

- Motion broken into
- Motion of CM
- Motion about CM


## Three Problem Types

- Real Pulleys
- Rolling
- Onset of Rolling


## 1. Real Fixed Pulleys

- Fixed pulley, no a. Interested only in $\tau$ 's and $\alpha$. TFBD only
- Friction btw string and pulley. Tension varies over pulley.
- Treat as two different T’s
- String relates $\mathrm{a}_{\text {block }}$ to $\alpha$ and R
- $R$ is lever arm for $T$ 's ( $T \perp R$ always)


$$
\begin{gathered}
\mathrm{RT}_{1}-\mathrm{RT}_{2}=-\mathrm{I} \alpha \\
\mathrm{I}=1 / 2 \mathrm{MR}^{2} \& \alpha=a / R \\
\mathrm{~T}_{1}-\mathrm{T}_{2}=-1 / 2 \mathrm{Ma} \\
\mathrm{~T}_{1}-M_{1} g=+M_{1} a \\
T_{2}-M_{2} g=-M_{2} a \\
a=\left(M_{2}-M_{1}\right) g /\left(M_{1}+M_{2}+1 / 2 M\right)
\end{gathered}
$$

## 2. Rolling Without Slipping



Moving right \& speeding up
Given $\mu_{\mathrm{s}}$ and $\mu_{\mathrm{k}}$. Find a and $\mathrm{f}_{\mathrm{s}}$.
If $F$ is too big, disk will slip. What is $\mathrm{F}_{\text {slip }}$ ?

- a and $\alpha$ dir ${ }^{n}$ s related
- $a=R \alpha$
- $f_{s}$, not $f_{k}$, not $f_{s}$ max
- Must find formula for $f_{s}$
- Dir ${ }^{n}$ of $f_{s}$ must be consistent with Newton's Laws
- $f_{s}$ and a must be expressed in terms of given quantities

Find consistent a \& $\alpha$

|  | Add all forces but $\mathrm{f}_{\mathrm{s}}$ <br> F, N, \& W cannot produce a torque. Some "other" force must! |
| :---: | :---: |
|  |  |
| $-\mathrm{Rf}_{\mathrm{s}}=-\mathrm{l} \alpha$ |  |
| $\mathrm{I}=1 / 2 \mathrm{MR}^{2} \& \alpha=a / R \Rightarrow f_{s}=1 / 2 \mathrm{Ma}$ |  |
| $\mathrm{F}-\mathrm{f}_{\mathrm{s}}=\mathrm{Ma}$ - \& | $\mathrm{N}-\mathrm{Mg}=0$ |
| $\mathrm{F}-1 / 2 \mathrm{Ma}=\mathrm{Ma}$ | $\Rightarrow \mathrm{a}=2 \mathrm{~F} / 3 \mathrm{M}$ |
| $\mathrm{f}_{\mathrm{s}}=1 / 2 \mathrm{Ma} \Rightarrow$ | = F/3 |

## Will start to slip ...

- Note $f_{s}=F / 3$ so $f_{s} \uparrow$ when $F \uparrow$
- But $f_{s}$ cannot exceed $f_{s}{ }^{\text {max }}$
- $\mathrm{f}_{\mathrm{s}}{ }^{\text {max }}=\mu_{\mathrm{s}} \mathrm{N}=\mu_{\mathrm{s}} \mathrm{Mg}$
- So will slip when $f_{s}=f_{s}{ }^{\text {max }}$ or
- $\mathrm{F} / 3=\mu_{\mathrm{s}} \mathrm{Mg}$ or $\mathrm{F}=3 \mu_{\mathrm{s}} \mathrm{Mg}$


## 3. Onset of Rolling



Note a $\neq R \alpha$. Must find equation for a and equation for $\alpha$. Then $v_{0}+$ at $=R\left(\omega_{0}+\alpha t\right)$. NB scalar eqn.

$f_{k}=M a \quad \& \quad N-M g=0 \quad \& f_{k}=\mu_{k} N$

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\Rightarrow \mathrm{a}=\mu_{\mathrm{k}} \mathrm{~g}
$$

$R f_{k}=I \alpha \& I=1 / 2 M R^{2}$

$$
\Rightarrow \alpha=2 \mathrm{f}_{\mathrm{k}} / \mathrm{MR} \Rightarrow \alpha=2 \mu_{\mathrm{k}} \mathrm{~g} / \mathrm{R}
$$

$v_{0}+\mu \mathrm{gt}=\mathrm{R}\left(\omega_{0}-2 \mu_{\mathrm{k}} \mathrm{gt} / \mathrm{R}\right) \Rightarrow \mathrm{t}=\mathrm{R} \omega_{0} / 3 \mu_{\mathrm{k}} \mathrm{g}$

