

# Mathematics Problem of the Week 9

## The winner of problem 268 is: Catherine Chow

Contact Tariq Nuruddin at Surrey MAC or Judy Bicep (Richmond,3335) for your prize or email [MathProblem@kpu.ca](mailto:MathProblem@kpu.ca).

We will start with one starry fish and then we will multiply the final result by 5.

Let  $S_n$  be the number of starry fish alive at the end of month  $n$ .

$$S_0 = S_1 = S_2 = S_3 = 1 \text{ starry fish}$$

$S_3 = 1$  as these starry fish cannot subdivide and produce new fish in the first three weeks of life.

$$S_4 = 1 + 2 \text{ new starry fishes} = \underline{3 \text{ starry fishes}}$$

$$S_5 = (1 + 2 \text{ new starry fishes}) + 2 \text{ starry fishes} = \underline{5 \text{ starry fishes}}$$

$$S_6 = (1 + 2 \text{ new starry fishes}) + 2 \text{ starry fishes} + 2 \text{ starry fishes} = \underline{7 \text{ starry fishes}}$$

$$S_7 = (1 + 2 \text{ new starry fishes}) + 2 \text{ starry fishes} + 2 \text{ starry fishes} + 2 \text{ starry fishes} = \underline{9 \text{ starry fishes}}$$

**The arrows denote that the starry fishes born in week 4 have lived for three weeks at the end of week 7.**

$$S_8 = (1 + 2 \text{ new starry fishes}) + 2 \text{ starry fishes} + 2 \text{ starry fishes} + 2 \text{ starry fishes} +$$

$$\underline{(2 + 2(2 \text{ new starry fishes}))} = \underline{15 \text{ starry fishes.}}$$

The underlined bold faced calculation shows the number of starry fishes that were reproduced by the starry fishes which were born in week 4.

$$\text{So our recursive formula becomes } S_n = S_{n-1} + 2 S_{n-4}, n \geq 4$$

$$\text{where } S_0 = S_1 = S_2 = S_3 = 1$$

Continuing in this way we get  $S_{15} = 329$  starry fishes

$$329 \text{ starry fishes times } 5 = 1645 \text{ starry fishes}$$

$$\text{Total number of starry fishes} = 1645.$$