MOTION OF A PROJECTILE

Today's Objectives:

Students will be able to:

1. Analyze the free-flight motion of a projectile.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Kinematic Equations for Projectile Motion
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The downward acceleration of an object in free-flight motion is

   A) zero   B) increasing with time

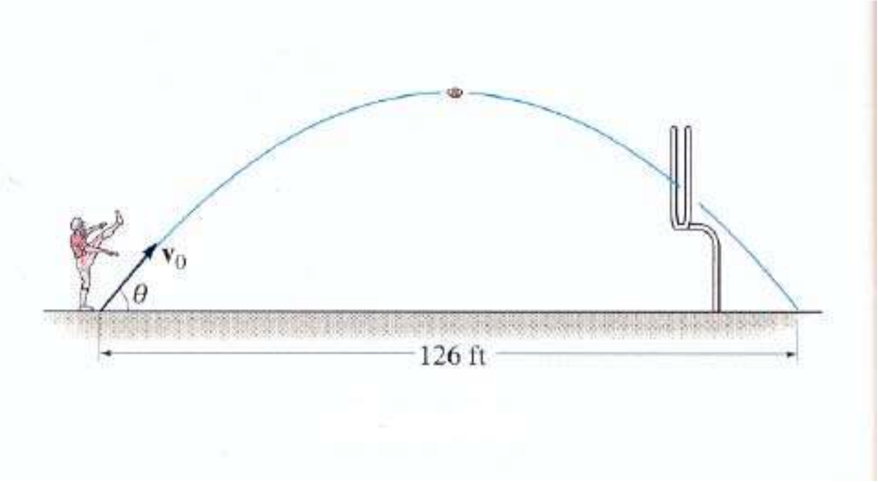
   C) 9.81 m/s^2 D) 9.81 ft/s^2

2. The horizontal component of velocity remains __________ during a free-flight motion.

   A) zero   B) constant

   C) at 9.81 m/s^2 D) at 32.2 ft/s^2

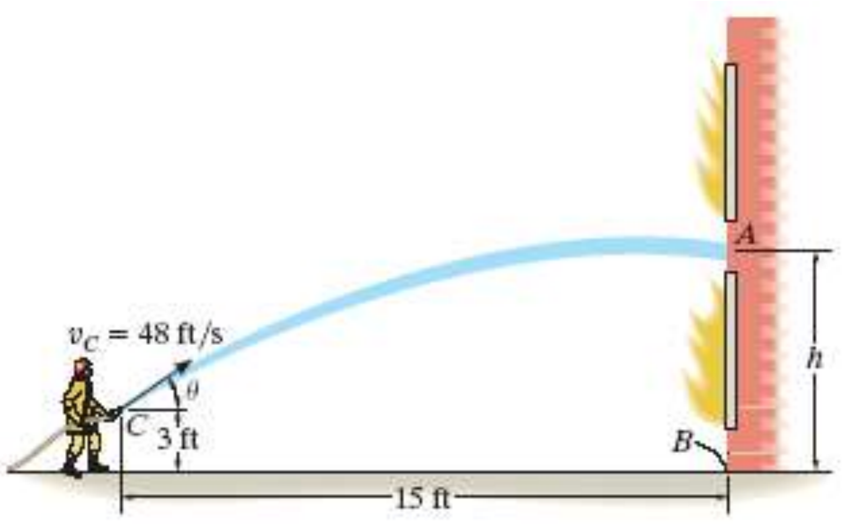
APPLICATIONS



A kicker should know at what angle, θ, and initial velocity, v_o, he must kick the ball to make a field goal.

For a given kick "strength", at what angle should the ball be kicked to get the maximum distance?

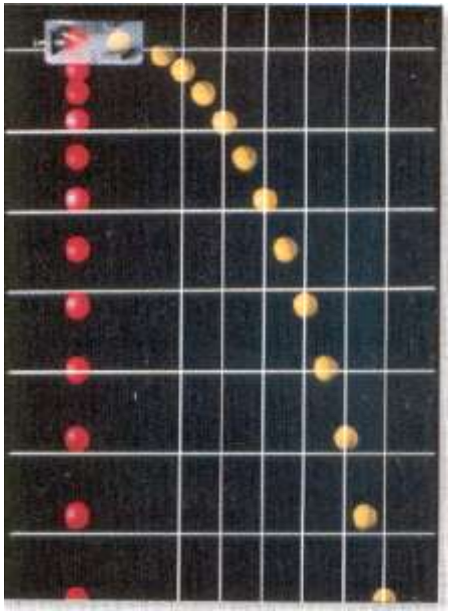
APPLICATIONS
(continued)



A fireman wishes to know the maximum height on the wall he can project water from the hose. At what angle, θ, should he hold the hose?

MOTION OF A PROJECTILE
(Section 12.6)

Projectile motion can be treated as two rectilinear motions, one in the horizontal direction experiencing zero acceleration and the other in the vertical direction experiencing constant acceleration (i.e., gravity).



For illustration, consider the two balls on the left. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity. Each picture in this sequence is taken after the same time interval. Notice both balls are subjected to the same downward acceleration since they remain at the same elevation at any instant. Also, note that the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction is constant.

KINEMATIC EQUATIONS: HORIZONTAL MOTION

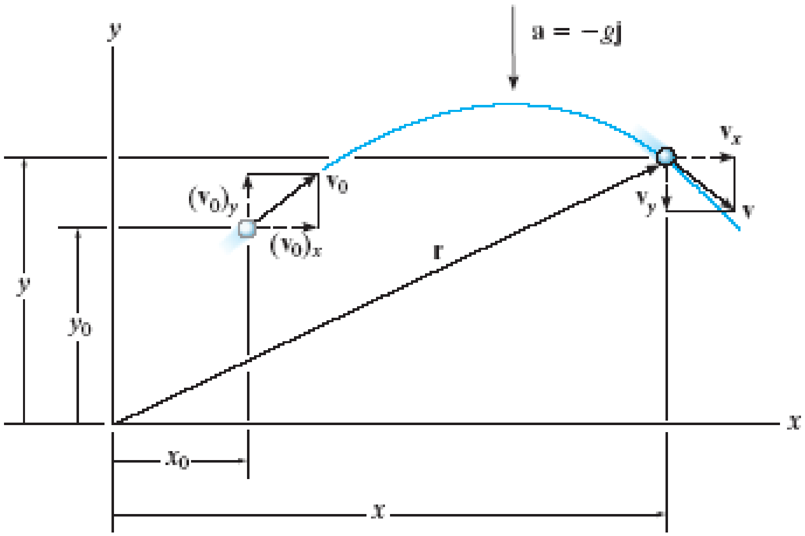


Fig. 12–20

Since $a_x = 0$, the velocity in the horizontal direction remains constant ($v_x = v_{ox}$) and the position in the x direction can be determined by:

$$x = x_o + (v_{ox})(t)$$

Why is a_x equal to zero (assuming movement through the air)?

KINEMATIC EQUATIONS: VERTICAL MOTION

Since the positive y-axis is directed upward, $a_y = -g$. Application of the constant acceleration equations yields:

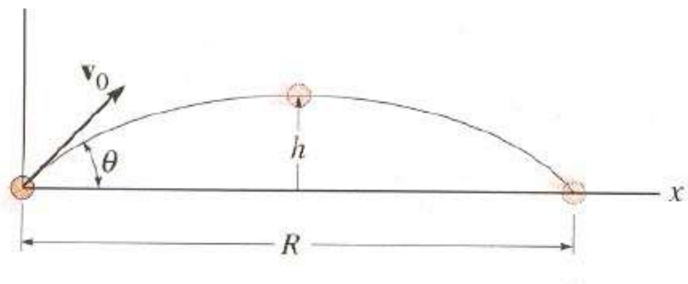
$$v_y = v_{oy} - g(t)$$

$$y = y_o + (v_{oy})(t) - \frac{1}{2}g(t)^2$$

$$v_y^2 = v_{oy}^2 - 2g(y - y_o)$$

For any given problem, only two of these three equations can be used. Why?

EXAMPLE



Given: v_o and θ

Find: The equation that defines y as a function of x.

Plan: Eliminate time from the kinematic equations.

Solution: Using $v_x = v_o \cos\theta$ and $v_y = v_o \sin\theta$

We can write: $x = (v_o \cos\theta)t$ or $t = \dfrac{x}{v_o \cos\theta}$

$$y = (v_o \sin\theta)t - ½\, g(t)^2$$

By substituting for t:

$$y = (v_o \sin\theta)\left(\frac{x}{v_o \cos\theta}\right) - \left(\frac{g}{2}\right)\left(\frac{x}{v_o \cos\theta}\right)^2$$

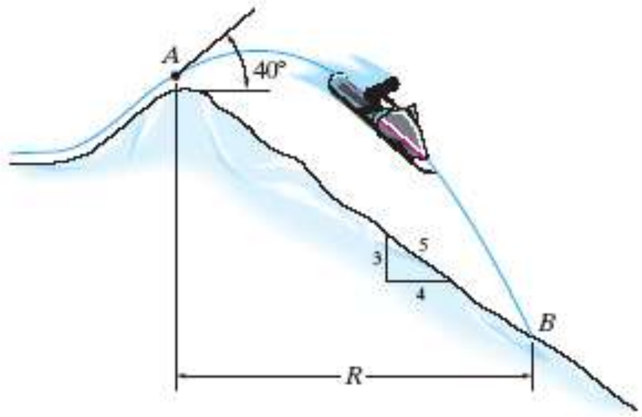
EXAMPLE
(continued)

Simplifying the last equation, we get:

$$y = (x \tan\theta) - \left(\frac{g\,x^2}{2v_o^{\,2}}\right)(1 + \tan^2\theta)$$

The above equation is called the "path equation" which describes the path of a particle in projectile motion. The equation shows that the path is parabolic.

EXAMPLE II



Given: Snowmobile is going 15 m/s at point A.

Find: The horizontal distance it travels (R) and the time in the air.

Solution:

First, place the coordinate system at point A. Then write the equation for horizontal motion.

$\xrightarrow{+}$ $x_B = x_A + v_{Ax} t_{AB}$ and $v_{Ax} = 15 \cos 40°$ m/s

Now write a vertical motion equation. Use the distance equation.

$+\uparrow$ $y_B = y_A + v_{Ay} t_{AB} - 0.5 g_c t_{AB}^2$ $v_{Ay} = 15 \sin 40°$ m/s

Note that $x_B = R$, $x_A = 0$, $y_B = -(3/4)R$, and $y_A = 0$.

Solving the two equations together (two unknowns) yields

$R = 42.8$ m $t_{AB} = 3.72$ s

CONCEPT QUIZ

1. In a projectile motion problem, what is the maximum number of unknowns that can be solved?

   A) 1 B) 2

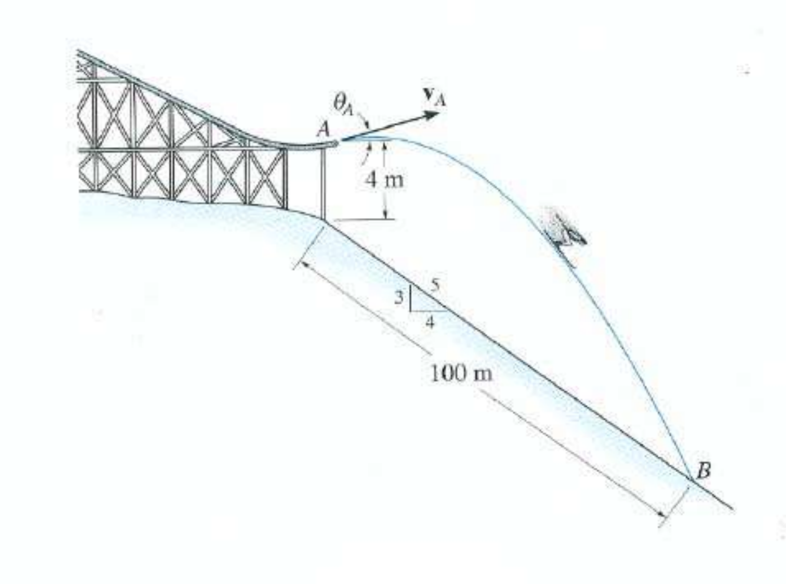
   C) 3 D) 4

2. The time of flight of a projectile, fired over level ground with initial velocity V_o at angle θ, is equal to

   A) $(v_o \sin \theta)/g$ B) $(2v_o \sin \theta)/g$

   C) $(v_o \cos \theta)/g$ D) $(2v_o \cos \theta)/g$

GROUP PROBLEM SOLVING



Given: Skier leaves the ramp at $\theta_A = 25^o$ and hits the slope at B.

Find: The skier's initial speed v_A.

Plan: Establish a fixed x,y coordinate system (in the solution here, the origin of the coordinate system is placed at A). Apply the kinematic relations in x and y-directions.

GROUP PROBLEM SOLVING
(continued)

Solution:

Motion in x-direction:

Using $x_B = x_A + v_{ox}(t_{AB})$

$$t_{AB} = \frac{(4/5)100}{v_A (\cos 25)} = \frac{88.27}{v_A}$$

Motion in y-direction:

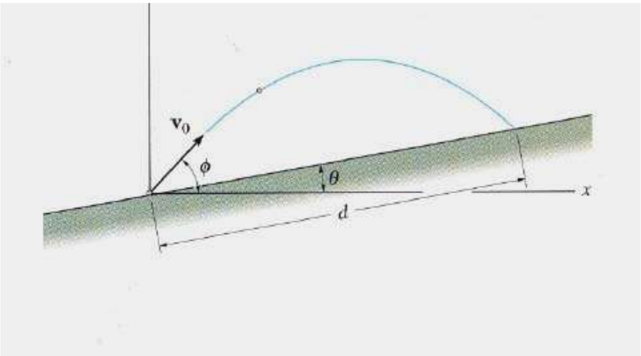
Using $y_B = y_A + v_{oy}(t_{AB}) – ½\, g(t_{AB})^2$

$$-64 = 0 + v_A(\sin 25)\left(\frac{80}{v_A (\cos 25)}\right) - ½\,(9.81)\left(\frac{88.27}{v_A}\right)^2$$

$$v_A = 19.42 \text{ m/s}$$

ATTENTION QUIZ

1. A projectile is given an initial velocity v_o at an angle ϕ above the horizontal. The velocity of the projectile when it hits the slope is ____________ the initial velocity v_o.



A) less than
B) equal to
C) greater than
D) None of the above.

2. A particle has an initial velocity v_o at angle θ with respect to the horizontal. The maximum height it can reach is when

A) $\theta = 30°$
B) $\theta = 45°$
C) $\theta = 60°$
D) $\theta = 90°$