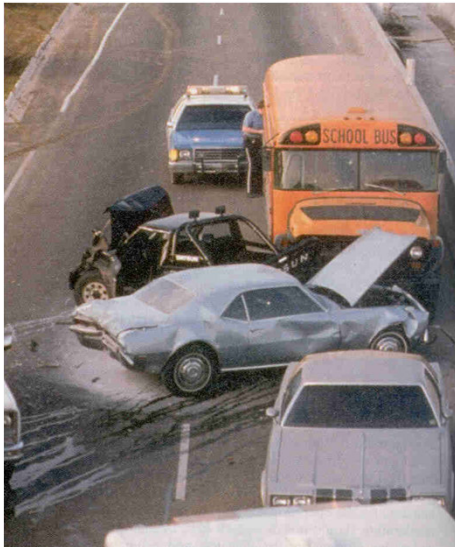


IMPACT

Today's Objectives:

Students will be able to:

1. Understand and analyze the mechanics of impact.
2. Analyze the motion of bodies undergoing a collision, in both central and oblique cases of impact.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Central Impact
- Coefficient of Restitution
- Oblique Impact
- Concept Quiz
- Group Problem Solving
- Attention Quiz

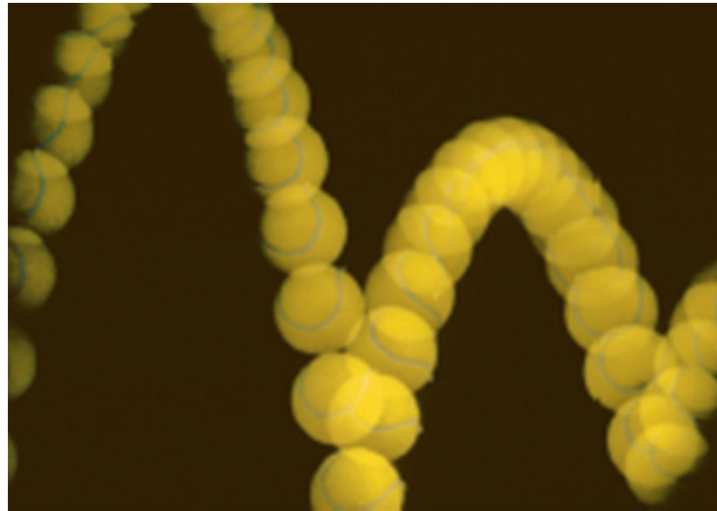


READING QUIZ

1. When the motion of one or both of the particles is at an angle to the line of impact, the impact is said to be _____.
A) central impact
B) oblique impact
C) major impact
D) None of the above.
2. The ratio of the restitution impulse to the deformation impulse is called _____.
A) impulse ratio
B) restitution coefficient
C) energy ratio
D) mechanical efficiency



APPLICATIONS



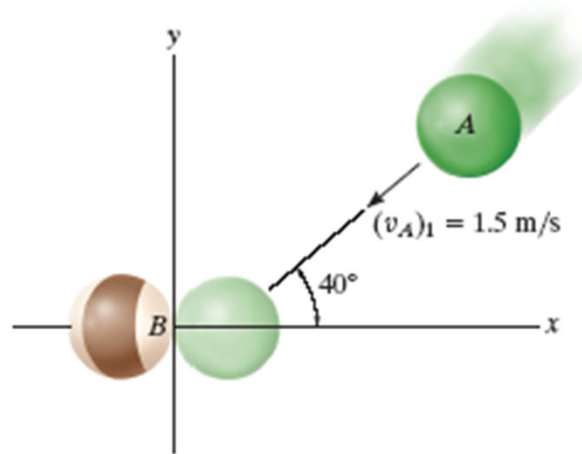
The quality of a tennis ball is measured by the height of its bounce. This can be quantified by the coefficient of restitution of the ball.

If the height from which the ball is dropped and the height of its resulting bounce are known, how can we determine the coefficient of restitution of the ball?



APPLICATIONS

(continued)



In a game of billiards, it is important to be able to predict the trajectory and speed of a ball after it is struck by another ball.

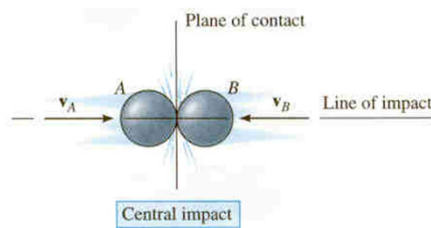
If we know the velocity of ball A before the impact, how can we determine the magnitude and direction of the velocity of ball B after the impact?



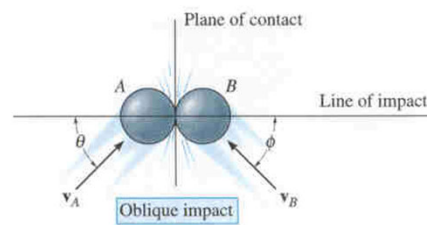
IMPACT

(Section 15.4)

Impact occurs when two bodies collide during a very **short** time period, causing large impulsive forces to be exerted between the bodies. Common examples of impact are a hammer striking a nail or a bat striking a ball. The **line of impact** is a line through the **mass centers** of the colliding particles. In general, there are **two** types of impact:



Central impact occurs when the directions of motion of the two colliding particles are along the line of impact.

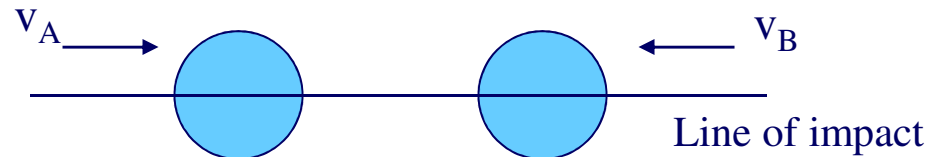


Oblique impact occurs when the direction of motion of one or both of the particles is at an angle to the line of impact.



CENTRAL IMPACT

Central impact happens when the velocities of the two objects are along the line of impact (recall that the line of impact is a line through the particles' mass centers).



Once the particles contact, they may **deform** if they are non-rigid. In any case, energy is transferred between the two particles.

There are two primary equations used when solving impact problems. The textbook provides extensive detail on their derivation.



CENTRAL IMPACT

(continued)

In most problems, the initial velocities of the particles, $(v_A)_1$ and $(v_B)_1$, are known, and it is necessary to determine the final velocities, $(v_A)_2$ and $(v_B)_2$. So the **first** equation used is the **conservation of linear momentum**, applied along the line of impact.

$$(m_A v_A)_1 + (m_B v_B)_1 = (m_A v_A)_2 + (m_B v_B)_2$$

This provides one equation, but there are usually two unknowns, $(v_A)_2$ and $(v_B)_2$. So another equation is needed. The **principle of impulse and momentum** is used to develop this equation, which involves the **coefficient of restitution**, or e .



CENTRAL IMPACT

(continued)

The **coefficient of restitution, e** , is the ratio of the particles' **relative separation velocity** after impact, $(v_B)_2 - (v_A)_2$, to the particles' **relative approach velocity** before impact, $(v_A)_1 - (v_B)_1$. The coefficient of restitution is also an indicator of the energy lost during the impact.

The equation defining the coefficient of restitution, e , is

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

If a value for e is specified, this relation provides the second equation necessary to solve for $(v_A)_2$ and $(v_B)_2$.



COEFFICIENT OF RESTITUTION

In general, e has a value between zero and one. The two limiting conditions can be considered:

- **Elastic impact ($e = 1$):** In a perfectly elastic collision, no energy is lost and the relative separation velocity equals the relative approach velocity of the particles. In practical situations, this condition cannot be achieved.
- **Plastic impact ($e = 0$):** In a plastic impact, the relative separation velocity is zero. The particles stick together and move with a common velocity after the impact.

Some **typical values** of e are:

Steel on steel: 0.5 – 0.8

Wood on wood: 0.4 – 0.6

Lead on lead: 0.12 – 0.18

Glass on glass: 0.93 – 0.95



IMPACT: ENERGY LOSSES

Once the particles' velocities before and after the collision have been determined, the **energy loss** during the collision can be calculated on the basis of the difference in the particles' **kinetic energy**. The energy loss is

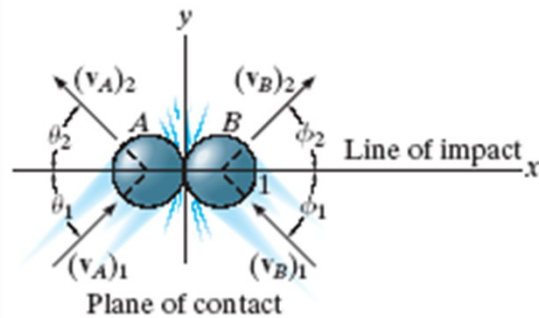
$$\sum U_{1-2} = \sum T_2 - \sum T_1 \quad \text{where } T_i = 0.5m_i(v_i)^2$$

During a collision, some of the particles' initial kinetic energy will be lost in the form of heat, sound, or due to localized deformation.

In a **plastic collision** ($e = 0$), the energy lost is a maximum, although it does not necessarily go to zero. Why?

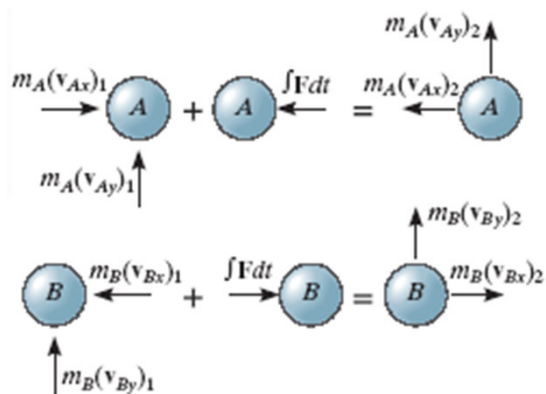


OBLIQUE IMPACT



In an **oblique impact**, one or both of the particles' motion is at an angle to the line of impact. Typically, there will be four unknowns: the **magnitudes** and **directions** of the final velocities.

The four equations required to solve for the unknowns are:



Conservation of momentum and the coefficient of restitution equation are applied **along** the line of impact (x-axis):

$$m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$e = [(v_{Bx})_2 - (v_{Ax})_2] / [(v_{Ax})_1 - (v_{Bx})_1]$$

Momentum of each particle is conserved in the direction **perpendicular** to the line of impact (y-axis):

$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \quad \text{and} \quad m_B(v_{By})_1 = m_B(v_{By})_2$$

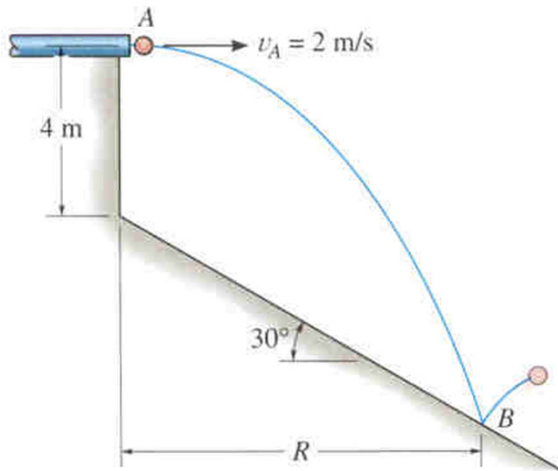


PROCEDURE FOR ANALYSIS

- In most impact problems, the initial velocities of the particles and the coefficient of restitution, e , are known, with the final velocities to be determined.
- Define the x-y axes. Typically, the **x-axis** is defined **along** the line of impact and the **y-axis** is in the plane of contact **perpendicular** to the x-axis.
- For both **central and oblique** impact problems, the following equations apply **along** the line of impact (x-dir.):
$$\sum m(v_x)_1 = \sum m(v_x)_2 \quad \text{and} \quad e = [(v_{Bx})_2 - (v_{Ax})_2] / [(v_{Ax})_1 - (v_{Bx})_1]$$
- For **oblique** impact problems, the following equations are also required, applied **perpendicular** to the line of impact (y-dir.):
$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \quad \text{and} \quad m_B(v_{By})_1 = m_B(v_{By})_2$$



EXAMPLE



Given: A 0.5 kg ball is ejected from the tube at A with a horizontal velocity $v_A = 2 \text{ m/s}$. The coefficient of restitution at B is $e = 0.6$.

Find: The horizontal distance R where the ball strikes the smooth inclined plane and the speed at which it bounces from the plane.

- Plan:**
- 1) Use **kinematics** to find the distance R (**projectile motion**).
 - 2) The collision at B is an **oblique impact**, with the line of impact perpendicular to the plane.
 - 3) Thus, the coefficient of restitution applies perpendicular to the incline and the momentum of the ball is conserved along the incline.



EXAMPLE

(continued)

Solution:

- 1) Apply the equations of projectile motion to determine R.

Place the origin at A ($x_o = y_o = 0$) with the initial velocity of

$$v_{y0} = 0, v_{x0} = v_A = 2 \text{ m/s:}$$

$$x = x_o + v_{x0}t \Rightarrow R = 0 + 2t$$

$$y = y_o + v_{y0}t - 0.5gt^2 \Rightarrow -(4 + R \tan 30^\circ) = 0 + 0 - 0.5(9.81)t^2$$

Solving these equations simultaneously yields

$$t = 1.028 \text{ s} \quad \text{and} \quad R = 2.06 \text{ m}$$

It is also necessary to calculate the velocity of the ball just before impact:

$$v_x = v_{x0} = 2 \text{ m/s} (\rightarrow)$$

$$v_y = v_{y0} - gt = 0 - 9.81(1.028) = -10.0886 \text{ m/s} (\downarrow)$$

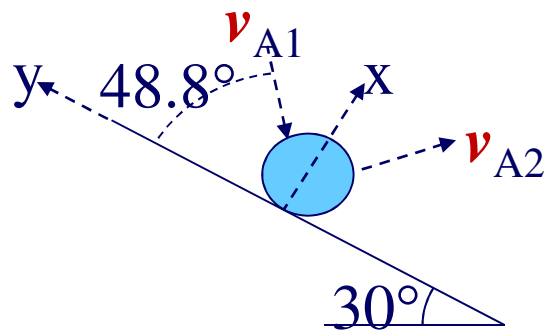
$$\Rightarrow v = 10.285 \text{ m/s} \searrow 78.8^\circ$$



EXAMPLE

(continued)

- 2) Solve the impact problem by using x-y axes defined along and perpendicular to the line of impact, respectively:



Denoting the ball as A and plane as B, the momentum of the ball is conserved in the y-dir:

$$m_A(-v_{Ay})_1 = m_A(-v_{Ay})_2$$
$$(v_{Ay})_2 = (v_{Ay})_1 = v_A \cos 48.8^\circ = 6.77 \text{ m/s}$$

The coefficient of restitution applies in the x-dir and

$$(v_{Bx})_1 = (v_{Bx})_2 = 0:$$

$$e = [(v_{Bx})_2 - (v_{Ax})_2] / [(v_{Ax})_1 - (v_{Bx})_1]$$

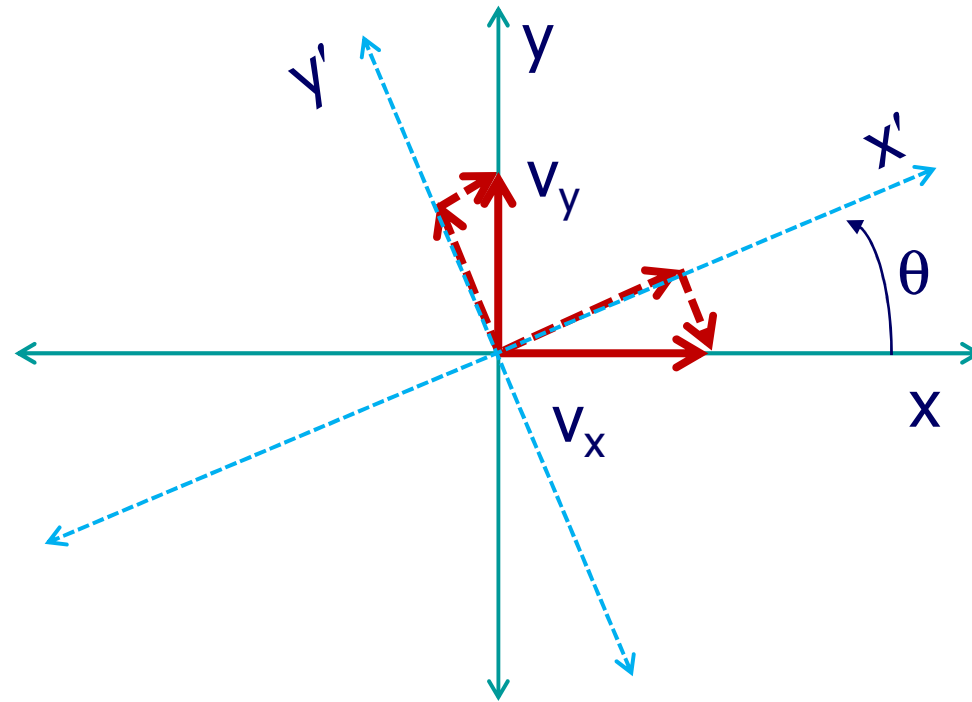
$$\Rightarrow 0.6 = [0 - (v_{Ax})_2] / [-10.285 \sin 48.8^\circ - 0]$$

$$\Rightarrow (v_{Ax})_2 = 4.64 \text{ m/s}$$

The speed is the **magnitude** of the velocity vector:

$$v_{A2} = \sqrt{((v_{Ax})_2)^2 + ((v_{Ay})_2)^2} = 8.21 \text{ m/s}$$





$$v_x' = v_x \cos\theta + v_y \sin\theta$$

$$v_y' = -v_x \sin\theta + v_y \cos\theta$$

$$\begin{pmatrix} v_x' \\ v_y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

After finding final velocities need to convert back

$$\begin{pmatrix} v_{xf} \\ v_{yf} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_{xf}' \\ v_{yf}' \end{pmatrix}$$

$$\mathbf{v} = 10.285 \text{ m/s} , -78.8^\circ \quad \theta = -30^\circ$$

$$\begin{pmatrix} v_x' \\ v_y' \end{pmatrix} = \begin{pmatrix} \cos(-30^\circ) & \sin(-30^\circ) \\ -\sin(-30^\circ) & \cos(-30^\circ) \end{pmatrix} \begin{pmatrix} 10.285 \cos 78.8^\circ \\ -10.285 \sin 78.8^\circ \end{pmatrix}$$

$$v_x' = 6.7746 \text{ and } v_y' = -7.7386$$

x' is parallel to incline so $v_{xf}' = 6.7746$, i.e. unchanged

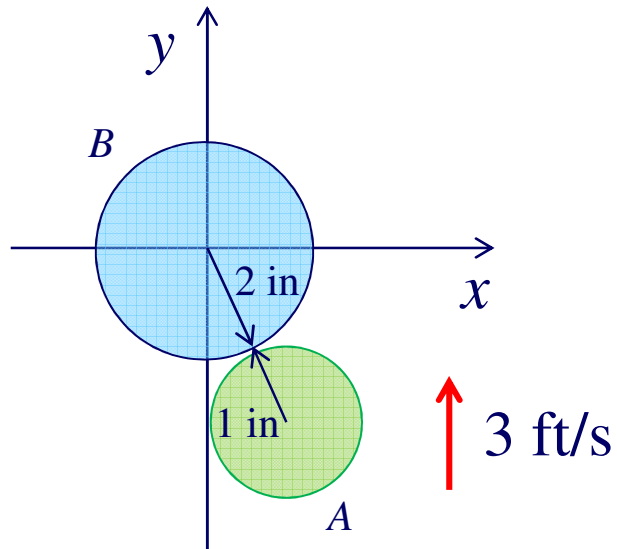
$$e = [(v_{Iy}')_f - (v_y')_f] / [(v_y')_i - (v_{Ix}')_i]$$

The incline is stationary, so $v_{yf}' = -e v_y' = 0.6 * 7.7386 = 4.6432$

$$\begin{pmatrix} v_{xf}' \\ v_{yf}' \end{pmatrix} = \begin{pmatrix} \cos(-30^\circ) & -\sin(-30^\circ) \\ \sin(-30^\circ) & \cos(-30^\circ) \end{pmatrix} \begin{pmatrix} -7.7386 \\ 4.6432 \end{pmatrix}$$

$$v_{xf} = 8.1886 \text{ and } v_{yf} = 0.6338$$

EXAMPLE



Given: Disk *A* weighs 2 lb and is moving at 3 ft/s. Disk *B* weighs 11 lb and is initially at rest. The coefficient of restitution is $e = 0.6$.

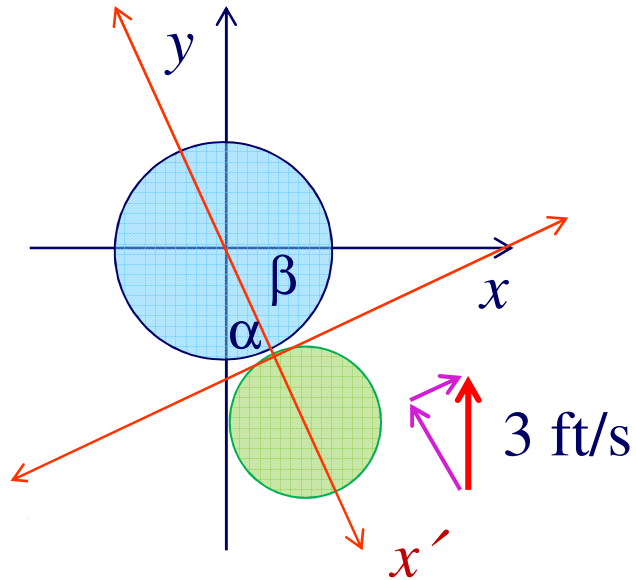
Find: The velocities after the collision.

Plan: The radii can be used to make a right triangle and determine the angle needed for rotating the axes.

The angle with the negative y axis is $\alpha = \arcsin(1/3) = 19.471^\circ$.

The angle with the positive x axis is $\beta = \arccos(1/3) = 70.529^\circ$.

EXAMPLE (cont'd)



Must first decide where the positive x' axis is.

Let's take $\theta = -\beta$. Note that CCW is + and CW is -.

You should pay attention to the components of each velocity vector in the new coordinate system.

EXAMPLE (cont'd)

$$\mathbf{v}_A = j3 \text{ ft/s}, \mathbf{v}_B = 0 \quad \theta = -70.529^\circ$$

$$\begin{pmatrix} v_{Ax}' \\ v_{Ay}' \end{pmatrix} = \begin{pmatrix} \cos(-70.529^\circ) & \sin(-70.529^\circ) \\ -\sin(-70.529^\circ) & \cos(-70.529^\circ) \end{pmatrix} \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -2.82843 \\ 1 \end{pmatrix}$$

Note $\mathbf{v}_B' = 0$. Also y' is \perp to collision axis so $v_{Ayf}' = 1$ & $v_{Byf}' = 0$

CM: $(2)v_{Axf}' + (11)v_{Bxf}' = (2)(-2.82843) + (11)(0) = -5.65685$

e: $v_{Axf}' - v_{Bxf}' = -(0.6)(-2.82843 - 0) = 1.6971$

$$\begin{pmatrix} 2 & 11 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_{Axf}' \\ v_{Bxf}' \end{pmatrix} = \begin{pmatrix} -5.65685 \\ 1.6971 \end{pmatrix} \Rightarrow \begin{pmatrix} v_{Axf}' \\ v_{Bxf}' \end{pmatrix} = \begin{pmatrix} 1.00083 \\ -0.69623 \end{pmatrix}$$

So $\mathbf{v}_{Af}' = \begin{pmatrix} 1.00083 \\ 1 \end{pmatrix}$ & $\mathbf{v}_{Bf}' = \begin{pmatrix} -0.69623 \\ 0 \end{pmatrix}$

EXAMPLE (cont'd)

Next, rotate back to the original axes for final answer.

$$\begin{pmatrix} V_{Axf} \\ V_{Ayf} \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} 1.00083 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.27642 \\ -0.61026 \end{pmatrix}$$

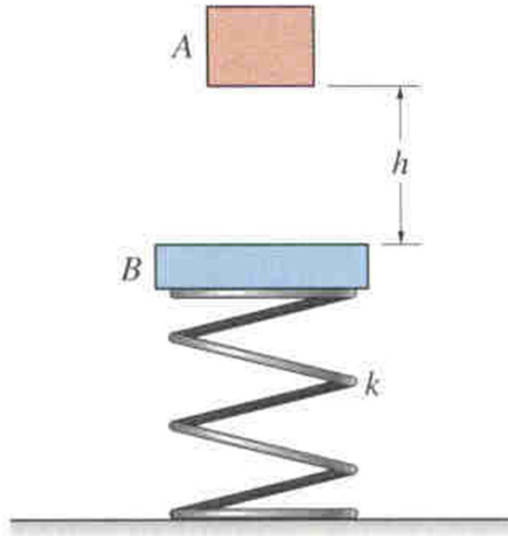
$$\begin{pmatrix} V_{Bxf} \\ V_{Byf} \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} -0.69623 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.23208 \\ 0.65461 \end{pmatrix}$$

CONCEPT QUIZ

- Two balls impact with a coefficient of restitution of 0.79. Can one of the balls leave the impact with a kinetic energy greater than before the impact?
 - Yes
 - No
 - Impossible to tell
 - Don't pick this one!
- Under what condition is the energy lost during a collision maximum?
 - $e = 1.0$
 - $e = 0.0$
 - $e = -1.0$
 - Collision is non-elastic.



GROUP PROBLEM SOLVING



Given: A 2 kg block A is released from rest, falls a distance $h = 0.5$ m, and strikes plate B (3 kg mass). The coefficient of restitution between A and B is $e = 0.6$, and the spring stiffness is $k = 30$ N/m.

Find: The velocity of block A just after the collision.

- Plan:**
- 1) Determine the speed of the block just before the collision using projectile motion or an energy method.
 - 2) Analyze the collision as a central impact problem.



GROUP PROBLEM SOLVING

(continued)

Solution:

- 1) Determine the speed of block A just before impact by using conservation of energy. Defining the gravitational datum at the initial position of the block ($h_1 = 0$) and noting the block is released from rest ($v_1 = 0$):

$$T_1 + V_1 = T_2 + V_2$$

$$0.5m(v_1)^2 + mgh_1 = 0.5m(v_2)^2 + mgh_2$$

$$0 + 0 = 0.5(2)(v_2)^2 + (2)(9.81)(-0.5)$$

$$v_2 = 3.132 \text{ m/s} \downarrow$$

This is the speed of the block just before the collision. Plate (B) is at rest, velocity of zero, before the collision.

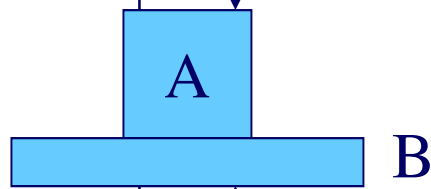


GROUP PROBLEM SOLVING

(continued)

2) Analyze the collision as a central impact problem.

$$(v_A)_2 \uparrow \quad (v_A)_1 = 3.132 \text{ m/s} \downarrow$$



$$(v_B)_2 \downarrow \quad (v_B)_1 = 0 \uparrow$$

Apply conservation of momentum to the system in the vertical direction:

$$+ \uparrow m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$
$$(2)(-3.132) + 0 = (2)(v_A)_2 + (3)(v_B)_2$$

Using the coefficient of restitution:

$$+ \uparrow e = [(v_B)_2 - (v_A)_2] / [(v_A)_1 - (v_B)_1]$$

$$\Rightarrow 0.6 = [(v_B)_2 - (v_A)_2] / [-3.132 - 0] \Rightarrow -1.879 = (v_B)_2 - (v_A)_2$$

Solving the two equations simultaneously yields

$$(v_A)_2 = -0.125 \text{ m/s} \downarrow, (v_B)_2 = -2.00 \text{ m/s} \downarrow$$

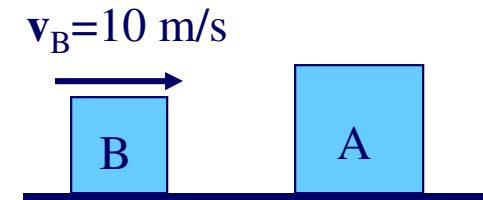
Both the block and plate will travel down after the collision.



ATTENTION QUIZ

1. Block B (1 kg) is moving on the smooth surface at 10 m/s when it squarely strikes block A (3 kg), which is at rest. If the velocity of block A after the collision is 4 m/s to the right, $(v_B)_2$ is

- A) 2 m/s \rightarrow B) 7 m/s \leftarrow
C) 7 m/s \rightarrow D) 2 m/s \leftarrow



2. A particle strikes the smooth surface with a velocity of 30 m/s. If $e = 0.8$, $(v_x)_2$ is _____ after the collision.

- A) zero B) equal to $(v_x)_1$
C) less than $(v_x)_1$ D) greater than $(v_x)_1$

