THE WORK OF A FORCE, PRINCIPLE OF WORK AND ENERGY, & PRINCIPLE OF WORK AND ENERGY FOR A SYSTEM OF PARTICLES

Today's Objectives:

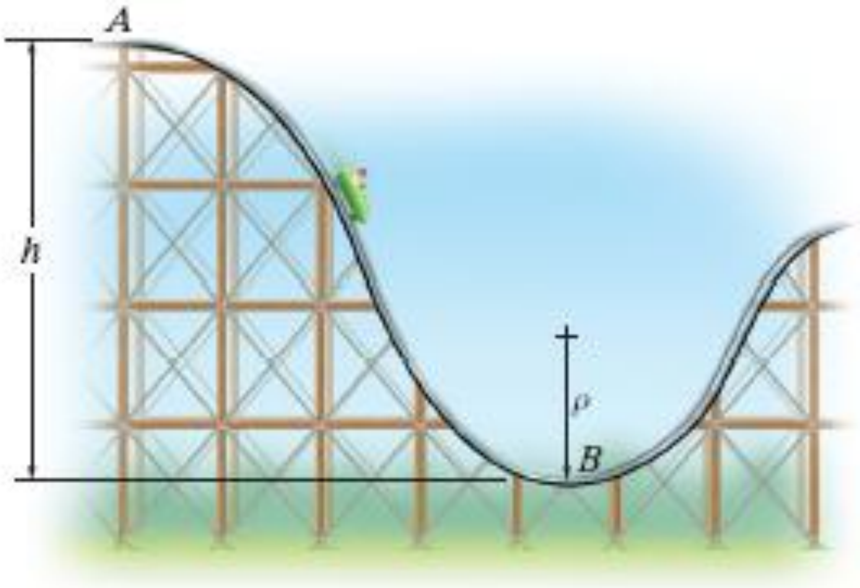
Students will be able to:

1. Calculate the work of a force.
2. Apply the principle of work and energy to a particle or system of particles.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Work of A Force
- Principle of Work And Energy
- Concept Quiz
- Group Problem Solving
- Attention Quiz

APPLICATIONS



A roller coaster makes use of gravitational forces to assist the cars in reaching high speeds in the "valleys" of the track.

How can we design the track (e.g., the height, h, and the radius of curvature, ρ) to control the forces experienced by the passengers?

APPLICATIONS
(continued)

Crash barrels are often used along roadways for crash protection. The barrels absorb the car’s kinetic energy by deforming.

If we know the typical velocity of an oncoming car and the amount of energy that can be absorbed by each barrel, how can we design a crash cushion?

WORK AND ENERGY

Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion ($\boldsymbol{F} = m\boldsymbol{a}$) with respect to displacement.

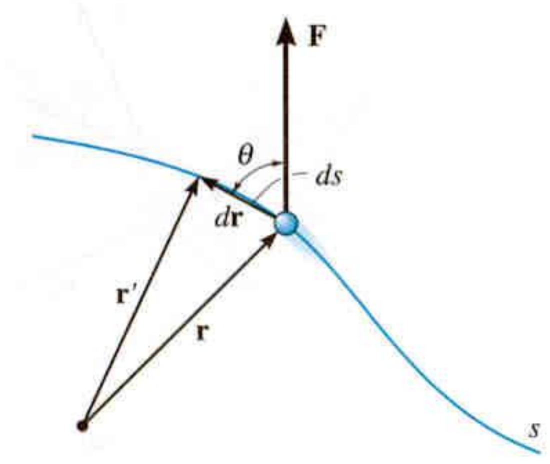
By substituting $a_t = v\,(dv/ds)$ into $F_t = ma_t$, the result is integrated to yield an equation known as the principle of work and energy.

This principle is useful for solving problems that involve force, velocity, and displacement. It can also be used to explore the concept of power.

To use this principle, we must first understand how to calculate the work of a force.

WORK OF A FORCE (Section 14.1)

A force does work on a particle when the particle undergoes a displacement along the line of action of the force.



Work is defined as the product of force and displacement components acting in the same direction. So, if the angle between the force and displacement vector is θ, the increment of work dU done by the force is

$$dU = F\, ds \cos\theta$$

By using the definition of the dot product and integrating, the total work can be written as

$$U_{1\text{-}2} = \int_{\boldsymbol{r}_1}^{\boldsymbol{r}_2} \boldsymbol{F} \bullet d\boldsymbol{r}$$

WORK OF A FORCE
(continued)

If ***F*** is a function of position (a common case) this becomes

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

If both F and θ are constant ($F = F_c$), this equation further simplifies to

$$U_{1\text{-}2} = F_c \cos\theta \, (s_2 - s_1)$$

Work is positive if the force and the movement are in the same direction. If they are opposing, then the work is negative. If the force and the displacement directions are perpendicular, the work is zero.

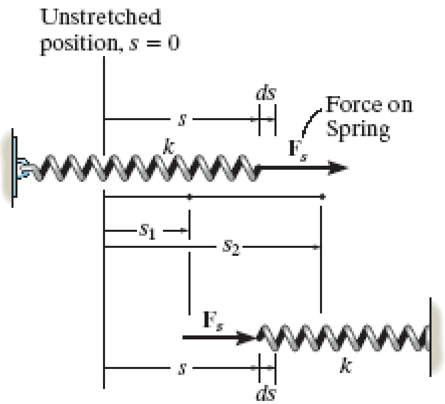
WORK OF A WEIGHT

The work done by the gravitational force acting on a particle (or weight of an object) can be calculated by using

$$U_{1\text{-}2} = \int_{y_1}^{y_2} -W\,dy = -W(y_2 - y_1) = -W\,\Delta y$$

The work of a weight is the product of the magnitude of the particle’s weight and its vertical displacement. If Δy is upward, the work is negative since the weight force always acts downward.

WORK OF A SPRING FORCE



When stretched, a linear elastic spring develops a force of magnitude $F_s = ks$, where k is the spring stiffness and s is the displacement from the unstretched position.

The work of the spring force moving from position s_1 to position s_2 is

$$U_{1\text{-}2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} k\, s\, ds = 0.5k(s_2)^2 - 0.5k(s_1)_2$$

If a particle is attached to the spring, the force F_s exerted on the particle is opposite to that exerted on the spring. Thus, the work done on the particle by the spring force will be negative or

$$U_{1\text{-}2} = -[\, 0.5k\,(s_2)^2 - 0.5k\,(s_1)^2 \,] \; .$$

SPRING FORCES

It is important to note the following about spring forces:

1. The equations just shown are for linear springs only! Recall that a linear spring develops a force according to F = ks (essentially the equation of a line).

2. The work of a spring is not just spring force times distance at some point, i.e., $(ks_i)(s_i)$. Beware, this is a trap that students often fall into!

3. Always double check the sign of the spring work after calculating it. It is positive work if the force put on the object by the spring and the movement are in the same direction.

PRINCIPLE OF WORK AND ENERGY
(Section 14.2 & Section 14.3)

By integrating the equation of motion, $\sum F_t = ma_t = mv(dv/ds)$, the principle of work and energy can be written as

$$\sum U_{1\text{-}2} = 0.5m(v_2)^2 - 0.5m(v_1)^2 \quad \text{or} \quad T_1 + \sum U_{1\text{-}2} = T_2$$

$\sum U_{1\text{-}2}$ is the work done by all the forces acting on the particle as it moves from point 1 to point 2. Work can be either a positive or negative scalar.

T_1 and T_2 are the kinetic energies of the particle at the initial and final position, respectively. Thus, $T_1 = 0.5\ m\ (v_1)^2$ and $T_2 = 0.5\ m\ (v_2)^2$. The kinetic energy is always a positive scalar (velocity is squared!).

So, the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle's final kinetic energy.

PRINCIPLE OF WORK AND ENERGY
(continued)

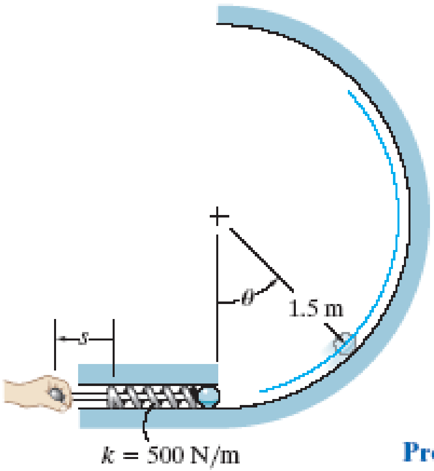
Note that the principle of work and energy ($T_1 + \sum U_{1\text{-}2} = T_2$) is not a vector equation! Each term results in a scalar value.

Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a joule (J), where 1 J = 1 N·m. In the FPS system, units are ft·lb.

The principle of work and energy cannot be used, in general, to determine forces directed normal to the path, since these forces do no work.

The principle of work and energy can also be applied to a system of particles by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system.

EXAMPLE



Prob. 14–10

Given: A 0.5 kg ball of negligible size is fired up a vertical track of radius 1.5 m using a spring plunger with k = 500 N/m. The plunger keeps the spring compressed 0.08 m when s = 0.

Find: The distance s the plunger must be pulled back and released so the ball will begin to leave the track when $\theta = 135$.

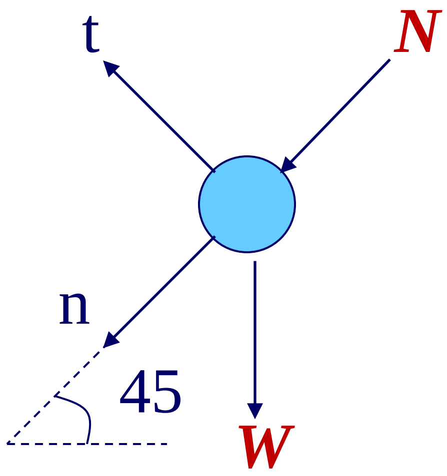
Plan:

1) Draw the FBD of the ball at $\theta = 135$.
2) Apply the equation of motion in the n-direction to determine the speed of the ball when it leaves the track.
3) Apply the principle of work and energy to determine s.

EXAMPLE
(continued)

Solution:

1) Draw the FBD of the ball at $\theta = 135$.



The weight (*W*) acts downward through the center of the ball. The normal force exerted by the track is perpendicular to the surface. The friction force between the ball and the track has no component in the n-direction.

2) Apply the equation of motion in the n-direction. Since the ball leaves the track at $\theta = 135$, set N = 0.

$$\Rightarrow \swarrow + \sum F_n = m a_n = m\,(v^2/\rho) \quad \Rightarrow \quad W \cos 45 = m\,(v^2/\rho)$$

$$\Rightarrow (0.5)(9.81)\cos 45 = (0.5/1.5)v^2 \quad \Rightarrow \quad v = 3.2257 \text{ m/s}$$

EXAMPLE
(continued)

3) Apply the principle of work and energy between position 1 ($\theta = 0$) and position 2 ($\theta = 135$). Note that the normal force (***N***) does no work since it is always perpendicular to the displacement direction. (Students: Draw a FBD to confirm the work forces).

$$T_1 + \sum U_{1\text{-}2} = T_2$$

$$0.5m\,(v_1)^2 - W\,\Delta y - (0.5k(s_2)^2 - 0.5k\,(s_1)^2) = 0.5m\,(v_2)^2$$

and $v_1 = 0$, $v_2 = 3.2257$ m/s

$s_1 = s + 0.08$ m, $s_2 = 0.08$ m

$\Delta y = 1.5 + 1.5 \sin 45 \;=\; 2.5607$ m

$$\Rightarrow\; 0 - (0.5)(9.81)(2.5607) - [0.5(500)(0.08)^2 - 0.5(500)(5 + 0.08)^2] = 0.5(0.5)(3.2257)^2$$

$$\Rightarrow\; s = 0.179 \text{ m} = 179 \text{ mm}$$

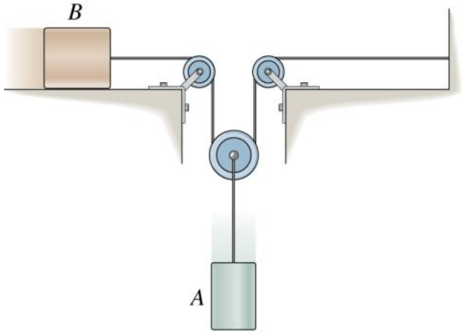
CONCEPT QUIZ

1. A spring with an un-stretched length of 5 in expands from a length of 2 in to a length of 4 in. The work done on the spring is __________ in ·lb .

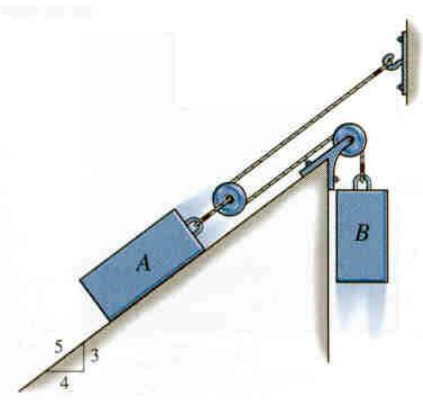
   A) $0.5\ k\ (2\ in)^2$
   B) $-[0.5\ k(4\ in)^2 - 0.5\ k(2\ in)^2]$
   C) $-[0.5\ k(3\ in)^2 - 0.5\ k(1\ in)^2]$
   D) $0.5\ k(3\ in)^2 - 0.5\ k(1\ in)^2$

2. Two blocks are initially at rest. How many equations would be needed to determine the velocity of block A after block B moves 4 m horizontally on the smooth surface?

   A) One
   B) Two
   C) Three
   D) Four



GROUP PROBLEM SOLVING



Given: Block A has a weight of 60 lb and block B has a weight of 10 lb. The coefficient of kinetic friction between block A and the incline is $\mu_k = 0.2$. Neglect the mass of the cord and pulleys.

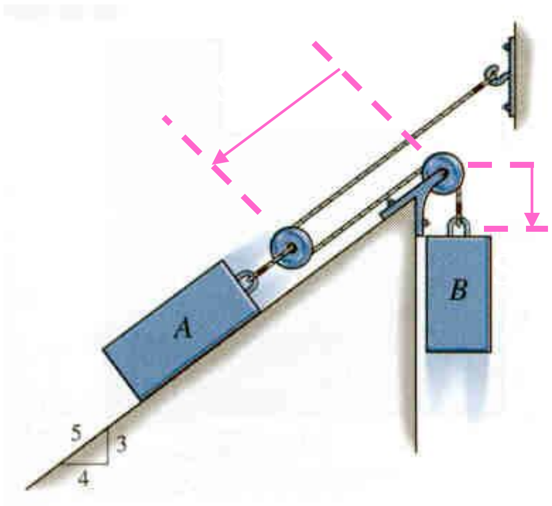
Find: The speed of block A after it moves 3 ft down the plane, starting from rest.

Plan:
1) Define the kinematic relationships between the blocks.
2) Draw the FBD of each block.
3) Apply the principle of work and energy to the system of blocks.

GROUP PROBLEM SOLVING
(continued)

Solution:

1) The kinematic relationships can be determined by defining position coordinates s_A and s_B, and then differentiating.



Since the cable length is constant:

$$2s_A + s_B = l$$

$$2\Delta s_A + \Delta s_B = 0$$

$$\Delta s_A = 3\text{ft} \quad \Rightarrow \quad \Delta s_B = -6 \text{ ft}$$

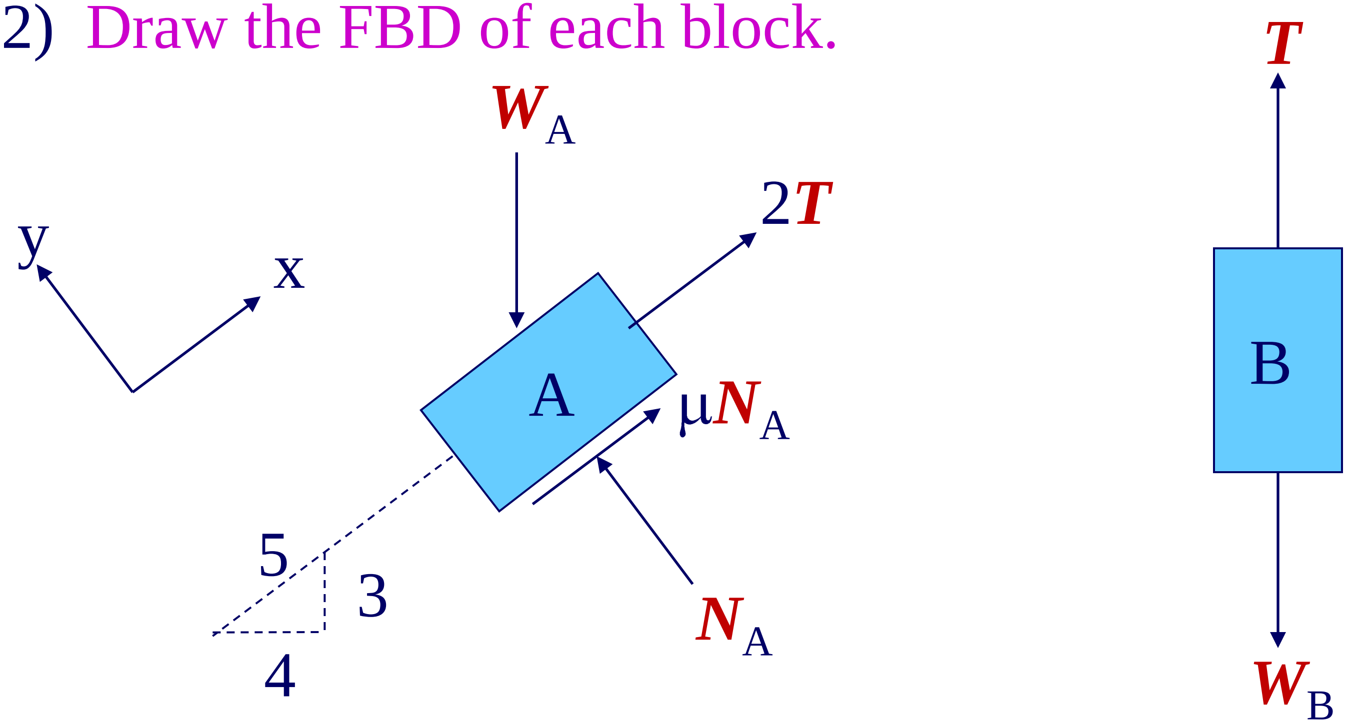
and $2v_A + v_B = 0$

$$\Rightarrow v_B = -2v_A$$

Note that, by this definition of s_A and s_B, positive motion for each block is defined as downwards.

GROUP PROBLEM SOLVING
(continued)

2) Draw the FBD of each block.



Sum forces in the y-direction for block A (note that there is no motion in this direction):

$\sum F_y = 0$: $N_A - (4/5)W_A = 0$ => $N_A = (4/5)W_A$

GROUP PROBLEM SOLVING
(continued)

3) Apply the principle of work and energy to the system (the blocks start from rest).

$$\sum T_1 + \sum U_{1\text{-}2} = \sum T_2$$

$$(0.5m_A(v_{A1})^2 + .5m_B(v_{B1})^2) + ((3/5)W_A - 2T - \mu N_A)\Delta s_A$$
$$+ (W_B - T)\Delta s_B = (0.5m_A(v_{A2})^2 + 0.5m_B(v_{B2})^2)$$

$v_{A1} = v_{B1} = 0$, $\Delta s_A = 3\text{ft}$, $\Delta s_B = -6\text{ ft}$, $v_B = -2v_A$, $N_A = (4/5)W_A$

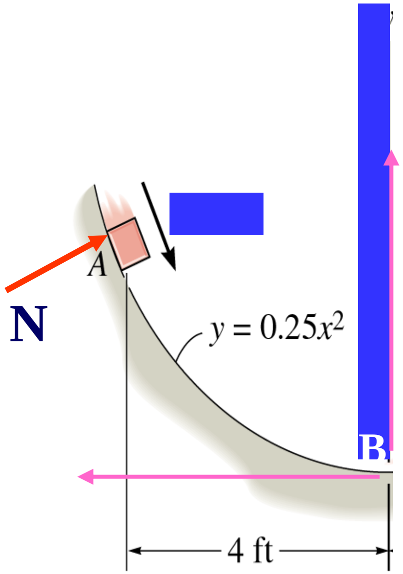
$$\Rightarrow \quad 0 + 0 + (3/5)(60)(3) - 2T(3) - (0.2)(0.8)(60)(3) + (10)(-6)$$
$$- T(-6) = 0.5(60/32.2)(v_{A2})^2 + 0.5(10/32.2)(-2v_{A2})^2$$

$$\Rightarrow \quad v_{A2} = 3.52\text{ ft/s}$$

Note that the work due to the cable tension force on each block cancels out.

ATTENTION QUIZ

1. What is the work done by the normal force N if a 10 lb box is moved from A to B ?



   A) - 1.24 lb · ft B) 0 lb · ft

   C) 1.24 lb · ft D) 2.48 lb · ft

2. If a spring force is $F = 5\ s^3$ N/m and the spring is compressed by s = 0.5 m, the work done on a particle attached to the spring will be

   A) 0.625 N · m B) – 0.625 N · m

   C) 0.0781 N · m D) – 0.0781 N · m