

Energy

- Many types
- Units 1 Joule (J) = 1 kg-m²/s²
- Kinetic Energy – $K = \frac{1}{2}mv^2$
 - (also rotational and vibrational)
- Gravitational Potential Energy – $U_g = mgh$
- Elastic (Spring) Potential Energy – $U_s = \frac{1}{2}kx^2$
- Internal or Thermal Energy – E_{TH}
- Chemical Energy – E_{chem}
- Mass Energy – $E = mc^2$

- Has Science identified all the possible energies?
- Probably not
- Dark Energy
 - Causes the accelerating expansion of universe
 - Not really known what it is

FIGURE 10.1 Energy transformations occur within the system.

The *environment* is everything that is *not* part of the system.

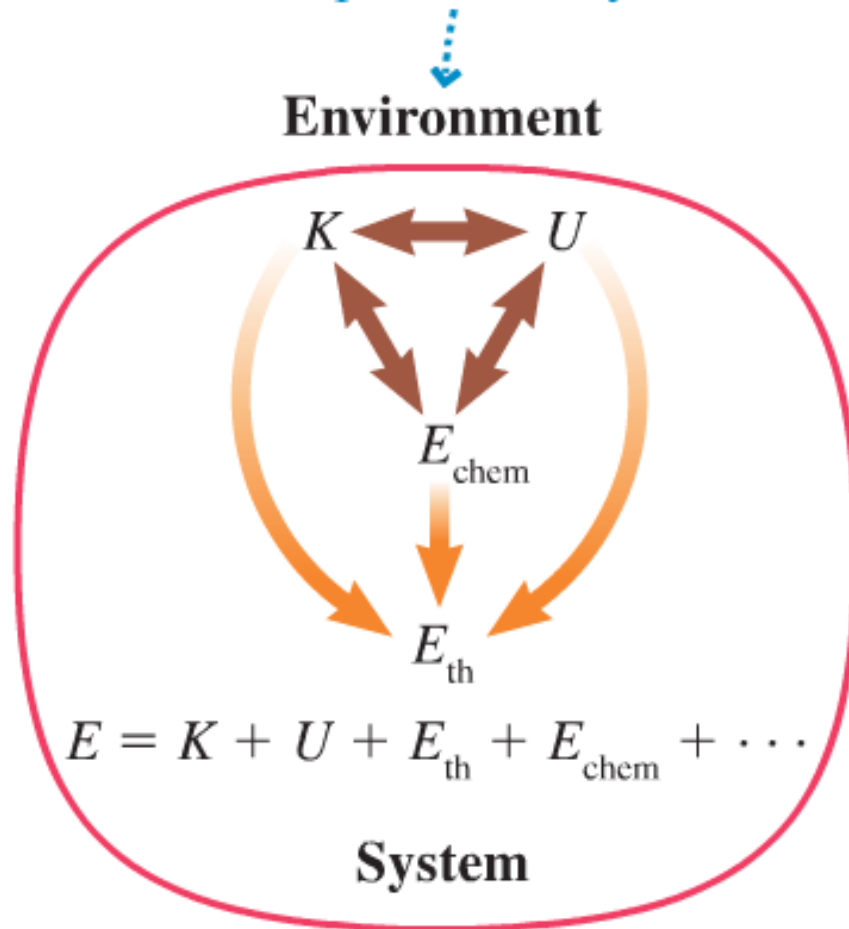
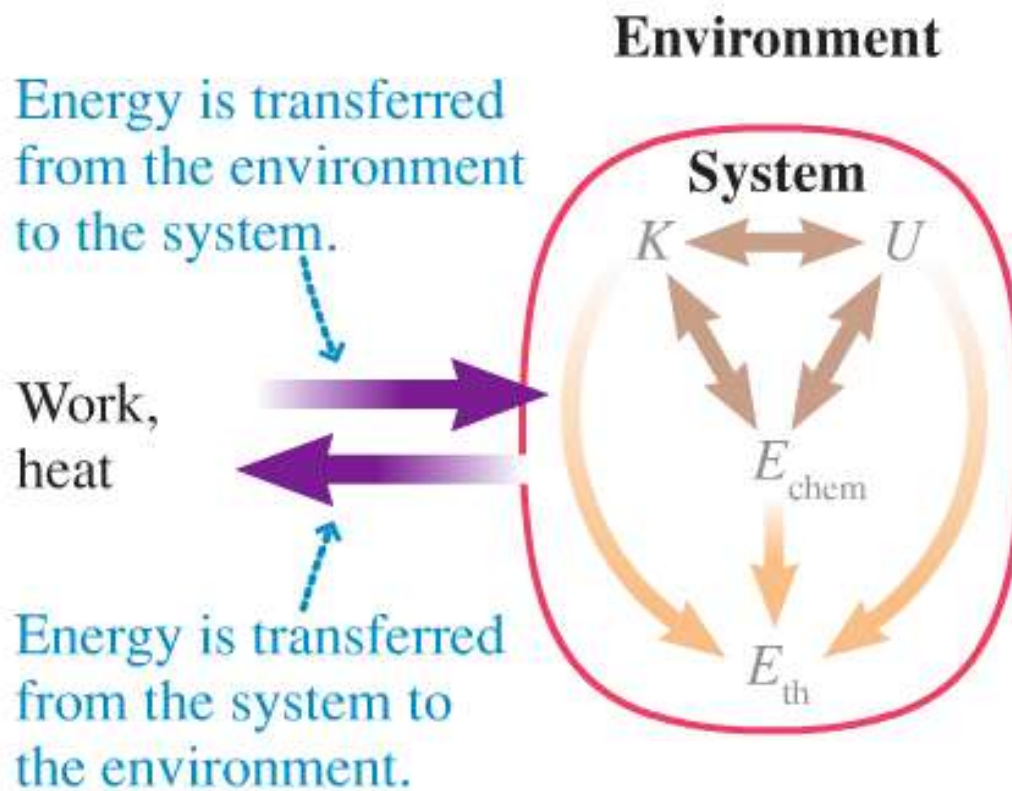
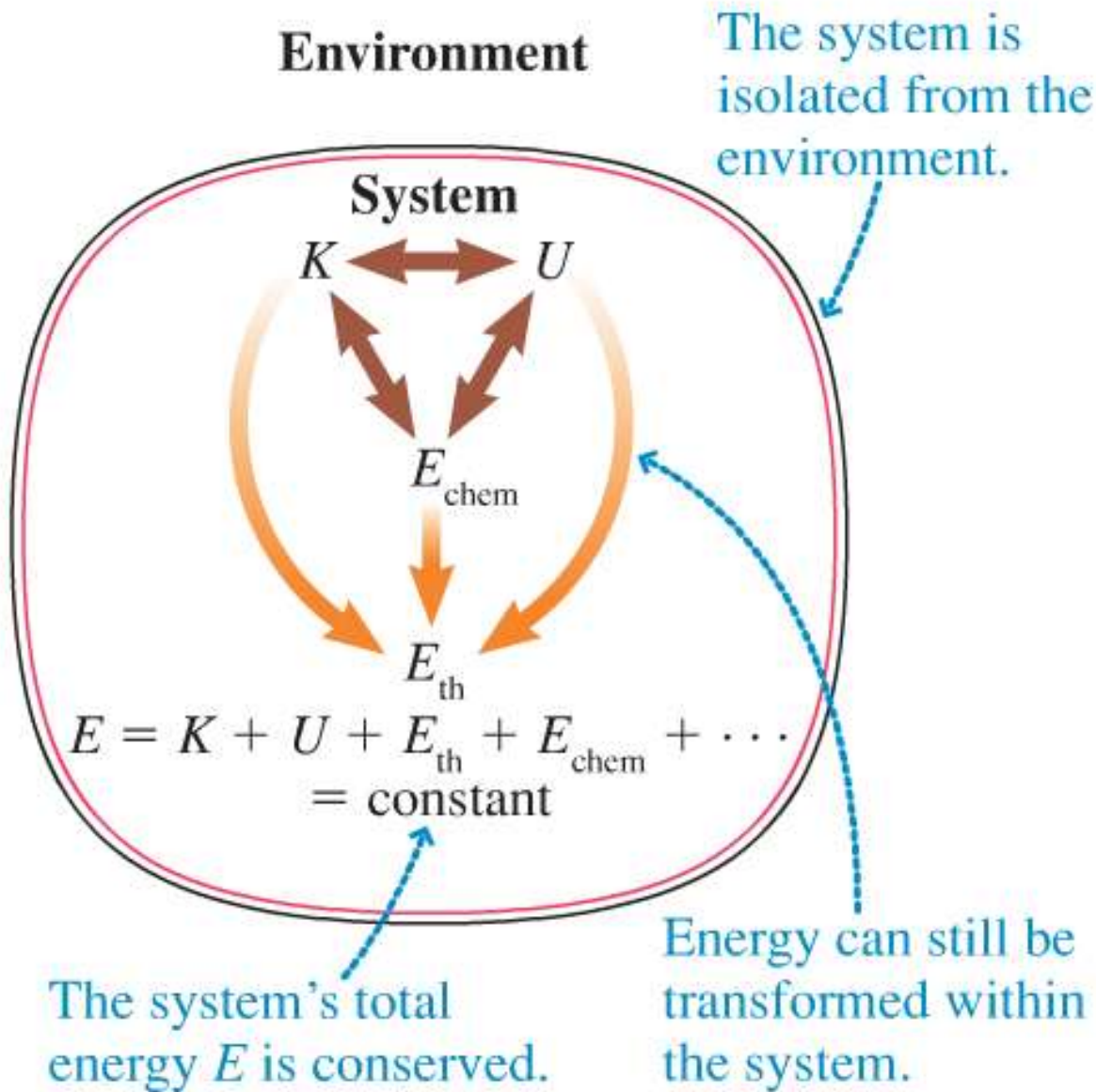


FIGURE 10.2 The basic energy model shows that work and heat are energy transfers into and out of the system, while energy transformations occur within the system.



$$W + Q = \Delta E_{\text{sys}}$$

FIGURE 10.3 An isolated system.



$$0 = \Delta E_{\text{sys}}$$

Heat and Work

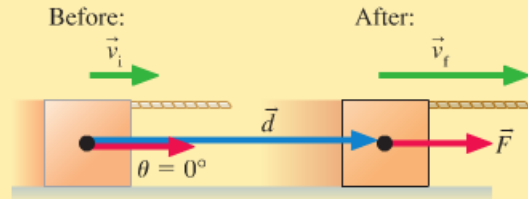
- Heat Q flows from a hotter to a cooler body (will discuss later)
- When an external object exerts a force on a system as it moves, work W is done.
- Internal forces occur in pairs (NIII), so no work is done by internal forces
- Work is a scalar.
- Magnitude depends on orientation of force and path.

Direction of force
relative to displacement

Angles and
work done

Sign of W

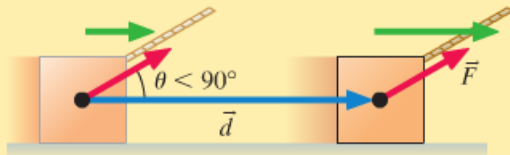
Energy transfer



$\theta = 0^\circ$
 $\cos \theta = 1$
 $W = Fd$

+

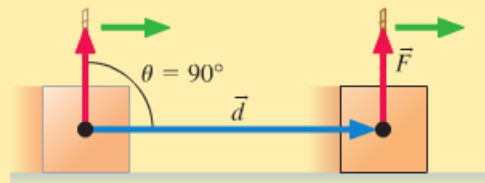
The force is in the direction of motion. The block has its greatest positive acceleration. K increases the most:
Maximum energy transfer to system.



$\theta < 90^\circ$
 $W = Fd \cos \theta$

+

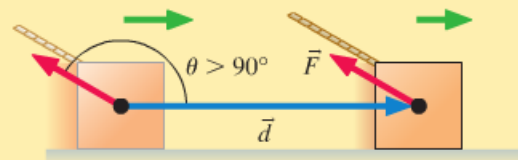
The component of force parallel to the displacement is less than F . The block has a smaller positive acceleration. K increases less:
Decreased energy transfer to system.



$\theta = 90^\circ$
 $\cos \theta = 0$
 $W = 0$

0

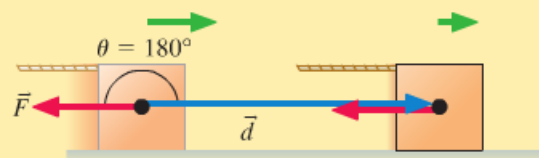
There is no component of force in the direction of motion. The block moves at constant speed. No change in K :
No energy transferred.



$\theta > 90^\circ$
 $W = Fd \cos \theta$

-

The component of force parallel to the displacement is opposite the motion. The block slows down, and K decreases:
Decreased energy transfer out of system.

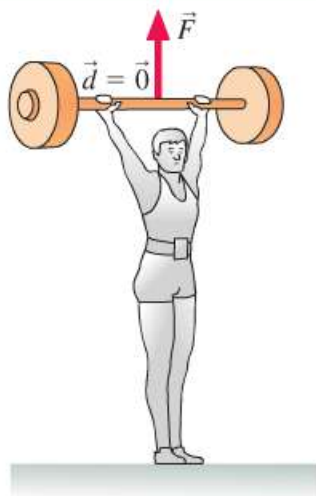


$\theta = 180^\circ$
 $\cos \theta = -1$
 $W = -Fd$

-

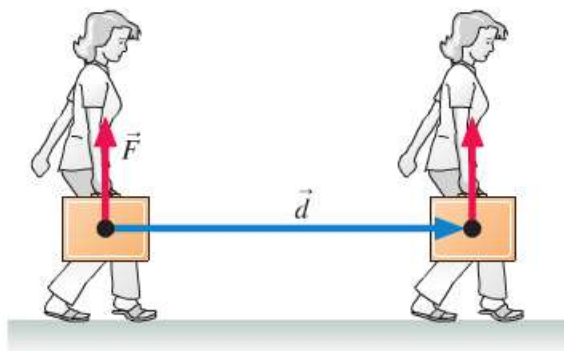
The force is directly opposite the motion. The block has its greatest deceleration. K decreases the most:
Maximum energy transfer out of system.

Forces that do no work



If the object undergoes no displacement while the force acts, no work is done.

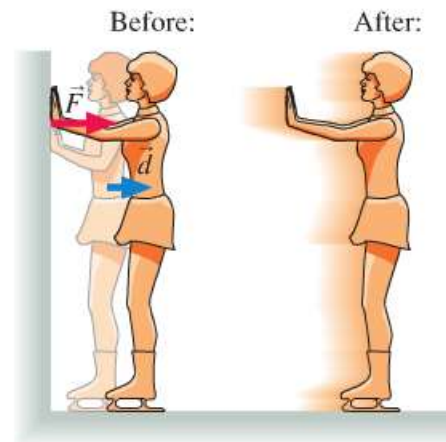
This can sometimes seem counterintuitive. The weightlifter struggles mightily to hold the barbell over his head. But during the time the barbell remains stationary, he does no work on it because its displacement is zero. Why then is it so hard for him to hold it there? We'll see in [Chapter 11](#) that it takes a rapid conversion of his internal chemical energy to keep his arms extended under this great load.



A force perpendicular to the displacement does no work.

The woman exerts only a vertical force on the briefcase she's carrying. This force has no component in the direction of the displacement, so the briefcase moves at a constant velocity and its kinetic energy remains constant. Since the energy of the briefcase doesn't change, it must be that no energy is being transferred to it as work.

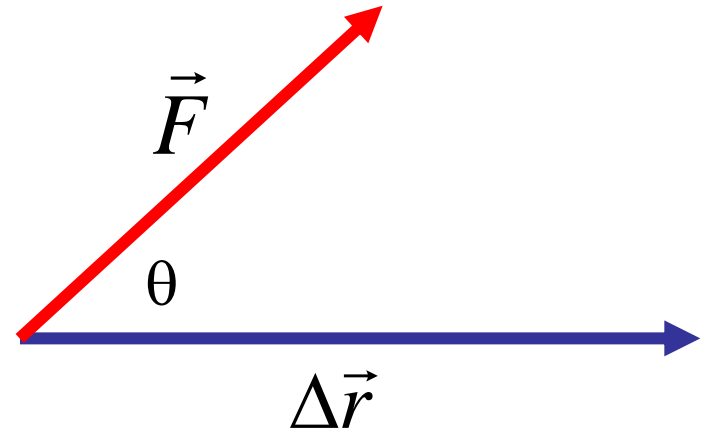
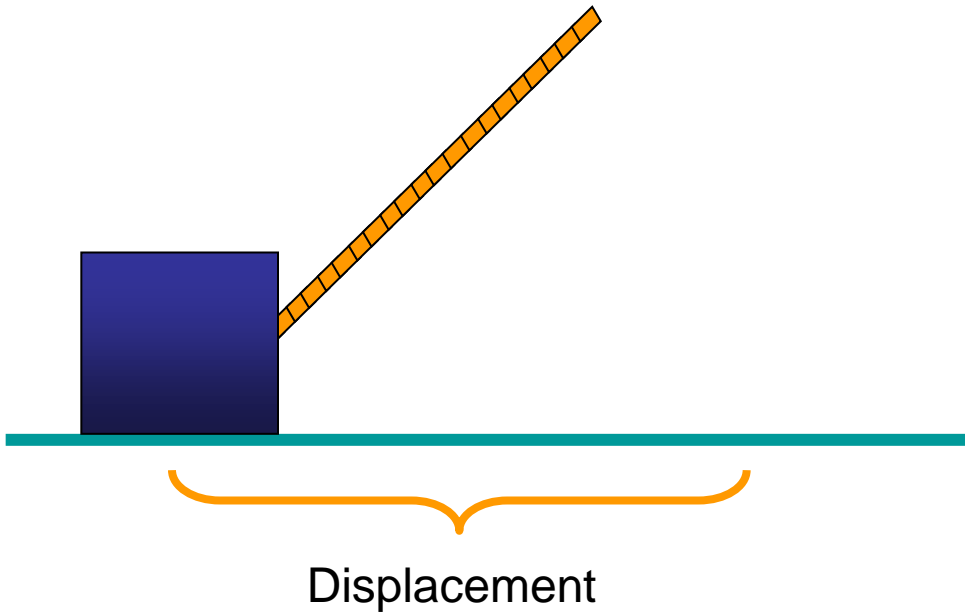
(This is the case where $\theta = 90^\circ$ in [Tactics Box 10.1](#).)



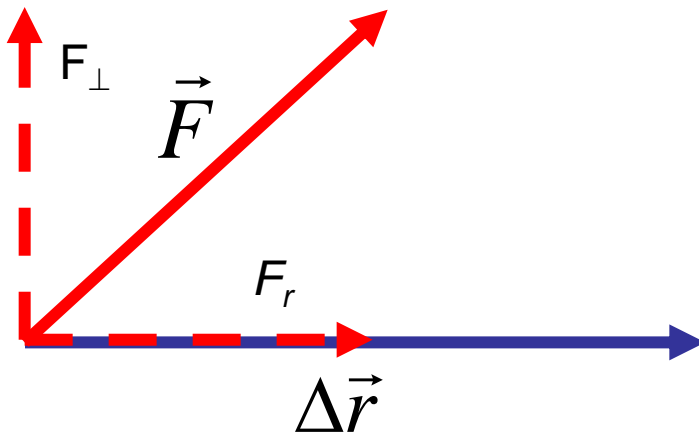
If the part of the object on which the force acts undergoes no displacement, no work is done.

Even though the wall pushes on the skater with a normal force \vec{n} and she undergoes a displacement \vec{d} , the wall does no work on her, because the point of her body on which \vec{n} acts—her hands—undergoes no displacement. This makes sense: How could energy be transferred as work from an inert, stationary object? So where does her kinetic energy come from? This will be the subject of much of [Chapter 11](#). Can you guess?

Work in Vector format



$$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r \cos \theta$$



$$\begin{aligned} W &= \vec{F} \cdot \Delta\vec{r} \\ &= F_r \Delta r \\ &= F_x \Delta x + F_y \Delta y + F_z \Delta z \end{aligned}$$

Work and Non-constant Forces

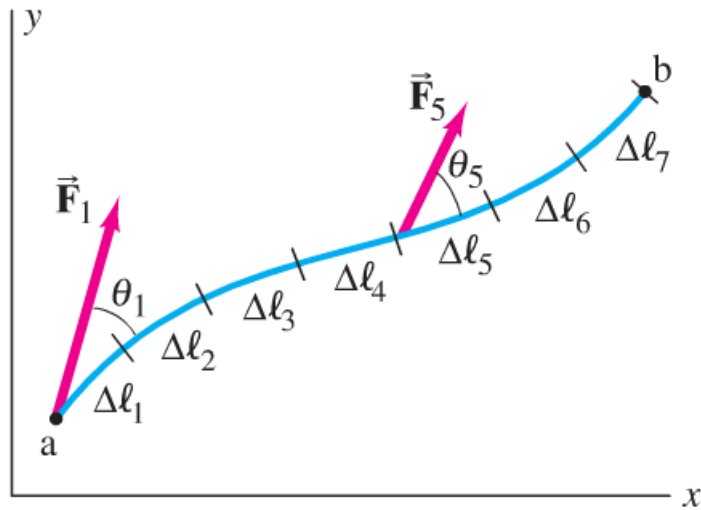
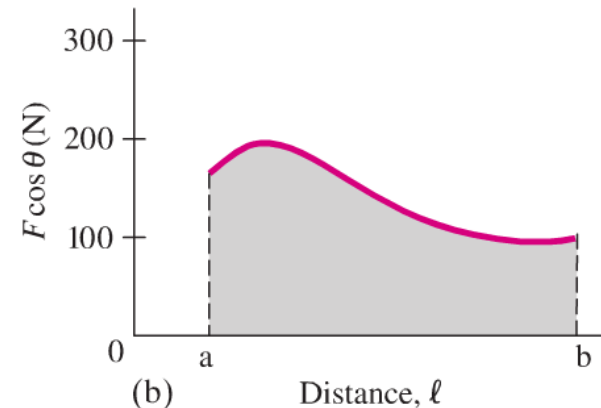
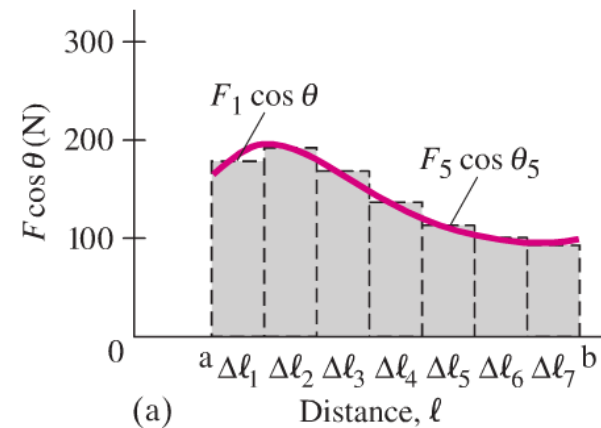


FIGURE 7-8 A particle acted on by a variable force, \vec{F} , moves along the path shown from point a to point b.

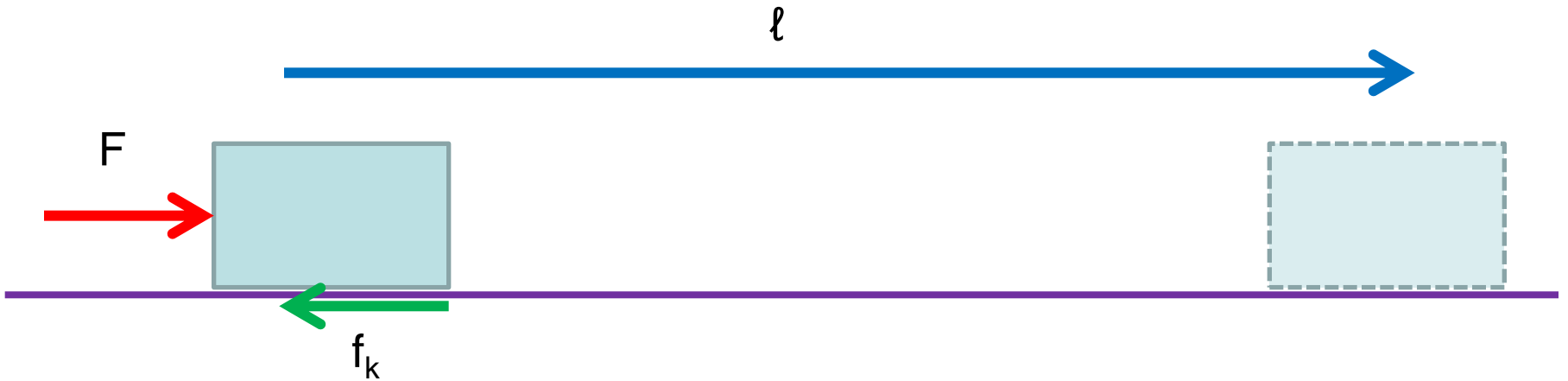
$$W = \int_a^b \vec{F} \cdot d\vec{\ell}.$$

$$W = \int_a^b F(x) dx$$

FIGURE 7-9 Work done by a force F is (a) approximately equal to the sum of the areas of the rectangles, (b) exactly equal to the area under the curve of $F \cos \theta$ vs. ℓ .



Work and Friction



Let $F = f_k$ so $a = 0$ and velocity does not change.

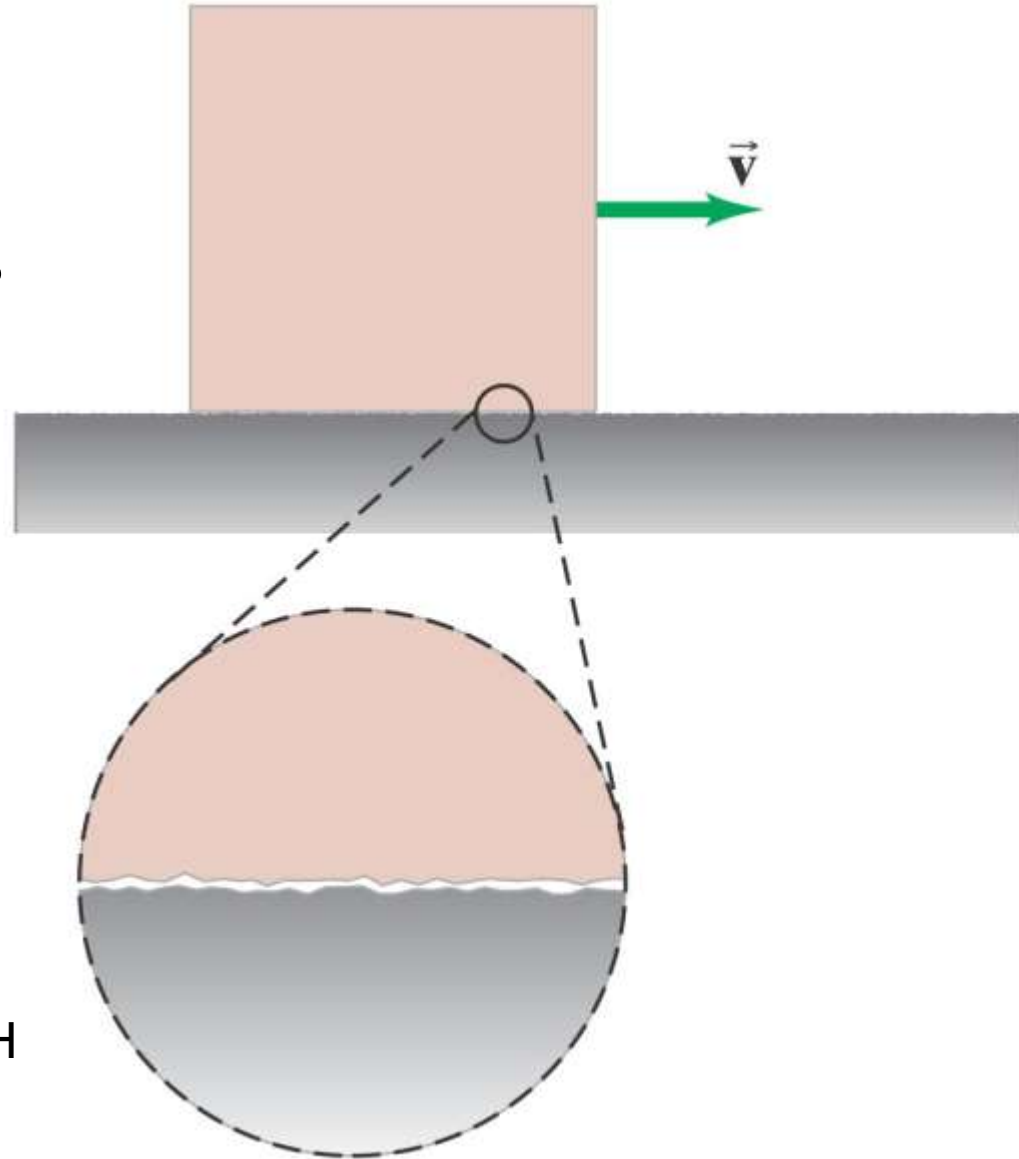
$$W_{\text{ext}} = \Delta E$$

$$W_{\text{ext}} = F\ell - f_k\ell \Rightarrow \Delta E = 0$$

But $\Delta E = \Delta E_{\text{TH}}$ increases, block gets warmer!

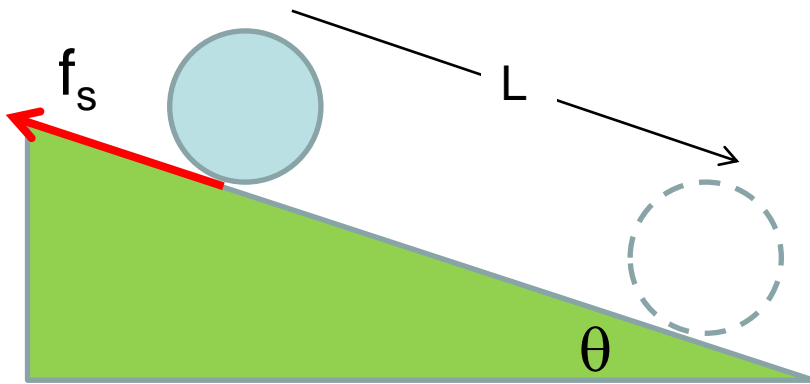
How can this be?

- Friction is a complex force made up of the forces between bumps on surfaces.
- Since the surfaces are not rigid, ℓ is not distance these forces all act over.
- For small distances, good enough and $\Delta E_{TH} \cong 0$



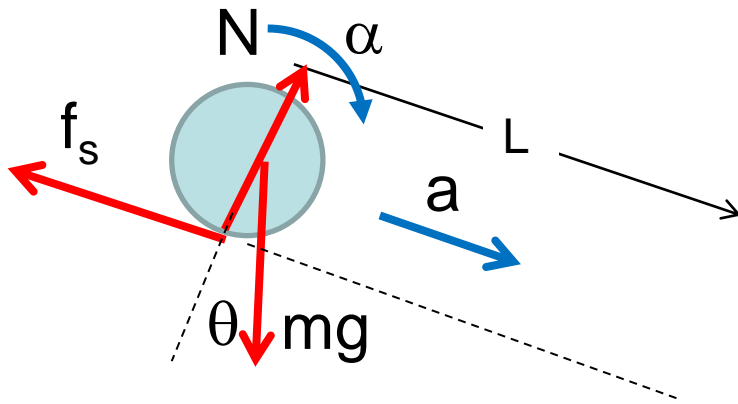
Static Friction and Work

- Work is only done by a force acting through a distance.
- When a wheel rolls without slipping, point of contact is at rest. No D for f_s .



- $W_f \neq -f_s L$
- $W_f = 0 !$
- Can you think of a case where f_s does do work?

Check!



- $mg\sin\theta - f_s = ma$
- $Rf_s = I\alpha = I(a/R)$
- $\Rightarrow f_s = (I/R^2) a$
- $mg\sin\theta = [m + (I/R^2)] a$
- From kinematics $a = v^2/2L$, so
- $mg\sin\theta = [m + (I/R^2)] (v^2/2L)$

- $W_{\text{ext}} = \Delta E_{\text{sys}}$
- $W_{\text{ext}} = -mgL\sin\theta + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
- $W_{\text{ext}} = -mgL\sin\theta + \frac{1}{2}mv^2 + \frac{1}{2}(I/R^2)v^2$
- $W_{\text{ext}} = -mgL\sin\theta + \frac{1}{2}[m + (I/R^2)]v^2$
- $W_{\text{ext}} = -L[m + (I/R^2)] (v^2/2L) + \frac{1}{2}[m + (I/R^2)]v^2$
- $W_{\text{ext}} = 0$

Energy Expressions



$$\mu = 0$$

$$W_{\text{ext}} = \Delta E_{\text{sys}}$$

$$+FL = \Delta E_{\text{sys}}$$

$$maL = \Delta E_{\text{sys}}$$

From kinematics; $m\frac{1}{2}(v_f^2 - v_0^2) = \Delta E_{\text{sys}}$

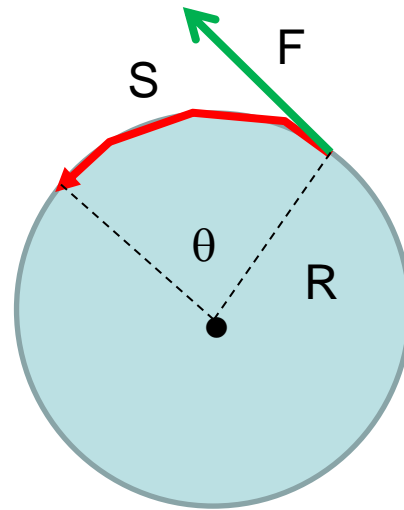
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \Delta E_{\text{sys}}$$

Define Kinetic Energy: $K_{\text{linear}} = \frac{1}{2}mv^2$

Rotational Kinetic Energy

$$\begin{aligned}\Delta E_{\text{sys}} &= W_{\text{ext}} \\ &= FS \\ &= FR\theta \\ &= \tau\theta \\ &= I\alpha\theta \\ &= I\left[\frac{1}{2}(\omega_f^2 - \omega_0^2)\right] \\ &= \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_0^2\end{aligned}$$

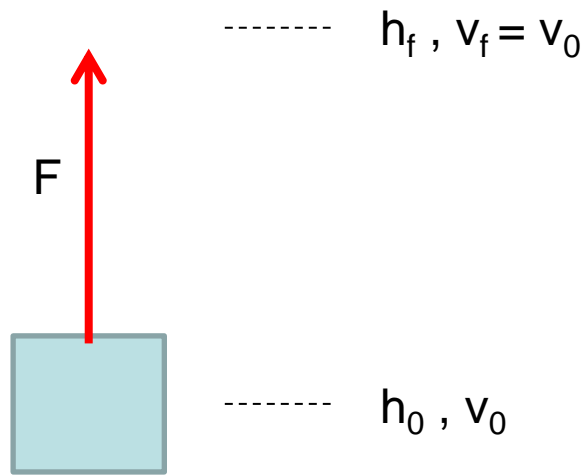
$$\text{So } K_{\text{rot}} = \frac{1}{2}I\omega^2$$



$$\begin{aligned}\text{Note: } K_{\text{rolling}} &= K_{\text{linear}} + K_{\text{rot}} \\ &= \frac{1}{2}mv_{\text{CM}}^2 + \frac{1}{2}I_{\text{CM}}\omega_{\text{CM}}^2\end{aligned}$$

Can use $v = R\omega$ to simplify.

Gravitational Potential Energy

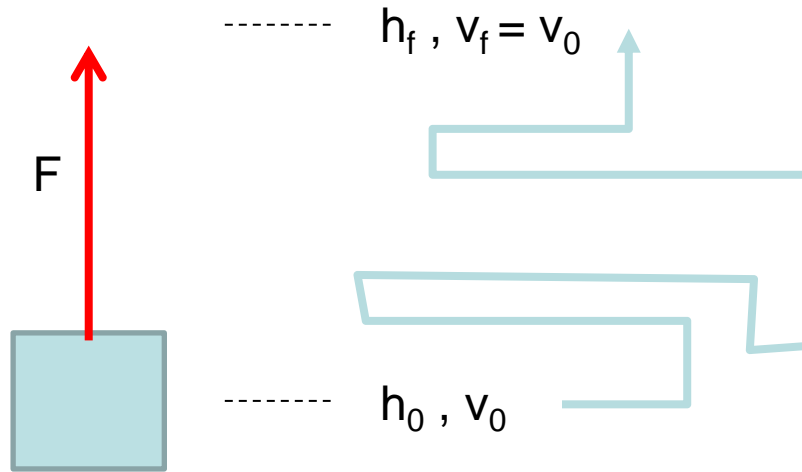


$$\begin{aligned}\Delta E_{\text{sys}} &= W_{\text{ext}} \\ &= F(h_f - h_0) \\ &= mg(h_f - h_0) \\ &= mgh_f - mgh_0\end{aligned}$$

$$\text{So } U_{\text{grav}} = mgh$$

Since $v_f = v_0$, $a = 0$. So $F_{\text{net}} = 0$. Thus $F = mg$

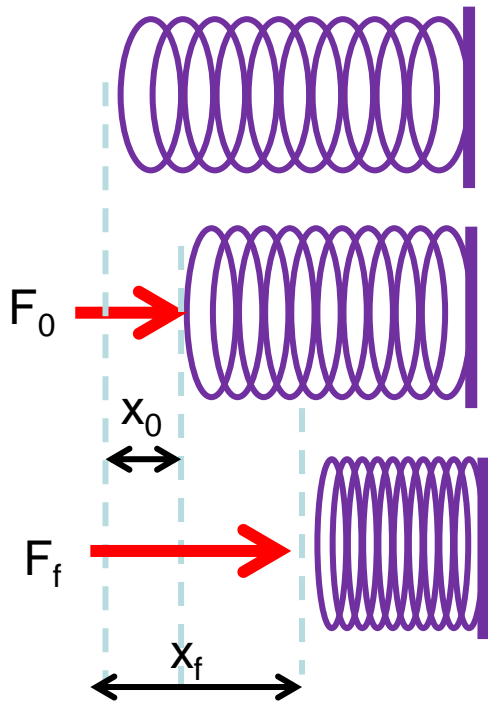
GPE continued



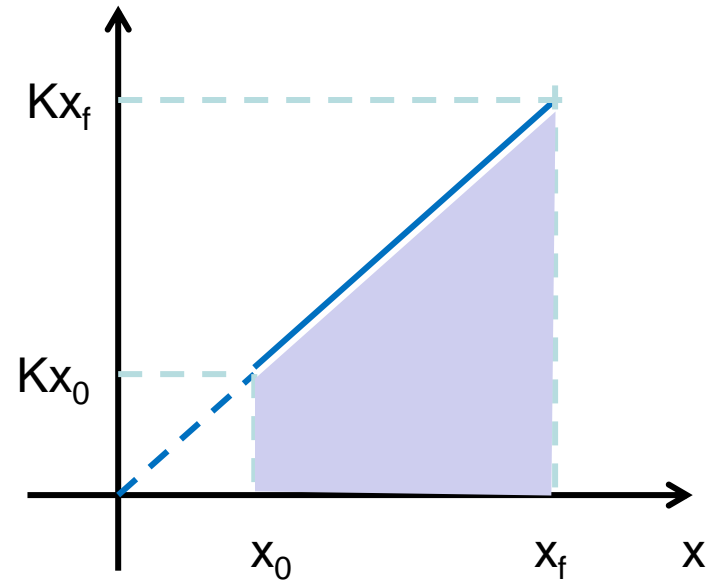
Any path produces the same change in GPE if start and end heights are same since F is \perp over horizontal pieces.

Gravity is said to be a conservative or path independent force.

Spring Potential Energy



$$\begin{aligned}\Delta E_{\text{sys}} &= W_{\text{ext}} \\ &= \int_{x_0}^{x_f} f(x) dx \\ &= \int_{x_0}^{x_f} kx dx\end{aligned}$$



Note: Variable force!
 $F = kx$ if spring is
massless.

SPE continued

- Area under curve is difference of triangles
- Area of a triangle is $A = \frac{1}{2}bh$
- $\Delta E_{\text{sys}} = \frac{1}{2}Kx_f^2 - \frac{1}{2}Kx_0^2$
- Define $U_{\text{spring}} = \frac{1}{2}Kx^2$

Using Work Energy Methods

- Use if interested in changes of height or speed
- Identify the system (often connected by ropes or springs!)
- Identify objects exerting forces on system
- Note gravity force accounted by mgh already.
- Internal system forces do no work.
- Look at $t = 0$ and t_{final} , how has the energy of each member of the system changed?

Power

- Rate of change of energy or at which work is done
- $P = \frac{\Delta E}{\Delta t}$, amount of energy transformed in time Δt
- $P = \frac{W}{\Delta t}$, amount of work done in time Δt
- $P = \frac{W}{\Delta t} = \frac{F_x \Delta x}{\Delta t} = F_x \frac{\Delta x}{\Delta t} = Fv$, rate of energy transfer by F acting on object travelling at v
- 1 Watt (W) = 1 Joule/second
- 1 Horsepower (Hp) = 745 W