POWER AND EFFICIENCY

<u>Today's Objectives</u>:

Students will be able to:

1. Determine the power generated by a machine, engine, or motor.
2. Calculate the mechanical efficiency of a machine.

<u>In-Class Activities</u>:

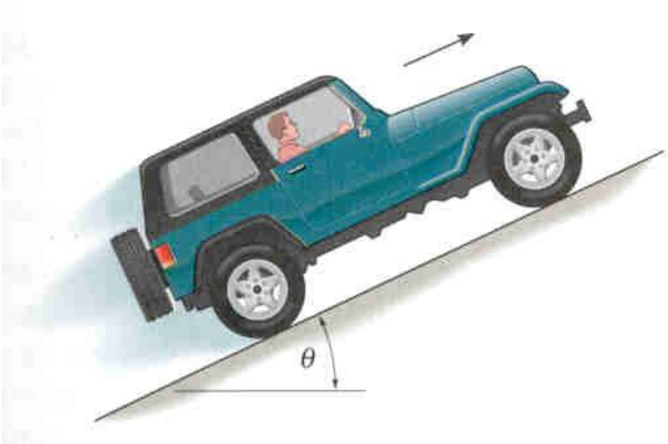
- Check Homework
- Reading Quiz
- Applications
- Define Power
- Define Efficiency
- Concept Quiz
- Group Problem Solving
- Attention Quiz

APPLICATIONS

Engines and motors are often rated in terms of their power output. The power requirements of the motor lifting this elevator depend on the vertical force ***F*** that acts on the elevator, causing it to move upwards.

Given the desired lift velocity for the elevator, how can we determine the power requirement of the motor?

APPLICATIONS
(continued)



The speed at which a vehicle can climb a hill depends in part on the power output of the engine and the angle of inclination of the hill.

For a given angle, how can we determine the speed of this jeep, knowing the power transmitted by the engine to the wheels?

POWER AND EFFICIENCY
(Section 14.4)

Power is defined as the amount of work performed per unit of time.

If a machine or engine performs a certain amount of work, dU, within a given time interval, dt, the power generated can be calculated as

$$P = dU/dt$$

Since the work can be expressed as $dU = \mathbf{F} \cdot d\mathbf{r}$, the power can be written

$$P = dU/dt = (\mathbf{F} \cdot d\mathbf{r})/dt = \mathbf{F} \cdot (d\mathbf{r}/dt) = \mathbf{F} \cdot \mathbf{v}$$

Thus, power is a scalar defined as the product of the force and velocity components acting in the same direction.

POWER

Using scalar notation, power can be written

$$P = \boldsymbol{F} \bullet \boldsymbol{v} = F\, v \cos \theta$$

where θ is the angle between the force and velocity vectors.

So if the velocity of a body acted on by a force ***F*** is known, the power can be determined by calculating the dot product or by multiplying force and velocity components.

The unit of power in the SI system is the watt (W) where

$$1\text{ W} = 1\text{ J/s} = 1\,(\text{N}\cdot\text{m})/\text{s}\,.$$

In the FPS system, power is usually expressed in units of horsepower (hp) where

$$1\text{ hp} = 550\,(\text{ft}\cdot\text{lb})/\text{s} = 746\text{ W}\,.$$

EFFICIENCY

The mechanical efficiency of a machine is the ratio of the useful power produced (output power) to the power supplied to the machine (input power) or

$$\varepsilon = (\text{power output})/(\text{power input})$$

If energy input and removal occur at the same time, efficiency may also be expressed in terms of the ratio of output energy to input energy or

$$\varepsilon = (\text{energy output})/(\text{energy input})$$

Machines will always have frictional forces. Since frictional forces dissipate energy, additional power will be required to overcome these forces. Consequently, the efficiency of a machine is always less than 1.

PROCEDURE FOR ANALYSIS

- Find the resultant external force acting on the body causing its motion. It may be necessary to draw a free-body diagram.
- Determine the velocity of the point on the body at which the force is applied. Energy methods or the equation of motion and appropriate kinematic relations, may be necessary.
- Multiply the force magnitude by the component of velocity acting in the direction of ***F*** to determine the power supplied to the body ($P = F\,v\cos\theta$).
- In some cases, power may be found by calculating the work done per unit of time ($P = dU/dt$).
- If the mechanical efficiency of a machine is known, either the power input or output can be determined.

EXAMPLE

Given: A sports car has a mass of 2 Mg and an engine efficiency of $\varepsilon = 0.65$. Moving forward, the wind creates a drag resistance on the car of $F_D = 1.2v^2$ N, where v is the velocity in m/s. The car accelerates at 5 m/s^2, starting from rest.

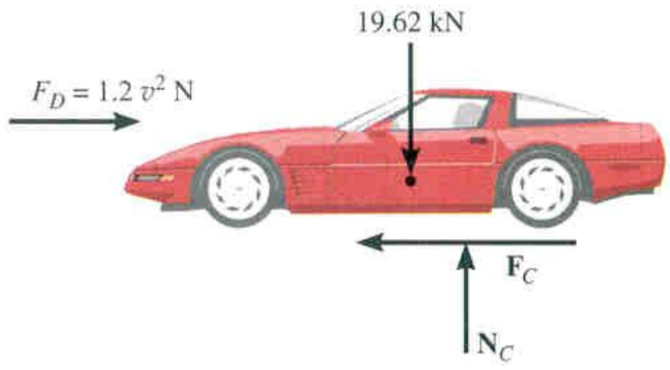
Find: The engine's input power when t = 4 s.

Plan:
1) Draw a free body diagram of the car.
2) Apply the equation of motion and kinematic equations to find the car's velocity at t = 4 s.
3) Determine the power required for this motion.
4) Use the engine's efficiency to determine input power.

EXAMPLE
(continued)

Solution:

1) Draw the FBD of the car.



The drag force and weight are known forces. The normal force ***N***$_c$ and frictional force ***F***$_c$ represent the resultant forces of all four wheels. The frictional force between the wheels and road pushes the car forward.

2) The equation of motion can be applied in the x-direction, with $a_x = 5$ m/s^2:

$$\overset{+}{\leftarrow}\ \sum F_x = m a_x \quad \Rightarrow \quad F_c - 1.2v^2 = (2000)(5)$$

$$\Rightarrow \quad F_c = (10{,}000 + 1.2v^2)\ \text{N}$$

EXAMPLE
(continued)

3) The constant acceleration equations can be used to determine the car's velocity.

$$v_x = v_{xo} + a_x t = 0 + (5)(4) = 20 \text{ m/s}$$

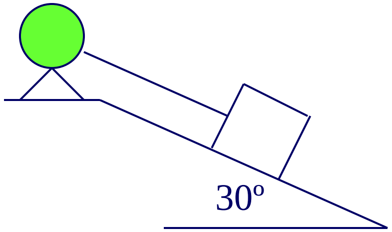
4) The power output of the car is calculated by multiplying the driving (frictional) force and the car's velocity:

$$P_o = (F_c)(v_x) = [10{,}000 + (1.2)(20)^2](20) = 209.6 \text{ kW}$$

5) The power developed by the engine (prior to its frictional losses) is obtained using the efficiency equation.

$$P_i = P_o/\varepsilon = 209.6/0.65 = 322 \text{ kW}$$

CONCEPT QUIZ



1. A motor pulls a 10 lb block up a smooth incline at a constant velocity of 4 ft/s. Find the power supplied by the motor.

   A) 8.4 ft·lb/s B) 20 ft·lb/s

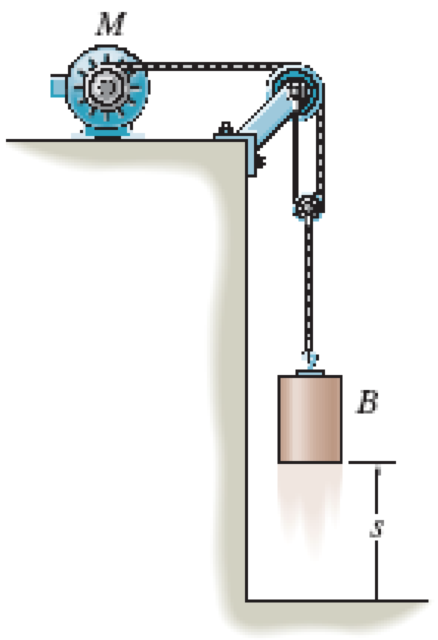
   C) 34.6 ft·lb/s D) 40 ft·lb/s

2. A twin engine jet aircraft is climbing at a 10 degree angle at 264 ft/s. The thrust developed by a jet engine is 1000 lb. The power developed by the engines is

   A) (1000 lb)(140 ft/s) B) (2000 lb)(140 ft/s) cos 10

   C) (1000 lb)(140 ft/s) cos 10 D) (2000 lb)(140 ft/s)

GROUP PROBLEM SOLVING



Given: A 50-lb load (B) is hoisted by the pulley system and motor M. The motor has an efficiency of 0.76 and exerts a constant force of 30 lb on the cable. Neglect the mass of the pulleys and cable.

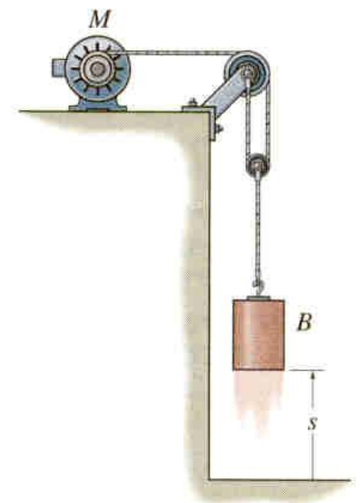
Find: The power supplied to the motor when the load has been hoisted 10 ft. The block started from rest.

Plan:
1) Relate the cable and block velocities by defining position coordinates. Draw a FBD of the block.
2) Use the equation of motion or energy methods to determine the block's velocity at 10 feet.
3) Calculate the power supplied by the motor and to the motor.

GROUP PROBLEM SOLVING
(continued)

Solution:

1) Define position coordinates to relate velocities.



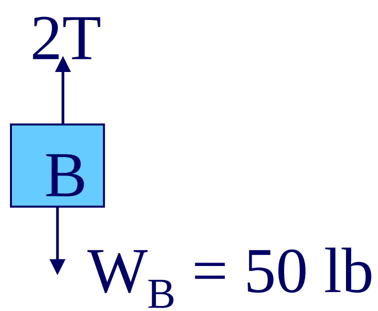
Here s_m is defined to a point on the cable. Also s_B is defined only to the lower pulley, since the block moves with the pulley. From kinematics,

$$s_m + 2s_B = l$$

$$\Rightarrow \quad v_m + 2v_B = 0$$

$$\Rightarrow \quad v_m = -2v_B$$

Draw the FBD of the block:



Since the pulley has no mass, a force balance requires that the tension in the lower cable is twice the tension in the upper cable.

GROUP PROBLEM SOLVING
(continued)

2) The velocity of the block can be obtained by applying the principle of work and energy to the block (recall that the block starts from rest).

$$+\uparrow \quad T_1 + \sum U_{1\text{-}2} = T_2$$

$$0.5m(v_1)^2 + [2T(s) - w_B(s)] = 0.5m\,(v_2)^2$$

$$0 + [2(30)(10) - (50)(10)] = 0.5(50/32.2)(v_2)^2$$

$$\Rightarrow \quad v_2 = v_B = 11.35 \text{ ft/s} \uparrow$$

Since this velocity is upwards, it is a negative velocity in terms of the kinematic equation coordinates.

The velocity of the cable coming into the motor (v_m) is calculated from the kinematic equation.

$$v_m = -2v_B = -(2)(-11.35) = 22.70 \text{ ft/s} \leftarrow$$

GROUP PROBLEM SOLVING
(continued)

3) The power supplied by the motor is the product of the force applied to the cable and the velocity of the cable:

$$P_o = \boldsymbol{F} \bullet \boldsymbol{v} = (30)(22.70) = 681 \text{ (ft} \cdot \text{lb)/s}$$

The power supplied to the motor is determined using the motor's efficiency and the basic efficiency equation.

$$P_i = P_o/\varepsilon = 681/0.76 = 896 \text{ (ft} \cdot \text{lb)/s}$$

Converting to horsepower

$$P_i = 896/550 = 1.63 \text{ hp}$$

ATTENTION QUIZ

1. The power supplied by a machine will always be __________ the power supplied to the machine.

   A) less than B) equal to

   C) greater than D) A or B

2. A car is traveling a level road at 88 ft/s. The power being supplied to the wheels is 52,800 ft·lb/s. Find the combined friction force on the tires.

   A) 8.82 lb B) 400 lb

   C) 600 lb D) 4.64×10^6 lb