Kwantlen Polytechnic University

Mathematics Problem of the Week 7

This week's winner is: Enguang Shen

Contact Tariq Nuruddin at Surrey MAC or Judy Bicep (Richmond,3335) for your prize or email MathProblem@kpu.ca.

Also submitting correct solutions to problem 266 was: Rixat, Catherine Chow, James Guerry, etc.

(Solution provided by James Guerry)

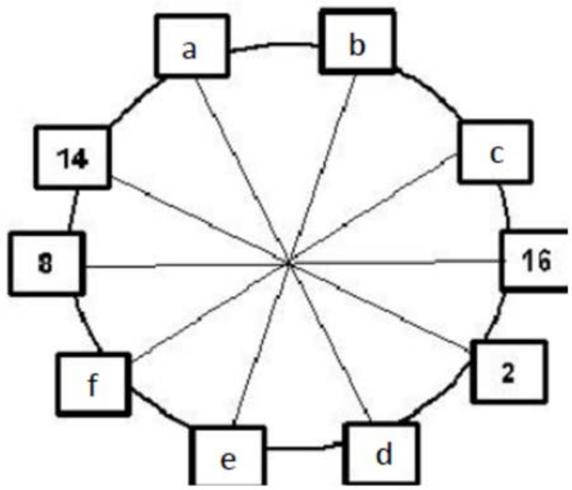
Let the six missing numbers be labeled a – f, as shown at the right. From the problem, we know:

$$14^2 + a^2 = 2^2 + d^2$$

$$a^2 + b^2 = d^2 + e^2$$

$$b^2 + c^2 = e^2 + f^2$$

$$c^2 + 16^2 = f^2 + 8^2$$



In solving these equations, we find that $d^2 - a^2 = b^2 - e^2 = f^2 - c^2 = 192$. Therefore, each of these differences of squares of diametrically opposite numbers must equal 192. Consider a general form of this scenario, $x^2 - y^2 = 192$. By factoring, we find that $(x + y)(x - y) = 192$, and that 192 may only be written as a product of the following factor pairs: $2 \cdot 96$, $3 \cdot 64$, $4 \cdot 48$, $6 \cdot 32$, $8 \cdot 24$, $12 \cdot 16$. We know $3 \cdot 64$ cannot equal $(x + y)(x - y)$ if x and y are integers, so we exclude that factor pair. For the rest of the factor pairs, we find the following x and y values:

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$$2 \cdot 96 = 49^2 - 47^2$$

$$4 \cdot 48 = 26^2 - 22^2$$

$$6 \cdot 32 = 19^2 - 13^2$$

$$8 \cdot 24 = 16^2 - 8^2$$

$$12 \cdot 16 = 14^2 - 2^2$$

16, 8, 14, and 2 have already been used in the circle, so we exclude those values. Therefore, the unique possibilities for the missing numbers are as follows:

$$(d, a), (b, e), (f, c) \in \{(49, 47), (26, 22), (19, 13)\}$$