

EQUATIONS OF MOTION: NORMAL AND TANGENTIAL COORDINATES

Today's Objectives:

Students will be able to:

1. Apply the equation of motion using normal and tangential coordinates.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equation of Motion In n-t Coordinates
- Concept Quiz
- Group Problem Solving
- Attention Quiz



READING QUIZ

1. The “normal” component of the equation of motion is written as $\Sigma \mathbf{F}_n = m\mathbf{a}_n$, where $\Sigma \mathbf{F}_n$ is referred to as the _____.
 - A) impulse
 - B) centripetal force
 - C) tangential force
 - D) inertia force
2. The positive \mathbf{n} direction of the normal and tangential coordinates is _____.
 - A) normal to the tangential component
 - B) always directed toward the center of curvature
 - C) normal to the bi-normal component
 - D) All of the above.



APPLICATIONS



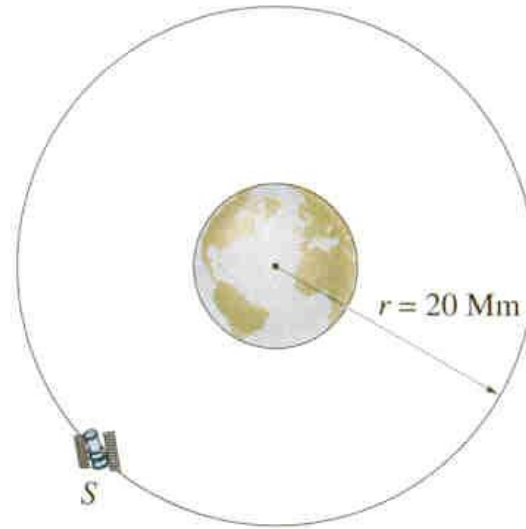
Race tracks are often banked in the turns to reduce the frictional forces required to keep the cars from sliding at high speeds.

If the car's maximum velocity and a minimum coefficient of friction between the tires and track are specified, how can we determine the minimum banking angle (θ) required to prevent the car from sliding?



APPLICATIONS

(continued)



Satellites are held in orbit around the earth by using the earth's gravitational pull as the centripetal force – the force acting to change the direction of the satellite's velocity.

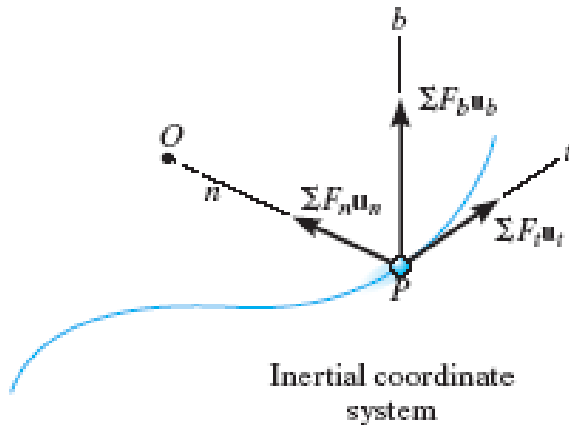
Knowing the radius of orbit of the satellite, how can we determine the required speed of the satellite to maintain this orbit?



NORMAL & TANGENTIAL COORDINATES

(Section 13.5)

When a particle moves along a **curved path**, it may be more convenient to write the equation of motion in terms of **normal** and **tangential** coordinates.

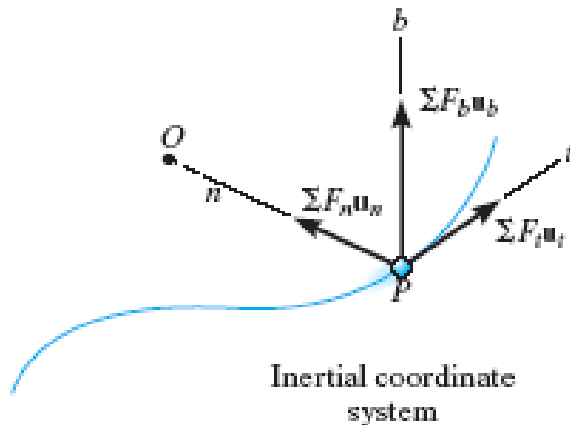


The normal direction (n) **always** points toward the path's **center of curvature**. In a circle, the center of curvature is the center of the circle.

The tangential direction (t) is **tangent** to the path, usually set as positive in the direction of motion of the particle.



EQUATIONS OF MOTION



Since the equation of motion is a **vector** equation, $\Sigma \mathbf{F} = m\mathbf{a}$, it may be written in terms of the n & t coordinates as

$$\Sigma F_t u_t + \Sigma F_n u_n + \Sigma F_b u_b = ma_t + ma_n$$

Here ΣF_t & ΣF_n are the sums of the force components acting in the t & n directions, respectively.

This vector equation will be satisfied provided the individual components on each side of the equation are equal, resulting in the two **scalar** equations: $\Sigma F_t = ma_t$ and $\Sigma F_n = ma_n$.

Since there is no motion in the binormal (b) direction, we can also write $\Sigma F_b = 0$.



NORMAL AND TANGENTIAL ACCELERATIONS

The **tangential acceleration**, $a_t = dv/dt$, represents the time rate of **change in the magnitude of the velocity**. Depending on the direction of $\sum F_t$, the particle's speed will either be increasing or decreasing.

The **normal acceleration**, $a_n = v^2/\rho$, represents the time rate of **change in the direction** of the velocity vector. Remember, a_n **always** acts toward the path's center of curvature. Thus, $\sum F_n$ will always be directed toward the center of the path.

Recall, if the path of motion is defined as $y = f(x)$, the **radius of curvature** at any point can be obtained from

$$\rho = \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|}$$



SOLVING PROBLEMS WITH n-t COORDINATES

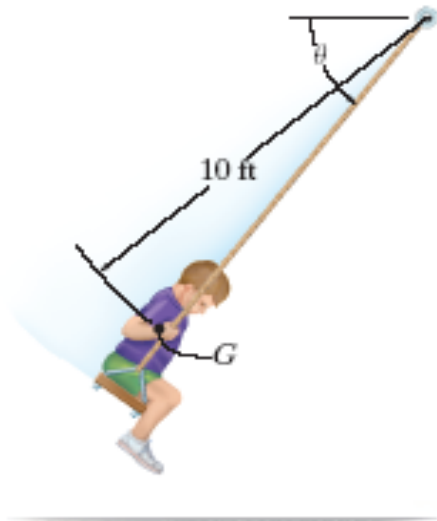
- Use n-t coordinates when a particle is moving along a known, **curved** path.
- Establish the **n-t coordinate system** on the particle.
- Draw **free-body** and **kinetic diagrams** of the particle. The **normal acceleration** (a_n) always acts “inward” (the positive n-direction). The **tangential acceleration** (a_t) may act in either the positive or negative t direction.
- Apply the **equations of motion** in scalar form and solve.
- It may be necessary to employ the **kinematic relations**:

$$a_t = dv/dt = v dv/ds$$

$$a_n = v^2/\rho$$



EXAMPLE

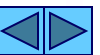


Given: At the instant $\theta = 60^\circ$, the boy's center of mass G is momentarily at rest. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.

Find: The boy's speed and the tension in each of the two supporting cords of the swing when $\theta = 90^\circ$.

Plan:

- 1) Since the problem involves a curved path and finding the force perpendicular to the path, use n - t coordinates. Draw the boy's free-body and kinetic diagrams.
- 2) Apply the equation of motion in the n - t directions.
- 3) Use kinematics to relate the boy's acceleration to his speed.



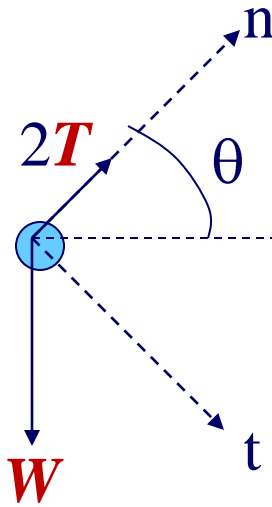
EXAMPLE

(continued)

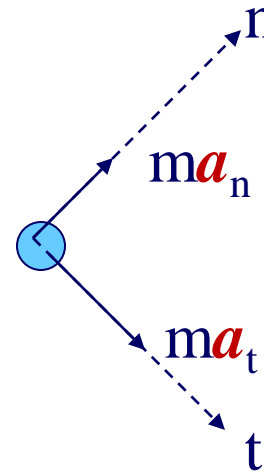
Solution:

- 1) The n-t coordinate system can be established on the boy at some arbitrary angle θ . Approximating the boy and seat together as a particle, the free-body and kinetic diagrams can be drawn.

Free-body diagram



Kinetic diagram



T = tension in each cord

W = weight of the boy



EXAMPLE

(continued)

2) Apply the **equations of motion** in the n-t directions.

$$(a) \sum F_n = ma_n \Rightarrow 2T - W \sin \theta = ma_n$$

Using $a_n = v^2/\rho = v^2/10$, $w = 60$ lb, and $m = w/g = (60/32.2)$,

$$\text{we get: } 2T - 60 \sin \theta = (60/32.2)(v^2/10) \quad (1)$$

$$(b) \sum F_t = ma_t \Rightarrow W \cos \theta = ma_t$$

$$\Rightarrow 60 \cos \theta = (60/32.2) a_t$$

$$\text{Solving for } a_t: a_t = 32.2 \cos \theta \quad (2)$$

Note that there are **2 equations and 3 unknowns** (T , v , a_t).

One more equation is needed.



EXAMPLE

(continued)

3) Apply **kinematics** to relate a_t and v .

$$v \, dv = a_t \, ds \quad \text{where} \quad ds = \rho \, d\theta = 10 \, d\theta$$

$$\Rightarrow v \, dv = 32.2 \cos\theta \, ds = 32.2 \cos\theta (10 \, d\theta)$$

$$\Rightarrow \int_0^v v \, dv = \int_{60}^{90} 322 \cos\theta \, d\theta$$

$$\Rightarrow \frac{v^2}{2} = 322 \sin\theta \Big|_{60}^{90} \Rightarrow v = 9.29 \text{ ft/s}$$

This v is the speed of the boy at $\theta = 90^\circ$. This value can be substituted into equation (1) to solve for T .

$$2T - 60 \sin(90^\circ) = (60/32.2)(9.29)^2/10$$

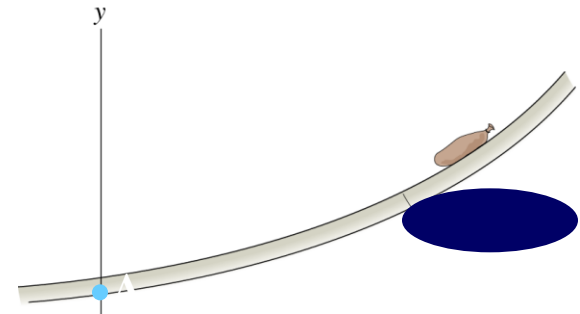
$$T = 38.0 \text{ lb} \quad (\text{the tension in each cord})$$



CONCEPT QUIZ

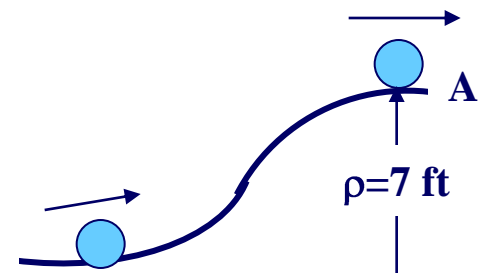
1. A 10 kg sack slides down a smooth surface. If the normal force on the surface at the flat spot, A, is 98.1 N (\uparrow), the radius of curvature is _____.

- A) 0.2 m B) 0.4 m
C) 1.0 m D) None of the above.

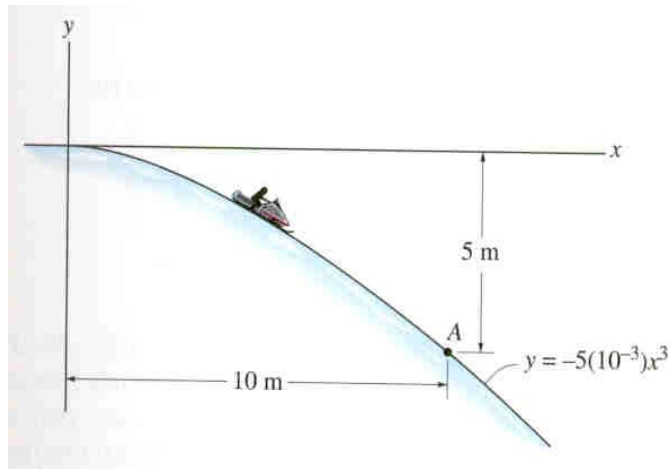


2. A 20 lb block is moving along a smooth surface. If the normal force on the surface at A is 10 lb, the velocity is _____.

- A) 7.6 ft/s B) 9.6 ft/s
C) 10.6 ft/s D) 12.6 ft/s



GROUP PROBLEM SOLVING



Given: A 200 kg snowmobile with rider is traveling down the hill. When it is at point A, it is traveling at 4 m/s and increasing its speed at 2 m/s^2 .

Find: The resultant normal force and resultant frictional force exerted on the tracks at point A.

- Plan:**
- 1) Treat the snowmobile as a particle. Draw the free-body and kinetic diagrams.
 - 2) Apply the equations of motion in the n-t directions.
 - 3) Use calculus to determine the slope and radius of curvature of the path at point A.

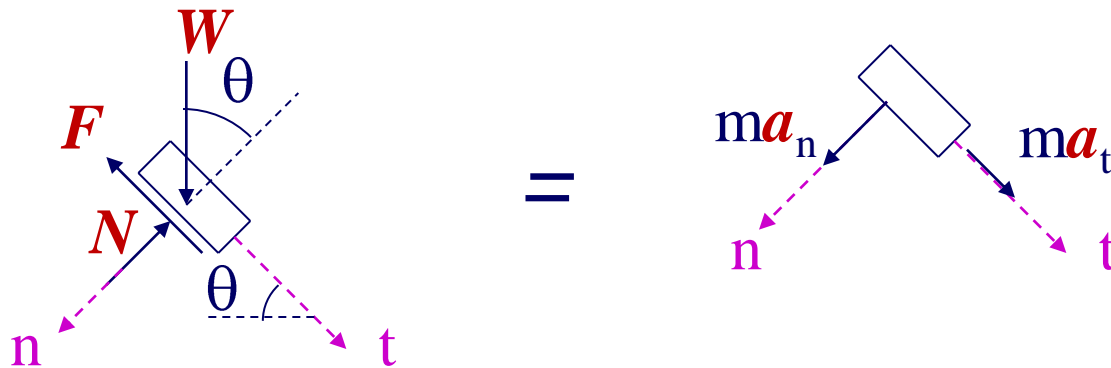


GROUP PROBLEM SOLVING

(continued)

Solution:

- 1) The n-t coordinate system can be established on the snowmobile at point A. Treat the snowmobile and rider as a particle and draw the free-body and kinetic diagrams:



$W = mg =$ weight of snowmobile and passenger

$N =$ resultant normal force on tracks

$F =$ resultant friction force on tracks



GROUP PROBLEM SOLVING

(continued)

2) Apply the **equations of motion** in the n-t directions:

$$\sum F_n = ma_n \Rightarrow W \cos \theta - N = ma_n$$

$$\text{Using } W = mg \text{ and } a_n = v^2/\rho = (4)^2/\rho$$

$$\Rightarrow (200)(9.81) \cos \theta - N = (200)(16/\rho)$$

$$\Rightarrow N = 1962 \cos \theta - 3200/\rho \quad (1)$$

$$\sum F_t = ma_t \Rightarrow W \sin \theta - F = ma_t$$

$$\text{Using } W = mg \text{ and } a_t = 2 \text{ m/s}^2 \text{ (given)}$$

$$\Rightarrow (200)(9.81) \sin \theta - F = (200)(2)$$

$$\Rightarrow F = 1962 \sin \theta - 400 \quad (2)$$



GROUP PROBLEM SOLVING

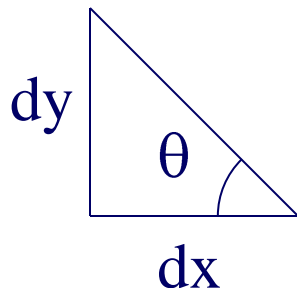
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3) Determine ρ by differentiating $y = f(x)$ at $x = 10$ m:

$$y = -5(10^{-3})x^3 \Rightarrow dy/dx = (-15)(10^{-3})x^2 \Rightarrow d^2y/dx^2 = -30(10^{-3})x$$

$$\rho \Big|_{x=10 \text{ m}} = \frac{[1 + (\frac{dy}{dx})^2]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} = \frac{[1 + (-1.5)^2]^{3/2}}{|-0.3|} = 19.53 \text{ m}$$

Determine θ from the slope of the curve at A:



$$\tan \theta = dy/dx \Big|_{x=10 \text{ m}}$$

$$\theta = \left| \tan^{-1} (dy/dx) \right| = \left| \tan^{-1} (-1.5) \right| = 56.31$$

$$\text{From Eq.(1): } N = 1962 \cos(56.31) - 3200/19.53 = 924 \text{ N}$$

$$\text{From Eq.(2): } F = 1962 \sin(56.31) - 400 = 1232 \text{ N}$$



ATTENTION QUIZ

- The tangential acceleration of an object
 - represents the rate of change of the velocity vector's direction.
 - represents the rate of change in the magnitude of the velocity.
 - is a function of the radius of curvature.
 - Both B and C.
- The ball has a mass of 20 kg and a speed of $v = 30 \text{ m/s}$ at the instant it is at its lowest point. Determine the tension in the cord at this instant.
 - 1596 N
 - 1796 N
 - 1996 N
 - 2196 N

