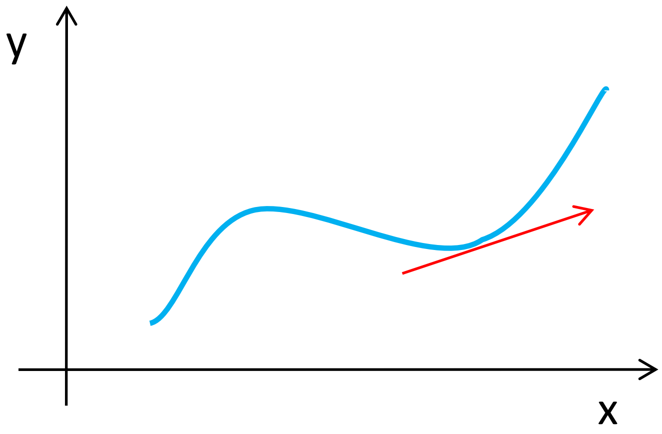
2D Paths and Direction
Curvilinear Unit Vectors



Can we find a unit vector that points in the direction of ***v***?

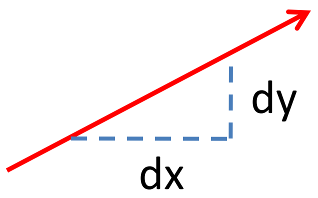
If we know v_x and v_y

$$\vec{v} = \hat{i}v_x + \hat{j}v_y$$

Then

$$\hat{u}_T = \frac{\vec{v}}{v} = \frac{\hat{i}v_x + \hat{j}v_y}{\sqrt{v_x^2 + v_y^2}}$$

If we know slope of tangent to curve $m = \frac{dy}{dx}$

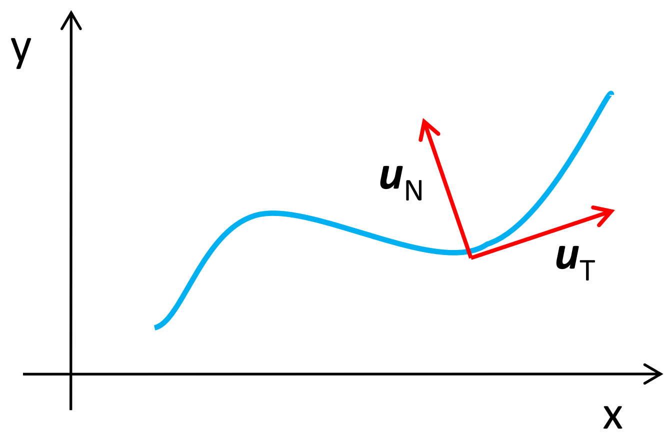


$$\hat{u}_T = \frac{\hat{i}dx + \hat{j}dy}{\sqrt{dx^2 + dy^2}}$$

Dividing top and bottom by dx, $\hat{u}_T = \dfrac{\hat{i}1 + \hat{j}\frac{dy}{dx}}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}$

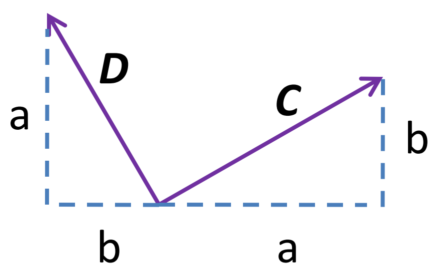
Dividing top and bottom by dt, $\hat{u}_T = \dfrac{\hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt}}{\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}} = \dfrac{\hat{i}v_x + \hat{j}v_y}{\sqrt{v_x^2 + v_y^2}}$

Can we find a unit vector $\boldsymbol{u}_N$?



Note if $\boldsymbol{C} = \boldsymbol{i}a + \boldsymbol{j}b$, and $\boldsymbol{D} = -\boldsymbol{i}b + \boldsymbol{j}a$.

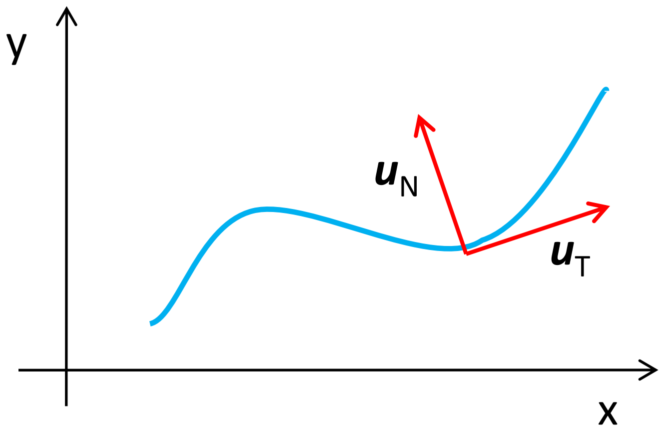
Then $\boldsymbol{C} \bullet \boldsymbol{D} = -ab + ba = 0$

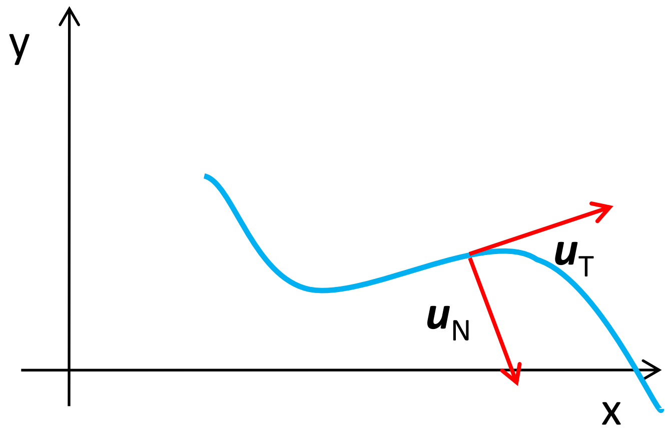


So if $\hat{u}_T = \dfrac{\hat{i}v_x + \hat{j}v_y}{\sqrt{v_x^2 + v_y^2}}$, then $\hat{u}_N = \dfrac{-\hat{i}v_y + \hat{j}v_x}{\sqrt{v_x^2 + v_y^2}}$

And if $\hat{u}_T = \dfrac{\hat{i}1 + \hat{j}\dfrac{dy}{dx}}{\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}}$, then $\hat{u}_N = \dfrac{-\hat{i}\dfrac{dy}{dx} + \hat{j}1}{\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}}$

Warning – normals depend on curvature





$$\hat{u}_N = \frac{-\hat{i}v_y + \hat{j}v_x}{\sqrt{v_x^2 + v_y^2}}$$

$$\hat{u}_N = \frac{\hat{i}v_y - \hat{j}v_x}{\sqrt{v_x^2 + v_y^2}}$$

$$\hat{u}_N = \frac{-\hat{i}\frac{dy}{dx} + \hat{j}1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$

$$\hat{u}_N = \frac{\hat{i}\frac{dy}{dx} - \hat{j}1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$