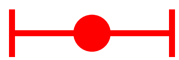
Uncertainty

- Measurement is a range not a single number
- Symbol 
- $L = L \pm \Delta L$, range is $L - \Delta L$ to $L + \Delta L$
  (absolute uncertainty)
- Also write $L = L \pm \ell\,\%$, where $\ell\,\% = \Delta L / L \times 100\%$
  (relative or percent uncertainty)

Significant digits

- ΔL given to 2 digits if leading non-zero digit is a 1 or 2 and only 1 digit otherwise
- e.g. 27.5967 $\pm$ 0.0176 $\Rightarrow$ 27.597 $\pm$ 0.018 (5 SF)
- e.g. 71.8523 $\pm$ 0.4571 $\Rightarrow$ 71.9 $\pm$ 0.5 (3 SF)

Estimating Uncertainty

- Two pieces
- Instrument uncertainty – how well you can read the scale
- Technique uncertainty – how hard it is to do the measurement
- Different people can have different but equally valid estimates of uncertainty

Math Operations with Uncertainty

- Suppose you have A = 16.4 $\pm$ 0.3 and B = 25.1 $\pm$ 0.2
- What is C = A + B?
- Might guess ΔC = ΔA + ΔB, so that C = 41.5 $\pm$ 0.5.
- This ΔC is too big if the uncertainties are independent
- $\Delta C = \sqrt{(\Delta A)^2 + (\Delta B)^2}$, C = 41.5 $\pm$ 0.4

Addition and Subtraction Rule

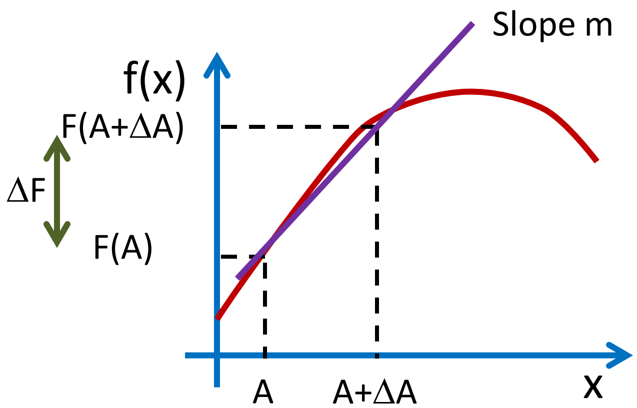
- For any addition or subtraction,
- e.g., if F = A + B − C + D − E,
- $\Delta F = \sqrt{(\Delta A)^2 + (\Delta B)^2 + (\Delta C)^2 + (\Delta D)^2 + (\Delta E)^2}$,
- Always + !!!

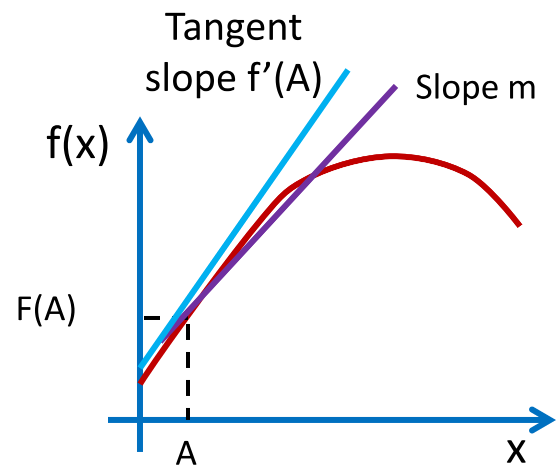
Multiplication and Division Rule

- For any combination of $\times$ and $\div$
- e.g., if F = ABC / DE,
- $\frac{\Delta F}{F} = \sqrt{(\frac{\Delta A}{A})^2 + (\frac{\Delta B}{B})^2 + (\frac{\Delta C}{C})^2 + (\frac{\Delta D}{D})^2 + (\frac{\Delta E}{E})^2}$,
- Then find ΔF = F $\times \frac{\Delta F}{F}$

Functions and Uncertainty

- Use a little calculus. Suppose F = f(x) and x = A±ΔA





- $m = \frac{F(A+\Delta A) - F(A)}{(A+\Delta A) - (A)}$
- $\Delta F = \Delta A \times m$
- $\Delta F \cong \Delta A \times f'(A)$

We apply quadrature to ensure the result is positive.

$$\Delta F \cong \sqrt{(\Delta A \times f'(A))^2} \cong |\Delta A \times f'(A)|$$

Remember

- A is the measured value
- ΔA is the uncertainty in the measurement
- f(x) is the function
- f'(x) is the derivative of the function (use following table)

e.g. evaluate F = ln(3.51 $\pm$ 0.17)

A = 3.51, ΔA = 0.17, f(x) = ln(x) , f'(x) = 1/x

ΔF = Δ A $\times$ 1/A = 0.17 / 3.51 = 0.0484

$\therefore$ F = 1.2556 $\pm$ 0.0484 = 1.26 +0.05

Function	Derivative
Kx (K exact)	K
x^z (z exact)	zx^{z-1}
ln(x)	$\frac{1}{x}$
e^x	e^x
sin(x)	cos(x)
cos(x)	-sin(x)
tan(x)	$\frac{1}{\cos^2(x)}$
arctan(x)	$\frac{1}{1+x^2}$

For trig functions, Δx must be given in radians

Combining Rules

- e.g. $F = A - B \ln(C)$

- Follow order of operations (functions, $\times$ and $\div$, $+$ and $-$)

- Use one rule at a time

Example: F = A – B ln(C)

- Let X = ln(C), ΔX = ΔC/C
- Now have F = A – BX
- Let Y = BX, $\frac{\Delta Y}{Y} = \sqrt{(\frac{\Delta B}{B})^2 + (\frac{\Delta X}{X})^2}$ or

$$\Delta Y = BX\sqrt{(\frac{\Delta B}{B})^2 + (\frac{\Delta X}{X})^2}$$

- Now F = A – Y.
- $\Delta F = \sqrt{(\Delta A)^2 + (\Delta Y)^2}$