ANGULAR MOMENTUM, MOMENT OF A FORCE AND ANGULAR IMPULSE AND MOMENTUM PRINCIPLES

Today's Objectives:

Students will be able to:

1. Determine the angular momentum of a particle and apply the principle of angular impulse & momentum.
2. Use conservation of angular momentum to solve problems.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Angular Momentum
- Angular Impulse & Momentum Principle
- Conservation of Angular Momentum
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

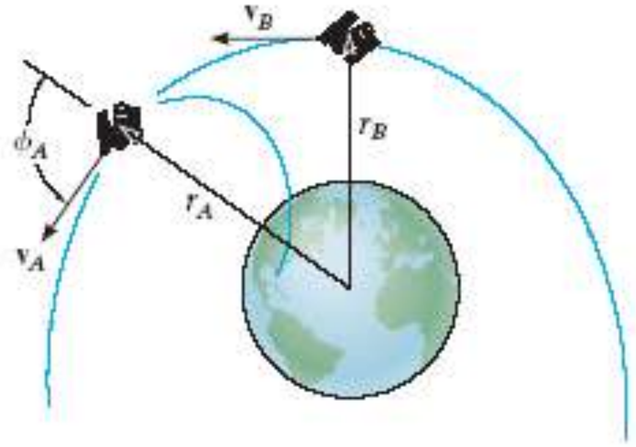
1. Select the correct expression for the angular momentum of a particle about a point.

   A) $\boldsymbol{r} \times \boldsymbol{v}$
   B) $\boldsymbol{r} \times (\mathrm{m}\, \boldsymbol{v})$
   C) $\boldsymbol{v} \times \boldsymbol{r}$
   D) $(\mathrm{m}\, \boldsymbol{v}) \times \boldsymbol{r}$

2. The sum of the moments of all external forces acting on a particle is equal to

   A) angular momentum of the particle.
   B) linear momentum of the particle.
   C) time rate of change of angular momentum.
   D) time rate of change of linear momentum.

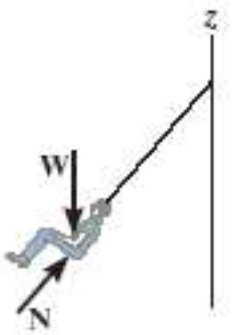
APPLICATIONS



Planets and most satellites move in elliptical orbits. This motion is caused by gravitational attraction forces. Since these forces act in pairs, the sum of the moments of the forces acting on the system will be zero. This means that angular momentum is conserved.

If the angular momentum is constant, does it mean the linear momentum is also constant? Why or why not?

APPLICATIONS (continued)

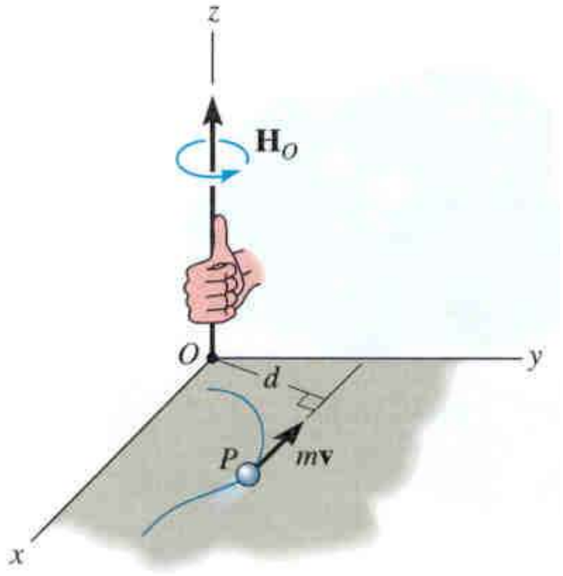


The passengers on the amusement-park ride experience conservation of angular momentum about the axis of rotation (the z-axis). As shown on the free body diagram, the line of action of the normal force, N, passes through the z-axis and the weight's line of action is parallel to it. Therefore, the sum of moments of these two forces about the z-axis is zero.

If the passenger moves away from the z-axis, will his speed increase or decrease? Why?

ANGULAR MOMENTUM
(Section 15.5)

The angular momentum of a particle about point O is defined as the "moment" of the particle's linear momentum about O.



$$\boldsymbol{H}_o = \boldsymbol{r} \times m\boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ r_x & r_y & r_z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

The magnitude of $\boldsymbol{H}_o$ is $(H_o)_z = mvd$

RELATIONSHIP BETWEEN MOMENT OF A FORCE AND ANGULAR MOMENTUM

(Section 15.6)

The resultant force acting on the particle is equal to the time rate of change of the particle's linear momentum. Showing the time derivative using the familiar "dot" notation results in the equation

$$\sum \boldsymbol{F} = \dot{\boldsymbol{L}} = m\dot{\boldsymbol{v}}$$

We can prove that the resultant moment acting on the particle about point O is equal to the time rate of change of the particle's angular momentum about point O or

$$\sum \boldsymbol{M}_o = \boldsymbol{r} \times \boldsymbol{F} = \dot{\boldsymbol{H}}_o$$

ANGULAR IMPULSE AND MOMENTUM PRINCIPLES

(Section 15.7)

Considering the relationship between moment and time rate of change of angular momentum

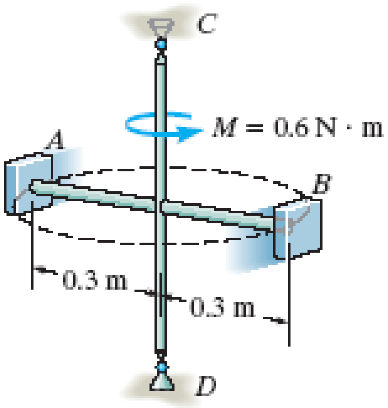
$$\sum \boldsymbol{M}_o = \dot{\boldsymbol{H}}_o = d\boldsymbol{H}_o/dt$$

By integrating between the time interval t_1 to t_2

$$\sum \int_{t_1}^{t_2} \boldsymbol{M}_o \, dt = (\boldsymbol{H}_o)_2 - (\boldsymbol{H}_o)_1 \quad \text{or} \quad (\boldsymbol{H}_o)_1 + \sum \int_{t_1}^{t_2} \boldsymbol{M}_o \, dt = (\boldsymbol{H}_o)_2$$

This equation is referred to as the principle of angular impulse and momentum. The second term on the left side, $\sum\int M_o \, dt$, is called the angular impulse. In cases of 2D motion, it can be applied as a scalar equation using components about the z-axis.

EXAMPLE



Given: Two 0.4 kg masses with initial velocities of 2 m/s experience a moment of 0.6 N·m.

Find: The speed of blocks A and B when t = 3 s.

Plan: Apply the principle of angular impulse and momentum. Since the velocities of A and B remain equal to each other at all times, the two momentum terms can be multiplied by two.

Solution:

$$2(0.3)(0.4)(2) + \int_{t_1}^{t_2} M\,dt = 2(0.3)(0.4)(v_2)$$

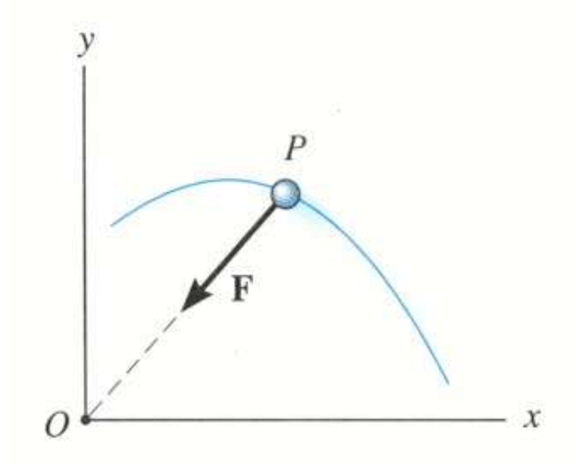
where $\int_{t_1}^{t_2} M\,dt = 0.6(3) = 1.8$ N·m·s

$0.48 + 1.8 = 0.24 v_2$ or $v_2 = 9.5$ m/s

CONSERVATION OF ANGULAR MOMENTUM

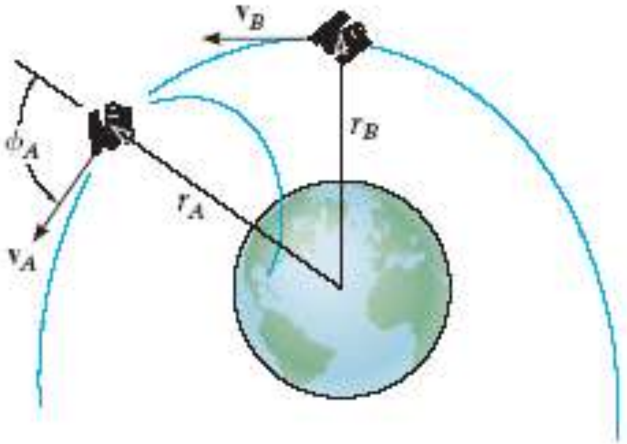
When the sum of angular impulses acting on a particle or a system of particles is zero during the time t_1 to t_2, the angular momentum is conserved. Thus,

$$(\boldsymbol{H}_O)_1 = (\boldsymbol{H}_O)_2$$



An example of this condition occurs when a particle is subjected only to a central force. In the figure, the force ***F*** is always directed toward point O. Thus, the angular impulse of ***F*** about O is always zero, and angular momentum of the particle about O is conserved.

EXAMPLE II



Given: A satellite has an elliptical orbit about earth.

$m_{satellite} = 700$ kg

$m_{earth} = 5.976 \times 10^{24}$ kg

$v_A = 10$ km/s

$r_A = 15 \times 10^6$ m

$\phi_A = 70$

Find: The speed, v_B, of the satellite at its closest distance, r_B, from the center of the earth.

Plan: Apply the principles of conservation of energy and conservation of angular momentum to the system.

EXAMPLE II
(continued)

Solution:

Conservation of energy: $T_A + V_A = T_B + V_B$ becomes

$$\frac{1}{2}\, m_s\, v_A^2 - \frac{G\, m_s\, m_e}{r_A} = \frac{1}{2}\, m_s\, v_B^2 - \frac{G\, m_s\, m_e}{r_B}$$

where $G = 66.73 \text{x} 10^{-12}\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$. Dividing through by m_s and substituting values yields:

$$0.5(10{,}000)^2 - \frac{66.73 \text{ x } 10^{-12}(5.976 \text{ x } 10^{24})}{15 \text{ x } 10^6} = 0.5\, v_B^2 - \frac{66.73 \text{ x } 10^{-12}(5.976 \text{ x } 10^{24})}{r_B}$$

or $23.4 \text{ x } 10^6 = 0.5\, (v_B)^2 - (3.99 \text{x} 10^{14})/r_B$

EXAMPLE II
(continued)

Solution:

Now use Conservation of Angular Momentum:

$$(r_A\, m_s\, v_A) \sin \phi_A = r_B\, m_s\, v_B$$

$$(15 \times 10^6)(10{,}000) \sin 70 \;\; = r_B\, v_B \quad \text{or} \quad r_B = (140.95 \times 10^9)/v_B$$

Solving the two equations for r_B and v_B yields

$$r_B = 13.8 \times 10^6 \text{ m} \qquad v_B = 10.2 \text{ km/s}$$

CONCEPT QUIZ

1. If a particle moves in the x - y plane, its angular momentum vector is in the

   A) x direction. B) y direction.

   C) z direction. D) x - y direction.

2. If there are no external impulses acting on a particle

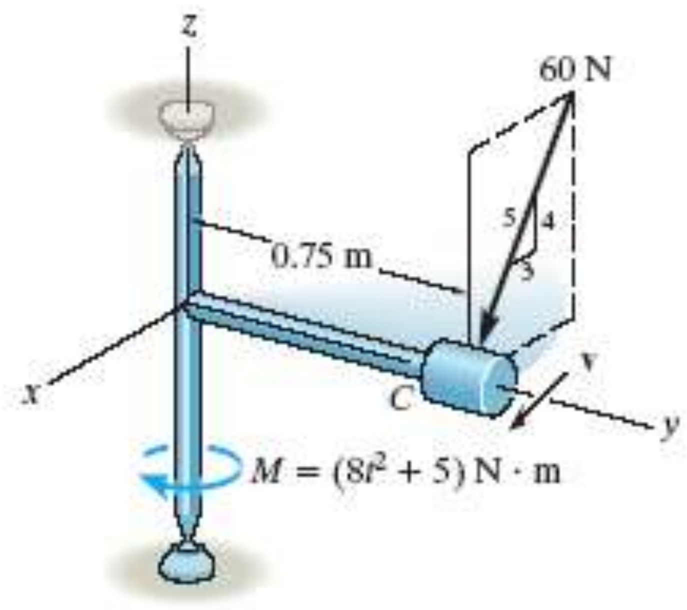
   A) only linear momentum is conserved.

   B) only angular momentum is conserved.

   C) both linear momentum and angular momentum are conserved.

   D) neither linear momentum nor angular momentum are conserved.

GROUP PROBLEM SOLVING



Given: A rod assembly rotates around its z-axis. The mass C is 10 kg and its initial velocity is 2 m/s. A moment and force both act as shown ($M = 8t^2 + 5$ N·m and F = 60 N).

Find: The velocity of the mass C after 2 seconds.

Plan: Apply the principle of angular impulse and momentum about the axis of rotation (z-axis).

GROUP PROBLEM SOLVING
(continued)

Solution:

Angular momentum: $\boldsymbol{H}_Z = \boldsymbol{r} \times m\boldsymbol{v}$ reduces to a scalar equation.

$(H_Z)_1 = 0.75(10)(2) = 7.5(2)$ and $(H_Z)_2 = 0.75(10)(v_2) = 7.5v_2$

Angular impulse: $\displaystyle\int_{t_1}^{t_2} M\,dt + \int_{t_1}^{t_2} (\boldsymbol{r} \times \boldsymbol{F})\,dt$

$$= \int_0^2 (8t^2 + 5)\,dt + \int_0^2 (0.75)(3/5)(60)\,dt$$

$$= (8/3)t^3 + 5t + 27t \Big|_0^2 = 85.33 \text{ N·m·s}$$

Apply the principle of angular impulse and momentum.

$7.5(2) + 85.33 = 7.5v \qquad v = 13.38$ m/s

ATTENTION QUIZ

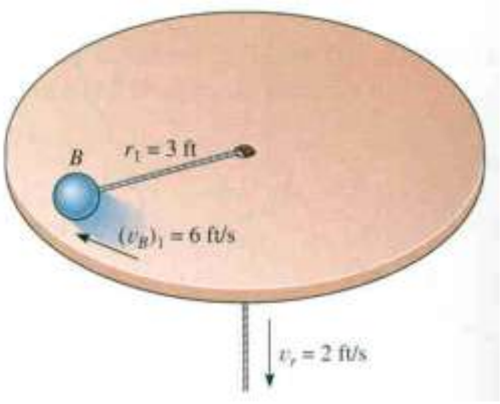
1. A ball is traveling on a smooth surface in a 3 ft radius circle with a speed of 6 ft/s. If the attached cord is pulled down with a constant speed of 2 ft/s, which of the following principles can be applied to solve for the velocity of the ball when r = 2 ft?

   A) Conservation of energy

   B) Conservation of angular momentum

   C) Conservation of linear momentum

   D) Conservation of mass



2. If a particle moves in the z - y plane, its angular momentum vector is in the

   A) x direction.

   B) y direction.

   C) z direction.

   D) z - y direction.