α in the phasor diagram, $\alpha = \arccos(0.75) = 41.41°$ or 0.7227 *rad*. The phase constant is therefore $\varphi_0 = \alpha + 270° = 311.41° = 5.435$ *rad*.

The locations of points *A*, *B*, *C*, and *D* are shown on the phasor diagram above.

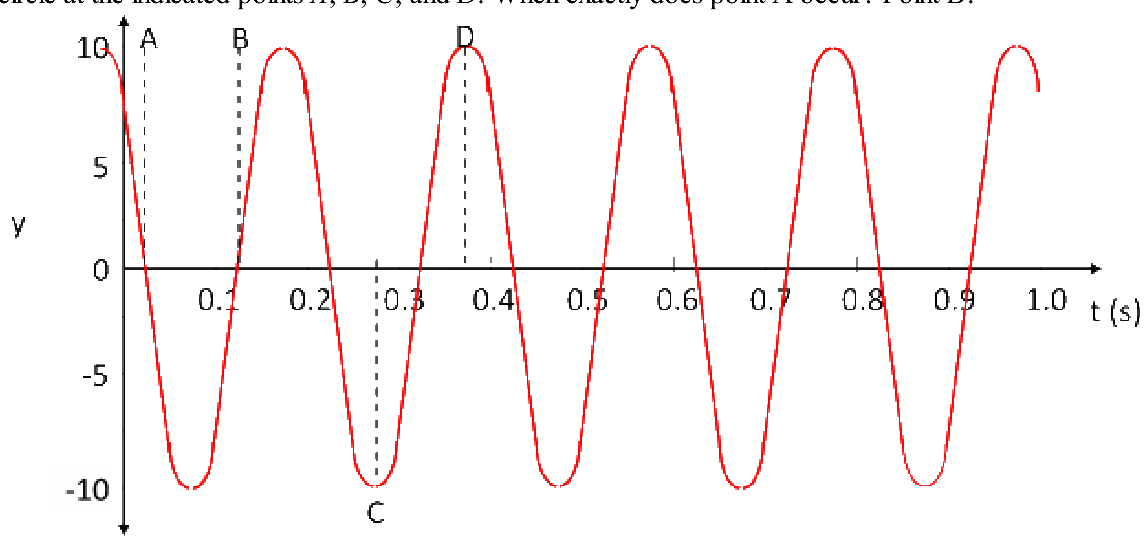
To find the exact value of *x* for point *A*, note that one full rotation of the vector about the reference circle is equivalent to a shift of one wavelength λ on the snapshot graph. The vector needs to rotate counter clockwise $90° - \alpha = 48.59°$ to get to point *A*. Therefore the location is $x_A = (48.59° / 360°)\lambda$. From the snapshot graph we can read that $\lambda = 0.20$ m. So $x_A = 0.0270$ m.

Similarly, to find the exact value of *x* for point *B*, the vector needs to rotate counter clockwise $90° - \alpha + 180° = 228.59°$. Therefore the location is $x_B = (228.59° / 360°)(0.20) = 0.1270$ m.

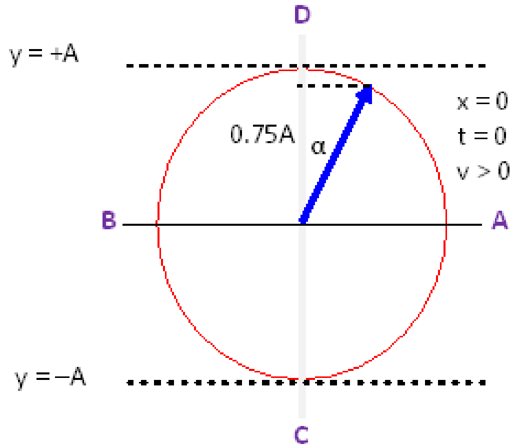
Top

---

6. Consider the history graph below for a wave moving to the right at x = 0. What is the phase constant φ_0? Sketch the reference circle at the indicated points *A*, *B*, *C*, and *D*. When exactly does point A occur? Point B?



The phase constant φ_0 is the angle measured counter clockwise from the positive *x*-axis to the vector when the phasor diagram shows $x = 0$ and $t = 0$. The history graph above shows that the initial amplitude is 7.5 or $0.75A$ where $A = 10$. The graph next has zero amplitude. Since the vector in a phasor diagram moves clockwise with increasing *t* for a wave travelling to the right, the vector must be in the first quadrant as shown in the diagram below.



We can find the angle α in the phasor diagram, α = arccos(0.75) = 41.41° or 0.7227 *rad*. The phase constant is therefore φ_0 = 90° − α = 48.59° = 0.8481 *rad*.

The locations of points *A*, *B*, *C*, and *D* are shown on the phasor diagram above.

To find the exact value of *t* for point *A*, note that one full rotation of the vector about the reference circle is equivalent to a shift of one period *T* on the history graph. The vector needs to rotate clockwise 48.59° to get to point *A*. Therefore the time is t_A = (48.59° / 360°)*T*. From the history graph we can read that *T* = 0.20 s. So t_A = 0.0270 s.

Similarly, to find the exact value of *t* for point *B*, the vector needs to rotate clockwise φ_0 + 180° = 228.59°. Therefore the time is t_B = (228.59° / 360°)(0.20) = 0.1270 s.

Top

Superposition in 1D and 2D

---

7. Two out-of-phase harmonic waves are traveling on a string. They each have an amplitude of 5.0 mm. The resultant wave has an amplitude of 3.5 mm. What is the phase difference between these two waves?

We saw that when two waves on a string interfere, we get another traveling wave whose amplitude is related to the amplitude of the original two waves by

$$A_{superposition} = 2A_o \cos(\delta/2).$$

This may be rearranged to yield

$$\delta = \cos^{-1}\left(A_s / 2A_0\right).$$

Using the given values, we find

$$\delta = \cos^{-1}\left(0.0035 / 2\times 0.005\right) = 139° = 2.43\ rad.$$

Top

---

8. Two speakers are in a line but one speaker is a distance d behind the other. You are 4.50 m in front of the nearest speaker. The sound is wavelength $\lambda = 1.50$ m.
(a) The speakers are in phase. What is the smallest non-zero d if you hear constructive interference?
(b) The speakers are in phase. What is the smallest non-zero d if you hear destructive interference?
(c) The front speaker leads the back speaker by 90°. What is the smallest non-zero d if you hear constructive interference?
(d) The front speaker leads the back speaker by 90°. What is the smallest non-zero d if you hear destructive interference?

(a) The equation for constructive interference is

$$\delta = 2\pi\Delta x/\lambda + \Delta\varphi_0 = n2\pi \qquad n = 0, 1, 2, 3, \ldots \qquad \text{C.I.}$$

The path difference Δx is d in this question and since the speakers are in phase $\Delta\varphi_0 = 0$. The above equation can be simplified to

$$d/\lambda = n \qquad n = 0, 1, 2, 3, \ldots \qquad \text{C.I.}$$

The smallest non-zero d for constructive intereference occurs for $n = 1$, so $d = \lambda = 1.50$ m.

(b) Similarly the general equation for Destructive Interference is given by

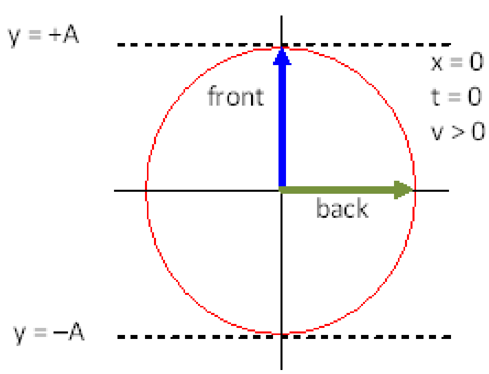
$$\delta = 2\pi\Delta x/\lambda + \Delta\varphi_0 = m\pi \qquad m = 1, 3, 5, \ldots \qquad \text{D.I.}$$

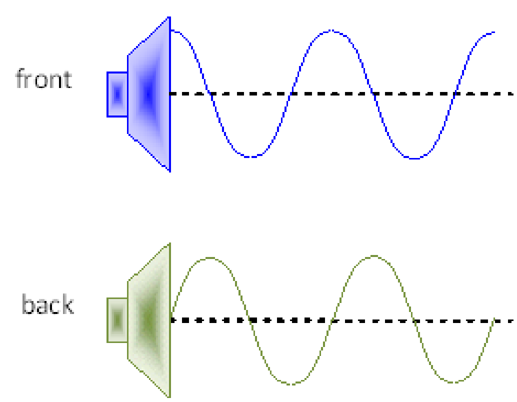
Simplifying again

$$2d/\lambda = m \qquad m = 1, 3, 5, \ldots \qquad \text{D.I.}$$

The smallest non-zero d for destructive intereference occurs for $m = 1$, so $d = \frac{1}{2}\lambda = 0.75$ m.

(c) Leading means the vector for the front speaker is 90° counter clockwise from the vector for the back speaker in the phasor diagram (shown below). For convenience, we can arbitrarily set the vectors at 90° and 0° in the phasor diagram. Since we draw a snapshot graph, the vectors in the phasor diagram rotate counter clockwise. The snapshot for the front speaker starts at maximum amplitude and will drop to zero. The back speaker starts at zero amplitude and rises to maximum. The diagram below shows the phasor diagram and the sound wave coming from each speaker but with the speakers shown side by side.





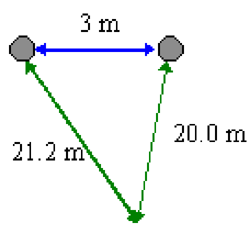
Clearly, the two waveforms will line up peak to peak, i.e. interfere constructively, when the back speaker is moved back ¼λ or $d = 0.375$ m.

(d) Similarly, the two waveforms will line up peak to trough, i.e. interfere destructively, when the back speaker is moved back ¾λ or $d = 1.125$ m.

Note in this problem, the location of the observor is irrelevant.

Top

9. Two loudspeakers are located 3.00 m apart on the stage of an auditorium. A detector is placed 20.0 m from one speaker and 21.2 m from the other. A signal generator drives the two speakers in phase and sweeps through the range (2 KHz − 20 KHz). Assume that the speed of sound in air is 343 m/s.
(a) What is lowest note (frequency) for which destructive interference is a maximum?
(b) What is highest note for which destructive interference is a maximum?
(c) What is the lowest note for which constructive interference is a maximum?
(d) What is the highest note for which constructive interference is a maximum?



(a) & (b) Since the sounds are in phase, destructive interference occurs when the difference in position is an odd number of half wavelengths,

$$\text{D.I.} \qquad \Delta x / \lambda = \tfrac{1}{2}n, \qquad \text{where } n = 1, 3, 5, \ldots$$

The difference in position is $\Delta x = 21.2\text{ m} - 20.0\text{ m} = 1.2\text{ m}$. We can use the relation $v = \lambda f$, to eliminate λ in favour of f. Thus we get

$$\text{D.I.} \qquad f = nv / 2\Delta x = n(142.9\text{ Hz}), \qquad \text{where } n = 1, 3, 5, \ldots$$

We next need to find the smallest and largest odd values of n in the given range of frequencies of 2,000 to 20,000 Hz. We find $n = 15$ yields $f = 2{,}144$ Hz and $n = 139$ yields $f = 19{,}865$ Hz.

(c) & (d) Similarly, constructive interference occurs when the difference in position is an integral number of full wavelengths,

$$\text{C.I.} \qquad \Delta x / \lambda = n, \qquad \text{where } n = 0, 1, 2, 3, \ldots$$

Using the difference in position is $\Delta x = 1.2$ m. and the relation $v = \lambda f$, we get

$$\text{C.I.} \qquad f = nv / \Delta x = n(285.8\text{ Hz}), \qquad \text{where } n = 0, 1, 2, 3, \ldots$$

We next need to find the smallest and largest values of n in the given range of frequencies of 2,000 to 20,000 Hz. We find $n = 14$ yields $f = 2{,}001$ Hz and $n = 69$ yields $f = 19{,}722$ Hz.

Top

10. Two stereo speakers, driven by the amplifier so that the sources are in phase, are arranged so that the sound is loudest at your position. What is the minimum non-zero distance that you are farther away from one speaker than the other? The frequency of the tone from the speakers is 750 Hz and the speed of sound in air is 340 m/s.

Loudest means constructive interference is occurring. There is no phase difference so the equation for superposition is

$$\delta = 2\pi\Delta x/\lambda = n2\pi \qquad n = 0, 1, 2, 3, \ldots \qquad \text{C.I.}$$

We can make this an equation for frequency rather than wavelength by using the equation $v = \lambda f$ to eliminate λ in the equation above.

$$\delta = 2\pi\Delta x f/v = n2\pi \qquad n = 0, 1, 2, 3, \ldots \qquad \text{C.I.}$$

Rearranging to isolate Δx yields

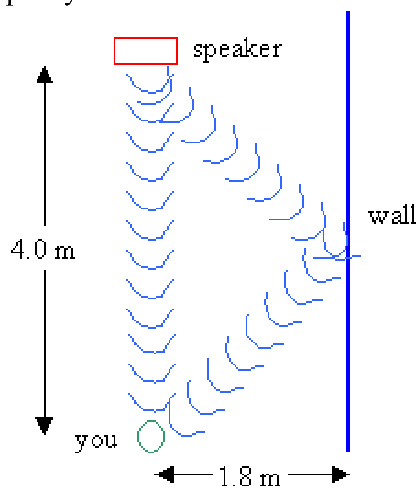
$$\Delta x = nv/f \qquad n = 0, 1, 2, 3, \ldots \qquad \text{C.I.}$$

The minimum non-zero distance occurs when $n = 1$, and we find $\Delta x = 1 \times (343 \text{ m/s}) / (750 \text{ Hz}) = 0.453$ m.

Top

---

11. As shown in the diagram below, a speaker plays a single frequency sound. On its way to you, some of the sound echoes (reflects) off a wall. As a result, the sound is quieter than it should be.
(a) If the wall is flexible, what is the lowest frequency that creates the lowest sound intensity at your position?
(b) If the wall is stiff, what is the lowest frequency that creates the lowest sound intensity at your position?



The speed of sound is 340 m/s. The lowest sound intensity occurs for destructive interference. If the wall is flexible, when the sound reflects the wave will not change phase. Thus the condition for destructive inference is

$$\text{D.I.} \qquad \Delta x / \lambda = \tfrac{1}{2}n, \qquad \text{where } n = 1, 3, 5, \ldots$$

We need to use the Pythagorean Theorem to get the difference in position, $\Delta x = 2[(1.8 \text{ m}) + (2.0 \text{ m})]^{\frac{1}{2}} - 4.0 \text{ m} = 1.381$ m. We can use the relation $v = \lambda f$, to eliminate λ in favour of f. Thus we get

$$\text{D.I.} \qquad f = nv / 2\Delta x = n(123 \text{ Hz}), \qquad \text{where } n = 1, 3, 5, \ldots$$

The lowest frequency is therefore 123 Hz.

Now if the wall is stiff, the phase of one wave is flipped, that is it is moved out of phase by one half wavelength. We must account for this initial difference

$$\text{D.I.} \qquad \Delta x / \lambda + \tfrac{1}{2} = \tfrac{1}{2}n, \qquad \text{where } n = 1, 3, 5, \ldots$$

Or in terms of frequency

$$\text{D.I.} \qquad f = \tfrac{1}{2}[n - 1]v / \Delta x = [n - 1](123 \text{ Hz}), \qquad \text{where } n = 1, 3, 5, \ldots$$

Here the $n = 1$ gives an unphysical solution, and the lowest frequency occurs for $n = 3$ and is 246 Hz.

Top

Doppler Shift

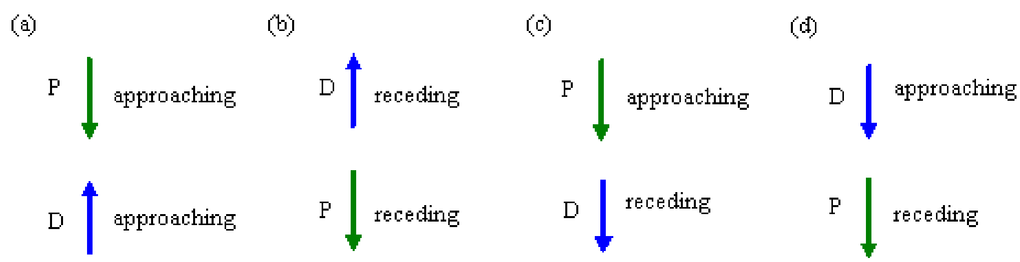
---

12. A driver travels north on a highway at a speed of 25 m/s. A police car, driving south at a speed of 40 m/s, approaches with its siren sounding at a base frequency of 2500 Hz. (a) What frequency is heard by the driver as the police car approaches? (b) What frequency is heard by the driver after the police car passes him? If the driver had been travelling south, what would your results have been for (a) and (b)? The speed of sound in air is v = 340 m/s.

The formula for the Doppler Shift is given by

$$f_{shift} = f_{siren} \, [(1 \pm u_{driver}/v)/(1 \pm u_{police}/v].$$

To use the above equation, we need to know how the driver and police constable are moving relative to one another, as is shown in the diagram below.



(a) f_a = (2500 Hz)[1 + 25/340]/[1 − 40/340] = 3042 Hz .

(b) f_b = (2500 Hz)[1 − 25/340]/[1 + 40/340] = 2072 Hz .

(c) f_c = (2500 Hz)[1 − 25/340]/[1 − 40/340] = 2625Hz .

(d) f_d = (2500 Hz)[1 + 25/340]/[1 + 40/340] = 2401 Hz .

Top

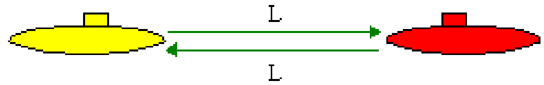
---

13. In sonar, an intermittent high frequency sound pulse is broadcast in all directions. The sound is reflected from solid objects and returns to broadcaster. The time it took for the echo to return and the direction from which the echo came are used to locate nearby objects. This is Echo Location. By measuring the Doppler Shift of the echo, the speed of the object can be found. There is an added complication in that the Doppler Shift occurs twice, once from the source to the receiver, and then from the receiver (now a source of the echo) back to the original source (which is now a receiver of the echo). The echo will have a frequency

$$f' = f_0 \, [(1 \pm u_r/v)/(1 \pm u_s/v][(1 \pm u_s/v)/(1 \pm u_r/v)] \, .$$

A submarine traveling at 17 km/h sends out pulses at 38.7 MHz. The delay in the echo off a second sub has been rapidly decreasing and is currently 75 ms. How far apart are the two subs? If the second sub is moving at 22 km/h, what is the frequency of the returned echo? The speed of sound in seawater is 1.54 km/s.

The sound is emitted by the first sub, hits the second and returns to the first. Sound travels much faster than subs, so we may assume that the distance the sound travels is 2L.



The distance the sound travels is related to t, by

$$d = 2L = v_{sound}\Delta t\ .$$

Thus the distance between the two sub is

$$L = ½(1.54 \times 10^{3}\ m/s)\ (75 \times 10^{-3}\ s) = 57.75\ m\ .$$

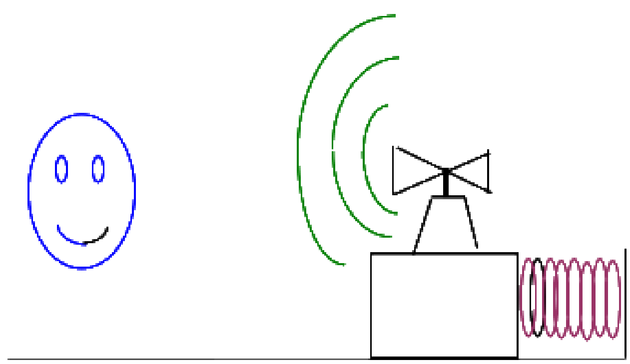
The altered frequency that the first sub hears is

$$f' = (38.7\ MHz)\left[\frac{1+{6.111}/{1540}}{1-{4.722}/{1540}}\right]\left[\frac{1+{4.722}/{1540}}{1-{6.111}/{1540}}\right] = 39.2\ MHz\ .$$

Top

---

14. A siren with a frequency f_0 = 2000 Hz is attached to a block. The siren and block together have a mass of 2.00 kg. The block is attached to a spring of unknown spring constant K. The spring is compressed an unknown distance and then released. The siren and block oscillate back and forth. A listener hears the source as emitting a varying frequency since the siren is moving. The highest frequency that the listener hears is 2060 Hz. The listener also determines that he hears this highest frequency repeat every 1.50 seconds. The speed of sound in air is 343 m/s.
(a) How fast is the block moving when the listener hears the highest frequency?
(b) Where in its motion is the block when the listener hears the highest frequency?
(c) What is the lowest frequency that the listener would hear?
(d) Where in its motion is the block when the listener hears the lowest frequency?
(e) What is the period and angular frequency of the block?
(f) What is the amplitude of the displacement of the block?
(g) What is the spring constant of the spring?



This problem combines SHM with the Doppler Effect.

(a) The moving source is approaching the stationary observor when the observed frequency is highest. The Doppler Effect equation for this case

$$f_{obs} = f_{source}\ (1 + v_{source}/v_{sound}),$$

We can rewrite the above equation to solve for the velocity of the source and use the given numerical values.

$$v_{source} = v_{sound}\ (f_{obs}/f_{source} - 1)$$

With the numbers we have, we get

$$v_{source} = (343 \text{ m/s})(2060/2000 - 1) = 10.29 \text{ m/s}$$

.

(b) Accoding to SHM, the block is moving fastest towards the observor when it passes to the equilibrium moving to the left.

(c) & (d) The Doppler Effect produces the lowest observed frequency when the block is moving away from the observor and passing through equilibrium. The block will have the same speed as in part (a). The equation for this is

$$f_{obs} = f_{source}\,(1 - v_{source}/v_{sound}),$$

and given a result which is 60 Hz below the source frequency, f_{obs} = 1940 Hz.

(e) The peak observed frequency occurs periodically and must match the period of the motion, so T = 1.50 s. The angular frequency is $\omega = 2\pi/T$ = 4.189 Hz.

(f) Maximum velocity is given by the formula $v_{max} = \omega A$. So $A = v_{max} / \omega$ = (10.29 m/s) / (4.189 Hz) = 2.457 m.

(g) Conservation of Energy says $\frac{1}{2}mv_{max}^2 = \frac{1}{2}KA^2$ where K is the spring constant or stiffness. Using $v_{max} = \omega A$, this equation reduces to $m\omega^2 = K$. With our values, $K = (2.00 \text{ kg})(4.189)^2 = 35.10$ N/m.

Top

Beats

---

15. The beat frequency between an unknown tuning fork and a 500 Hz tuning fork is 12 Hz. Compared with a 504 Hz tuning fork, the beat frequency is 16 Hz. What is the frequency of the unknown tuning fork?

Let f be the unknown frequency, the beat frequency is defined as $f_{beat} = |f - f_{known}|$. Therefore, in the two stated cases, we have

$$12 \text{ Hz} = |f - 500 \text{ Hz}|, \text{ and}$$

$$16 \text{ Hz} = |f - 504 \text{ Hz}|.$$

Each of these equations has two possible solutions. However, since the beat frequency increased as the know frequency increases, the solution must be lower than 500 Hz. Hence

$$f = 500 \text{ Hz} - 12 \text{ Hz} = 504 \text{ Hz} - 16 \text{ Hz} = 488 \text{ Hz}.$$

The unknown frequency is 488 Hz.

Top

---

16. You have two 400-Hz tuning forks which you ring together. You drop one fork down a well. Before the fork hits bottom and stops ringing you hear a beat frequency of 15 Hz. How deep is the well?

You hear a beat frequency because the falling tuning fork is Doppler shifted to a lower frequency as it is moving away. In fact it must be at 400 − 15 = 385 Hz.

Using the Doppler Shift equation for the case when the source is moving away,

$$f_{obs} = f_{source} (1 - v_{source}/v_{sound}),$$

we can find the speed of the moving tuning fork by rearranging the above equation.

$$v_{source} = v_{sound} (1 - f_{obs}/f_{source})$$

With the numbers we have and taking v_{sound} = 340 m/s, we get

$$v_{source} = (343 \text{ m/s})(1 - 385/400) = 12.75 \text{ m/s}$$

.

Using simple kinematics for an object dropped from rest under the influence of gravity, the distance the tuning fork fell was

$$d = v_{source}^2 / 2g = (12.75 \text{ m/s})^2/2/(9.81 \text{ m/s}^2) = 8.29 \text{ m.}$$

Questions?mike.coombes@kwantlen.ca

KPU KWANTLEN POLYTECHNIC UNIVERSITY