

Mathematics Problem of the Week (249)

This week's winner is: **Matt Potma**

Contact Lin Hammill (Surrey Fir 348) or Judy Bicep (Richmond,3335) for your prize or email MathProblem@kpu.ca.

Also submitting correct solutions to problem 249 were:

Tom Ouellette, David Luna and Anthony Roberts

Problem 249 solution:

(1) $x^2 + ax + 1 = 0$

(2) $x^2 + x + a = 0$

Solution method 1:

Solve (2) for a: $a = -x^2 - x$. Substitute into (1) and solve: $x^2 + (-x^2 - x)x + 1 = 0$
 $-x^3 + 1 = 0 \Rightarrow x = 1$

Then $a = -1^2 - 1 = -2$

Solution method 2:

Using the quadratic formula the solutions to (1) and (2) are: $x = \frac{-a \pm \sqrt{a^2 - 4}}{2}$ and $x = \frac{-1 \pm \sqrt{1 - 4a}}{2}$

$$-a \pm \sqrt{a^2 - 4} = -1 \pm \sqrt{1 - 4a}$$

$$\pm \sqrt{a^2 - 4} = a - 1 \pm \sqrt{1 - 4a}$$

$$a^2 - 4 = a^2 - 2a + 1 \pm 2(a - 1)\sqrt{1 - 4a} + 1 - 4a \quad \text{squaring both sides}$$

$$6a - 6 = \pm 2(a - 1)\sqrt{1 - 4a}$$

$$3 = \pm \sqrt{1 - 4a}$$

$$9 = 1 - 4a$$

$$a = -2$$

squaring both sides