Kwantlen Polytechnic University

Mathematics Problem of the Week (259)

This week's winner is:

Gurmehak Sandhu

Contact Lin Hammill (Surrey Fir 348) or Judy Bicep (Richmond,3335) for your prize or email MathProblem@kpu.ca.

Also submitting correct solutions to problem 259 were:

Harpreet Kaur, Brady Schmidt, Pratham Barghouti, Tierney Wisniewski, and Suzanne Pearce

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Problem 259 solution:

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$
$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ has exactly one zero. The zero occurs because $5 \times 2 = 10$.

We get as many zeros at the end of $n!$ as there are factors of 10. A factor of 10 occurs whenever there is a 5 multiplied by an even number. Thus to determine how many zeros at the end of $n!$ were need only count the number of 5s (since there are lots of even numbers).

5! has one 5. There is not another 5 until we reach 10!
10! has two 5s (5 and $10 = 2 \times 5$) and so ends in 2 zeros. There is not another 5 until we reach 15!
15! has three 5s and so is the first to end in 3 zeros.
20! has four 5s and so is the first to end in 4 zeros.
25! has six 5s (because $25 = 5 \times 5$) and so ends in 6 zeros.

Thus 25! is the first to end in exactly 6 zeros.

In fact 25! = 15511210043330985984000000.

Using the same reasoning 100! ends in 24 zeros, one for each multiple of 5 (5,10,15, ...95, 100) and an extra one for 25, 50, 75 and 100, each of which has a factor of 5^2.

In fact 100! =9332621544394415268169923885626670049071596826438162146859296389521
75999932299156089414639761565182862536979208272237582511852109168640000000000000000
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