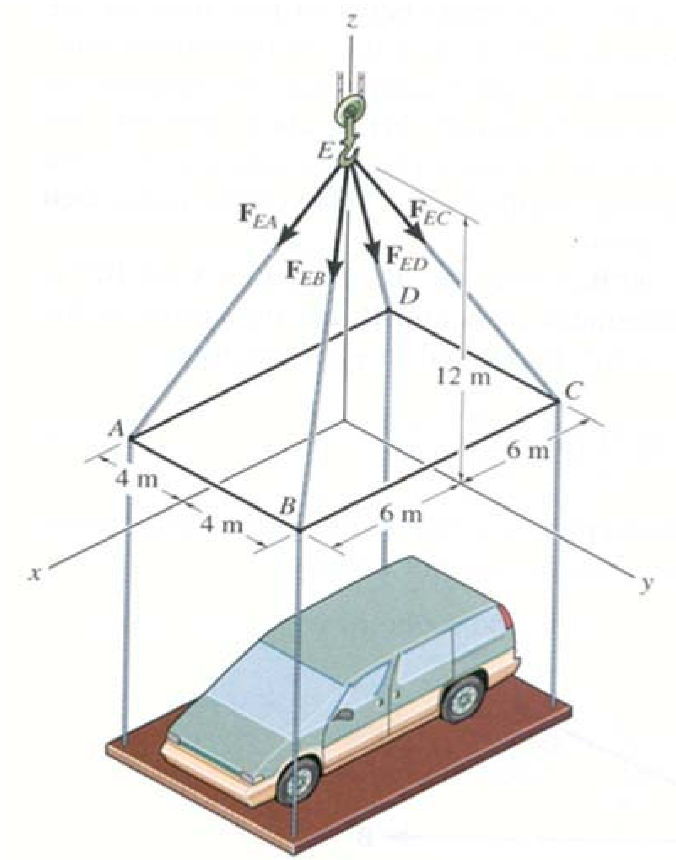
3D APPLICATIONS

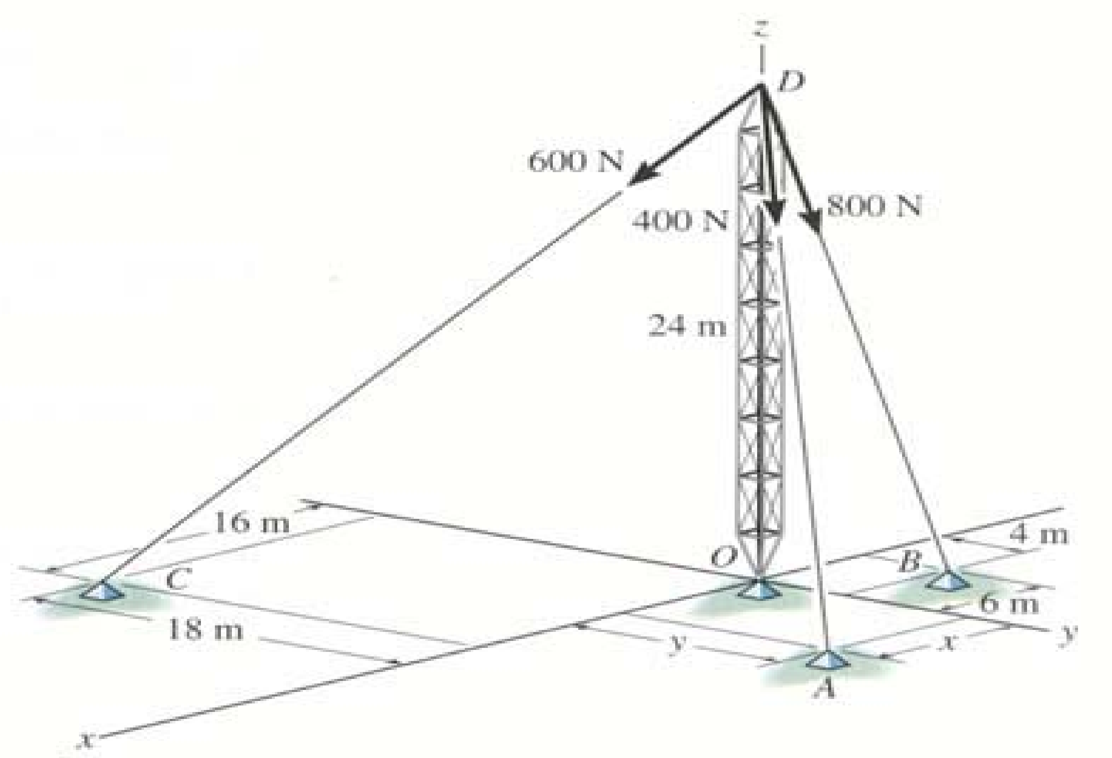


Many problems in real-life involve 3-Dimensional Space.

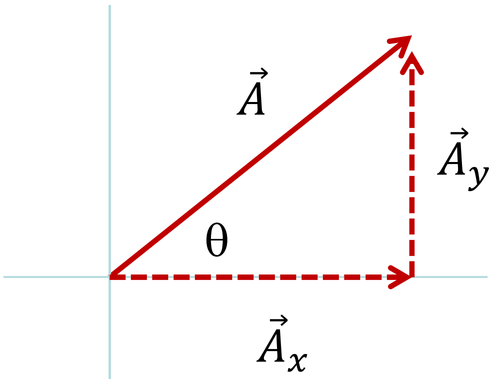
How will you represent each of the cable forces in Cartesian vector form?

3D APPLICATIONS
(continued)

Given the forces in the cables, how will you determine the resultant force acting at D, the top of the tower?



In 2D



Can state vector $\vec{A}$ in two ways:

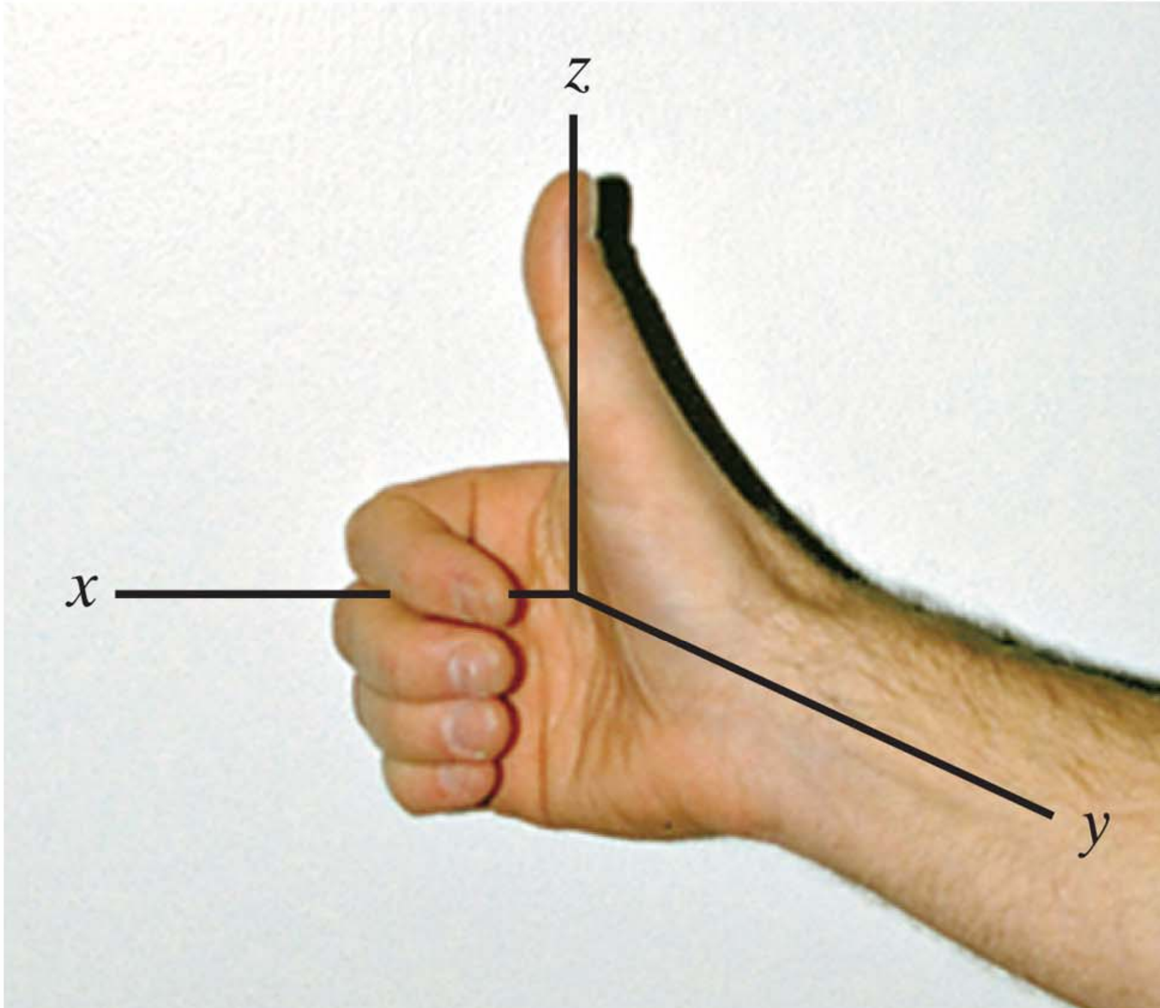
$\vec{A} = A\ , \angle\theta$ (magnitude and direction)

$\vec{A} = \hat{\imath}\, A_x + \hat{\jmath}\, A_y$ (Cartesian or component)

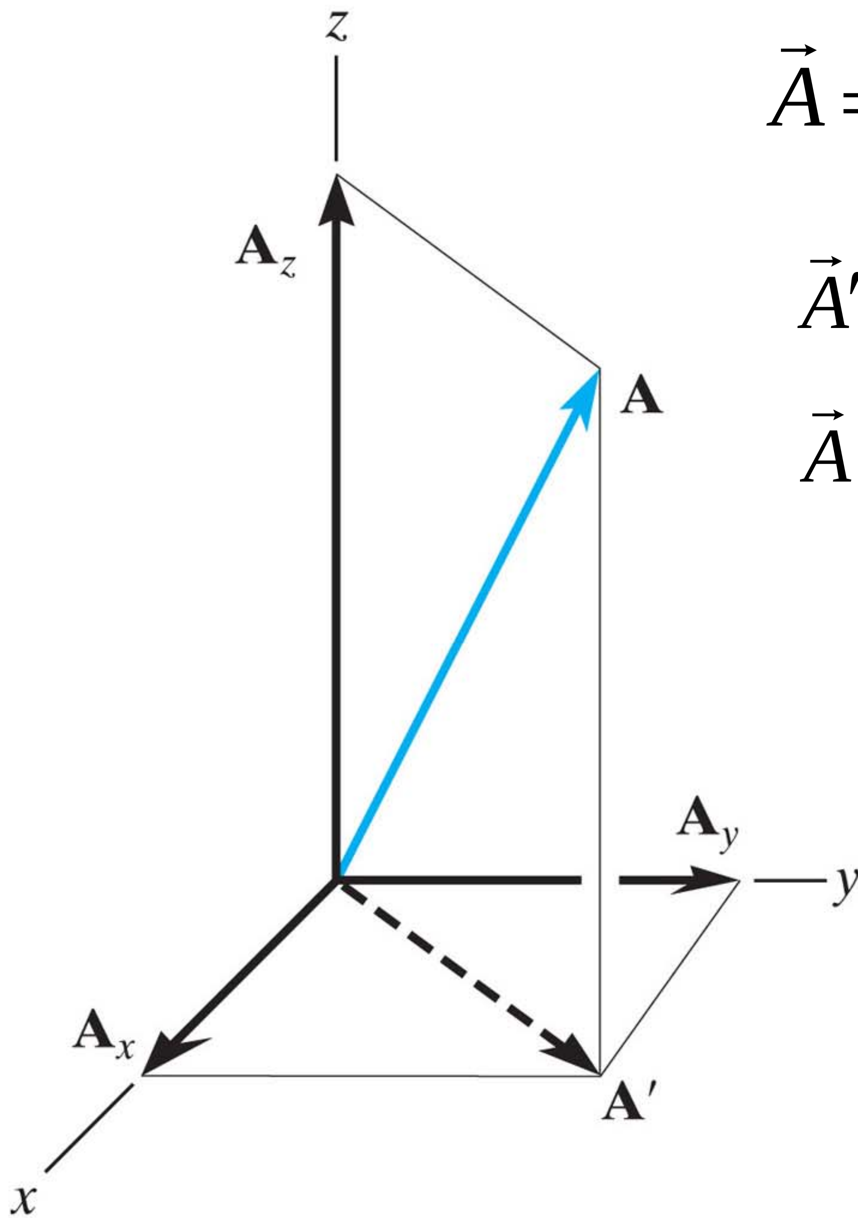
Can convert from one form to the other using some simple trig.

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad tan\theta = \frac{A_y}{A_x}$$

How do we do the equivalent in 3D?



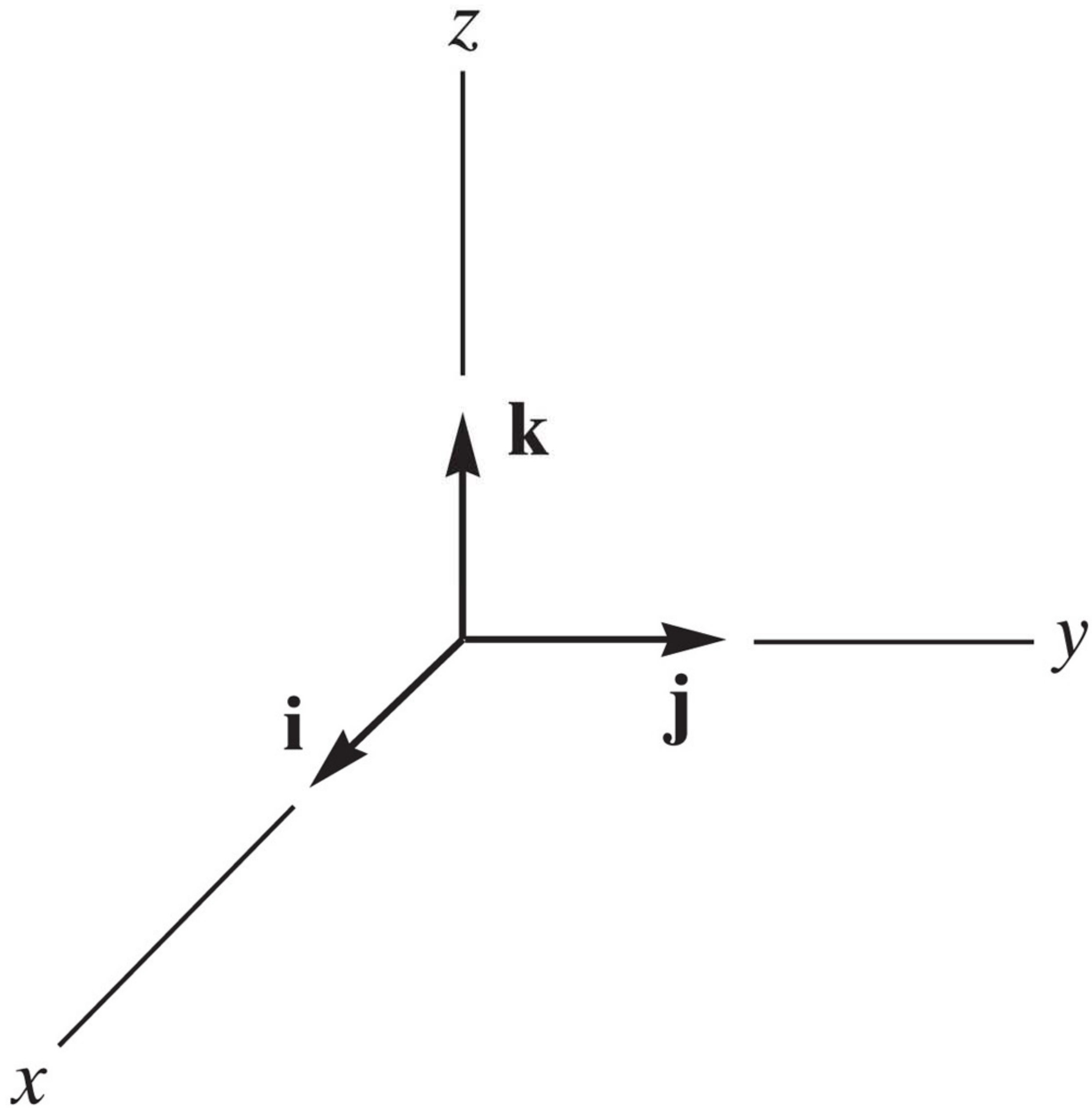
Positive *x*, *y*, and *z* directions defined
by a Right Hand Rule

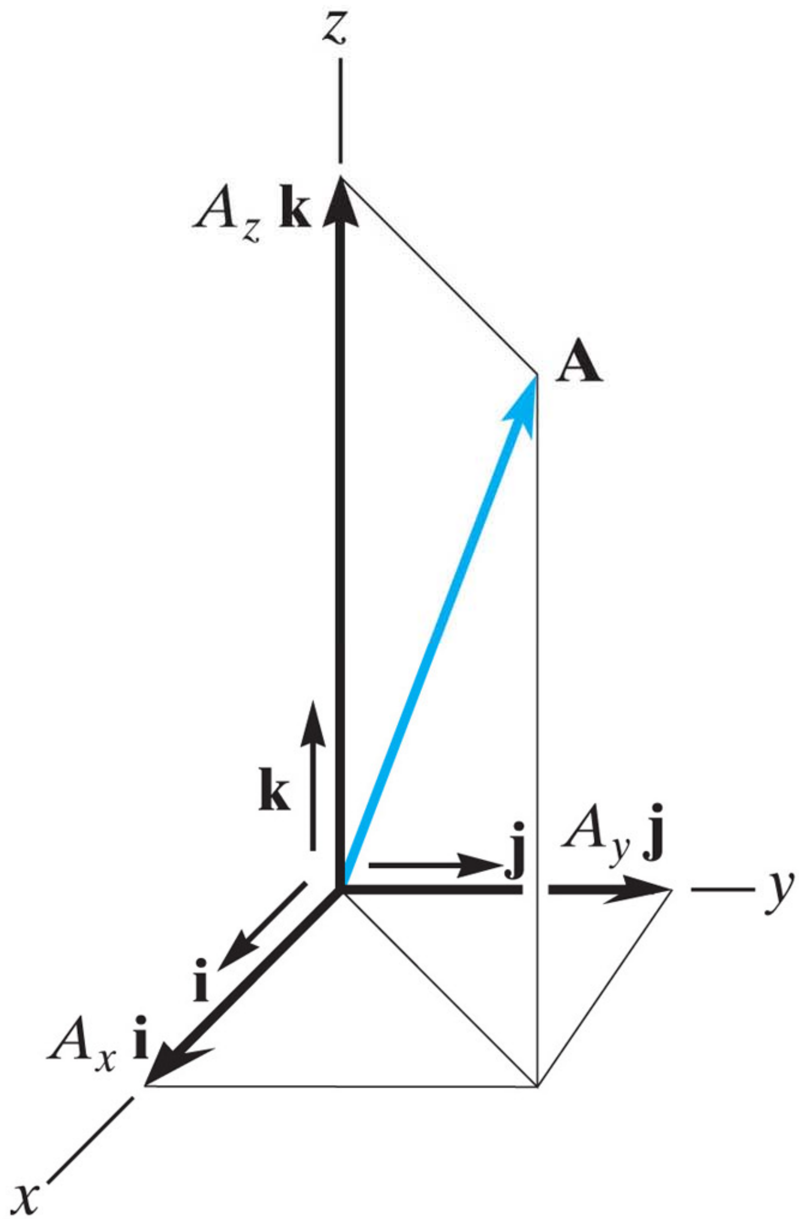


$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

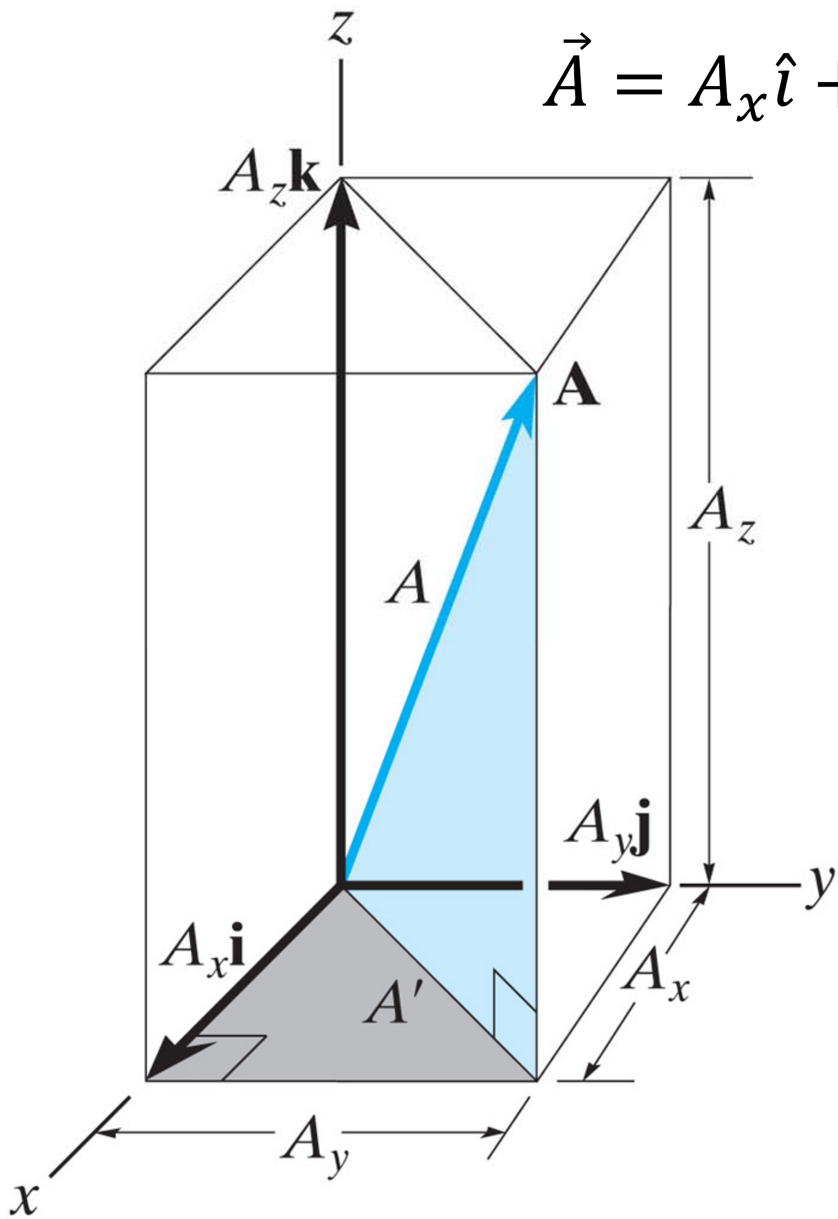
$\vec{A}'$ is the projection of

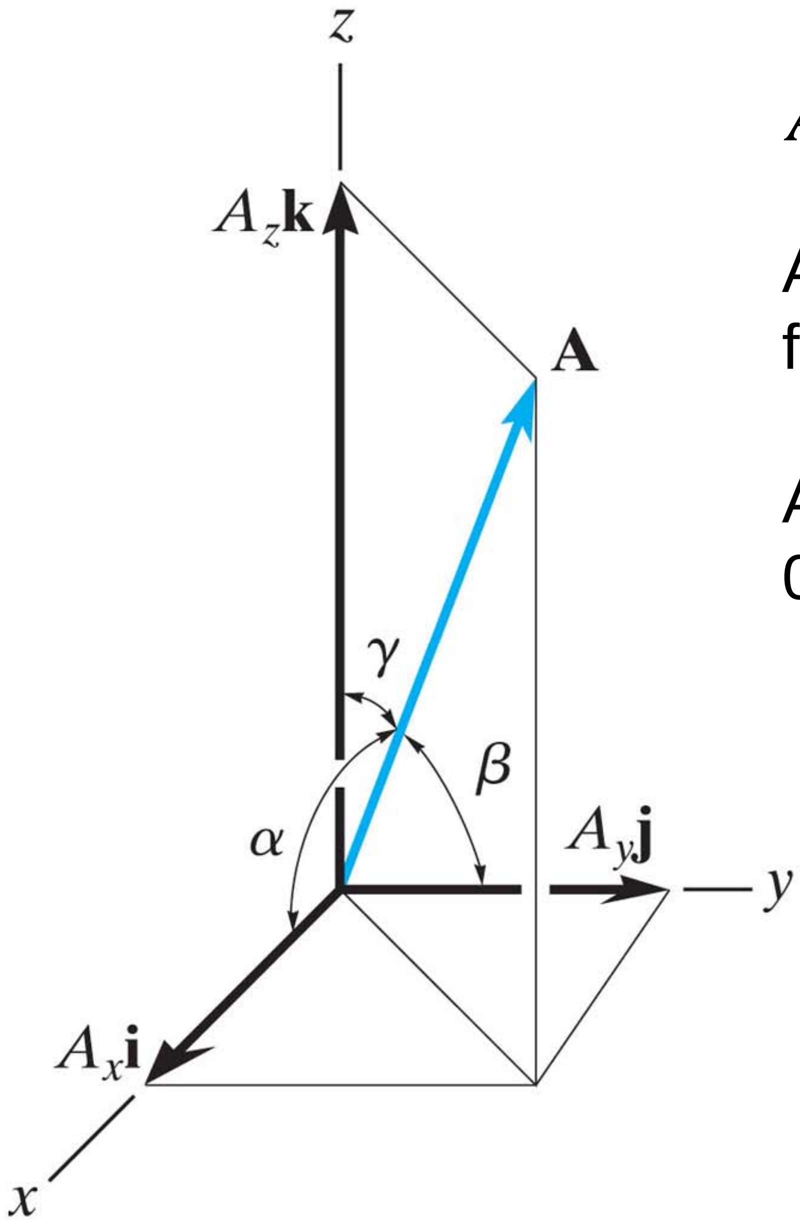
$\vec{A}$ in x-y plane


z
k
y
j
i
x


z
A_z **k**
A
k
j
A_y **j**
y
i
A_x **i**
x

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$





$\vec{A} = A, \angle\alpha, \angle\beta, \angle\gamma$

Angles measured from positive axes.

Angles range from 0° to 180°

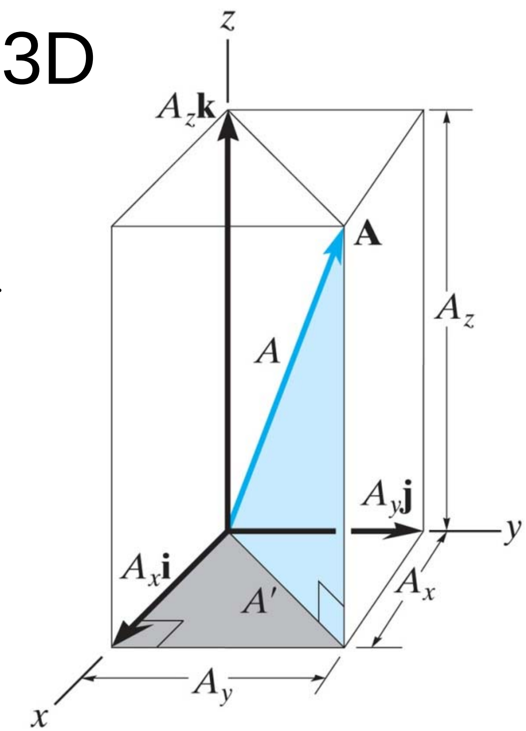
How do we convert between the two 3D forms?

Magnitude of a 3D Vector

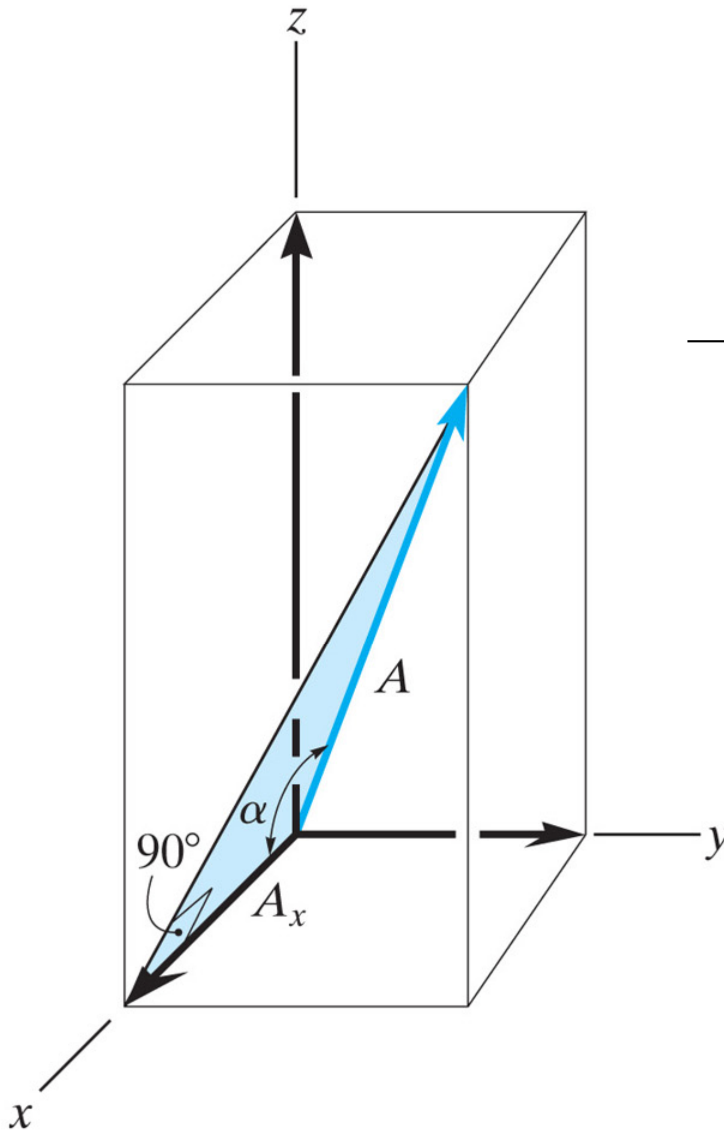
- Pythagorean Theorem works in 3D
- Proof – Apply PT twice

$$A' = \sqrt{A_x^2 + A_y^2} \qquad A = \sqrt{(A')^2 + A_z^2}$$

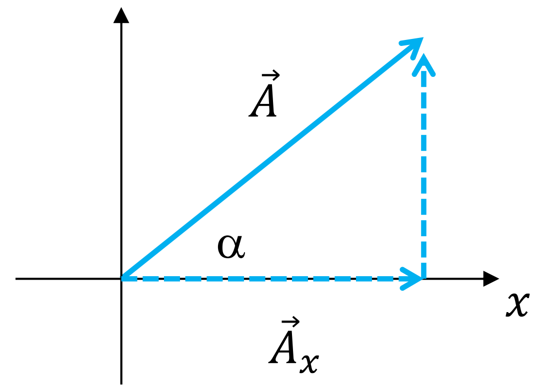
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



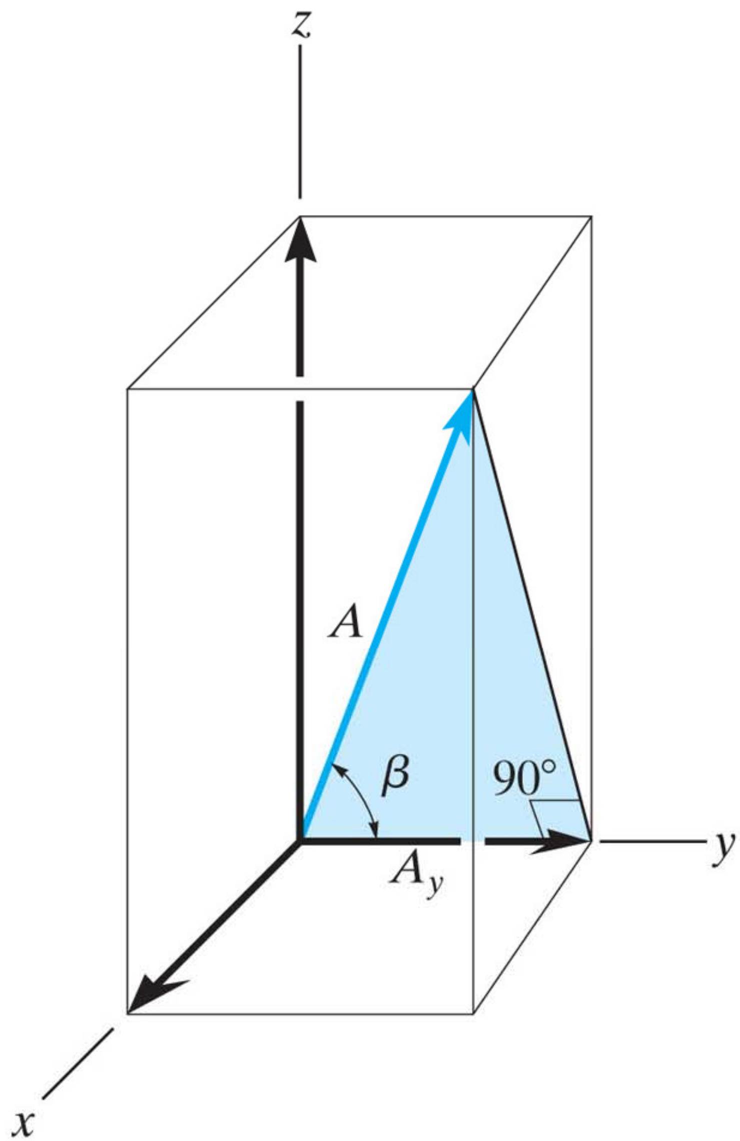
- What about direction?
- How do you find the three angles α, β, and γ? (Actually just need 2)



(a)

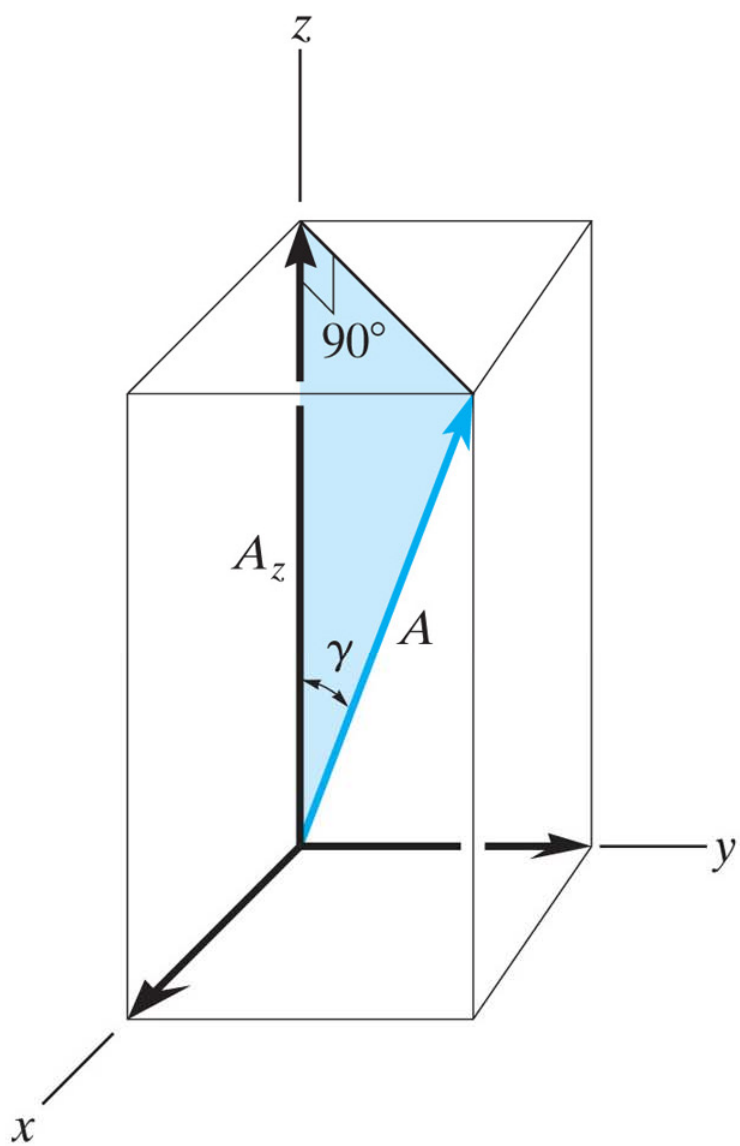


$$\cos\alpha = \frac{A_x}{A}$$



(b)

$$\cos\beta = \frac{A_y}{A}$$



(c)

$$\cos\gamma = \frac{A_z}{A}$$

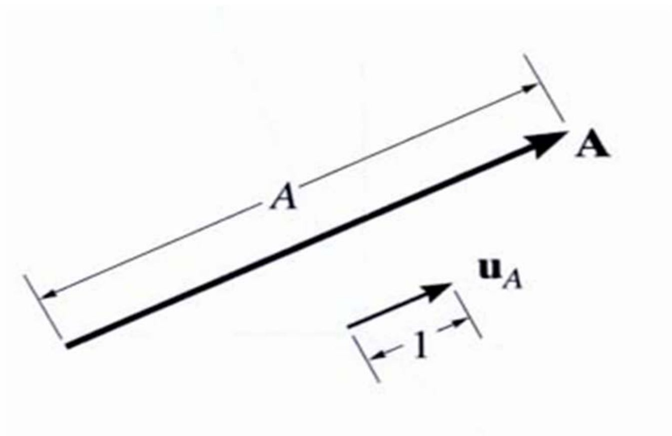
UNIT VECTORS

Characteristics of a unit vector**:**

a) Its magnitude is 1.

b) It is dimensionless.

c) It points in the same direction as the original vector $\vec{\boldsymbol{A}}$.



d) Think of $\hat{u}_A$ as direction of vector $\vec{\boldsymbol{A}}$.

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{A} = A, \angle\alpha, \angle\beta, \angle\gamma$$

$$\vec{A} = A\hat{u}_A$$

$$\hat{u}_A = \frac{\vec{A}}{A}$$

$$\hat{u}_A = \frac{A_x}{A}\hat{\imath} + \frac{A_y}{A}\hat{\jmath} + \frac{A_z}{A}\hat{k}$$

Using trigonometry, “direction cosines” are found using the formulas

$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

So $\hat{u}_a$ can be written another way

$$\hat{u}_a = cos\alpha\,\hat{\imath} + cos\beta\,\hat{\jmath} + cos\gamma\,\hat{k}$$

Applying the Pythagorean Equation,

$$1 = cos^2\alpha + cos^2\beta + cos^2\gamma$$

So the angles are not independent. If you have two, can use this equation to find the third.

Solving Problems

Sometimes 3-D vector information is given as:

a) Magnitude and the 3 coordinate direction angles, or

b) Magnitude and the 2 coordinate direction angles, use

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ to find 3rd angle, or

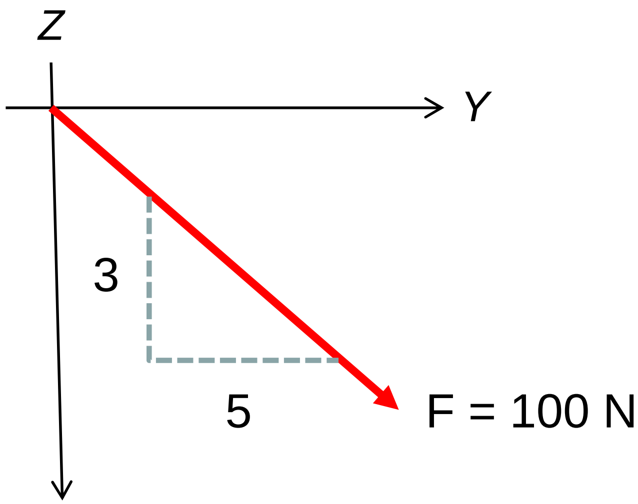
c) Magnitude and projection angles. A projection angle is

the angle in a plane. Use trig to get A_x, A_y, and A_z.

You should be able to use these types of information to change the representation of the vector into the Cartesian form, i.e.,

$\boldsymbol{F} = \{10\,\boldsymbol{i} - 20\,\boldsymbol{j} + 30\,\boldsymbol{k}\}$ N .

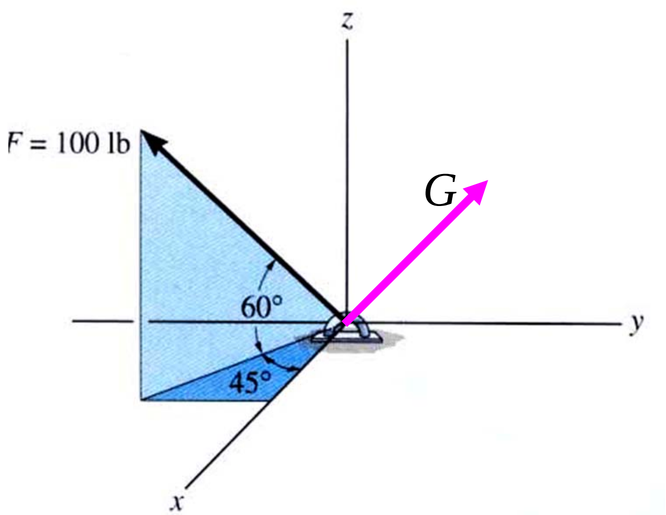
Slope Information



$$\sqrt{3^2 + 5^2} = 5.83$$

$$\vec{F} = (100\ N)\left\{\frac{5}{5.83}\hat{j} - \frac{3}{5.83}\hat{k}\right\}$$

EXAMPLE



Given: Two forces ***F*** and ***G*** are applied to a hook. Force ***F*** is shown in the figure and it makes 60° angle with the *x*-*y* plane. Force ***G*** is pointing up and has a magnitude of 80 lb with α = 111° and β = 69.3°.

Find: The resultant force in the Cartesian vector form.

Plan:

1) Using geometry and trigonometry, write ***F*** and ***G*** in the Cartesian vector form.
2) Then add the two forces.

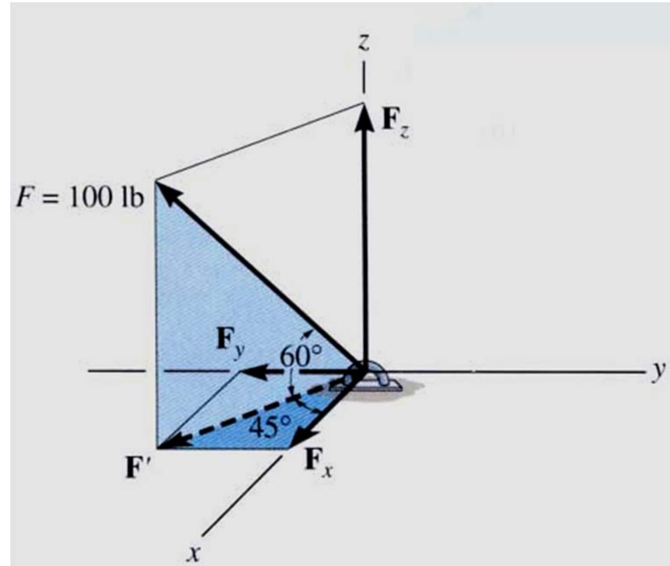
Solution : First, resolve force ***F***.

$F_z = 100 \sin 60° = 86.60$ lb

$F' = 100 \cos 60° = 50.00$ lb

$F_x = 50 \cos 45° = 35.36$ lb

$F_y = 50 \sin 45° = 35.36$ lb



Now, you can write:

$\boldsymbol{F} = \{35.36\,\boldsymbol{i} - 35.36\,\boldsymbol{j} + 86.60\,\boldsymbol{k}\}$ lb

Now resolve force ***G***.

We are given only α and β. Hence, first we need to find the value of γ. Recall the formula $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$.

Now substitute what we know. We have

$\cos^2(111°) + \cos^2(69.3°) + \cos^2(\gamma) = 1$ or $\cos(\gamma) = \pm 0.8641$

Solving, we get $\gamma = 30.22°$ (+) or $149.78°$ (–). Since the vector is pointing up, $\gamma = 30.22°$

Now using the coordinate direction angles, we can get $\boldsymbol{u}_{G}$, and determine $\boldsymbol{G} = 80\ \boldsymbol{u}_G$ lb.

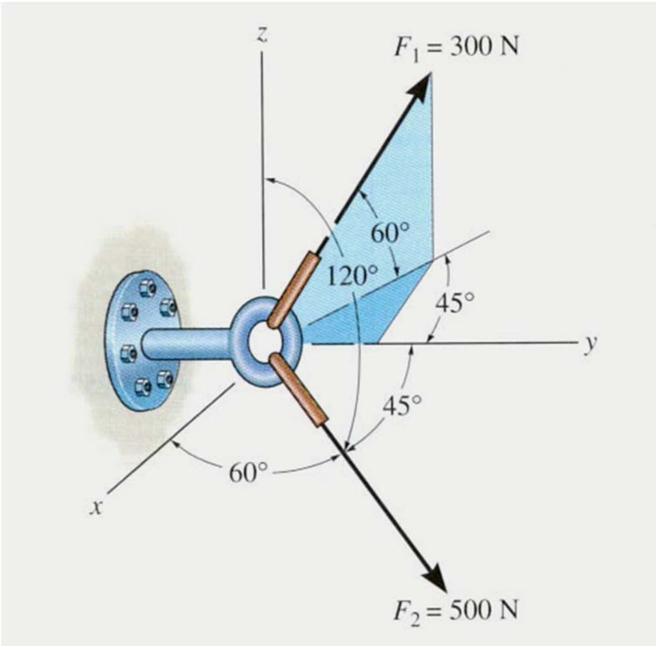
$\boldsymbol{G} = \{80\ (\cos(111°)\ \boldsymbol{i} + \cos(69.3°)\ \boldsymbol{j} + \cos(30.22°)\ \boldsymbol{k})\}$ lb

$\boldsymbol{G} = \{-28.67\ \boldsymbol{i} + 28.28\ \boldsymbol{j} + 69.13\ \boldsymbol{k}\}$ lb

Now, $\boldsymbol{R} = \boldsymbol{F} + \boldsymbol{G}$ **or**

$\boldsymbol{R} = \{6.69\ \boldsymbol{i} - 7.08\ \boldsymbol{j} + 156\ \boldsymbol{k}\}$ lb

Example



Given: The screw eye is subjected to two forces.

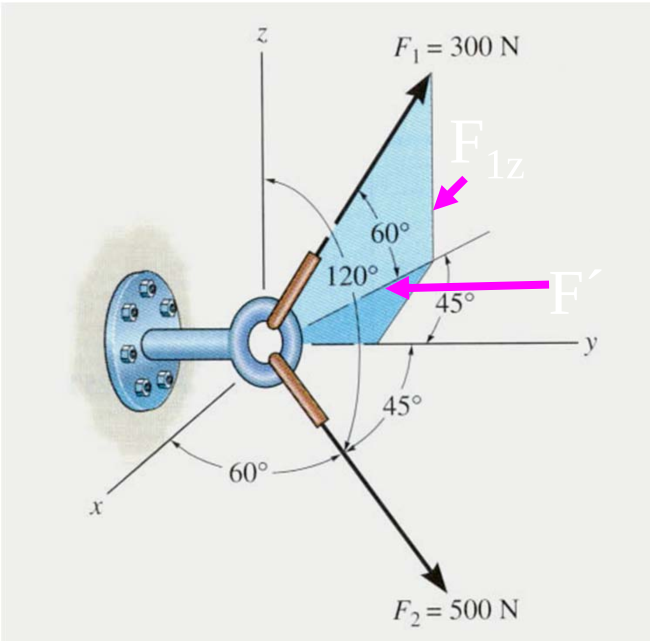
Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1) Using the geometry and trigonometry, write $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in the Cartesian vector form.

2) Add $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ to get $\boldsymbol{F_R}$.

3) Determine the magnitude and α, β, γ .



First resolve the force $\boldsymbol{F_1}$.

F_{1z} = 300 sin 60° = 259.8 N

F´ = 300 cos 60° = 150.0 N

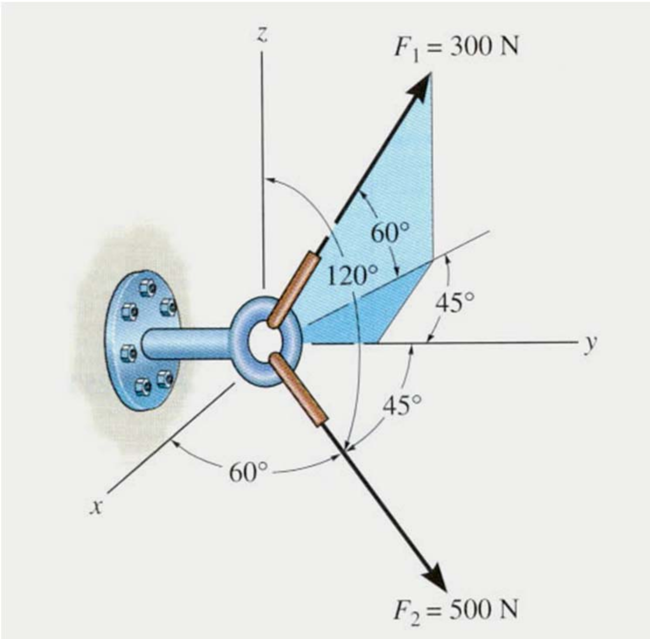
F′ can be further resolved as,

F_{1x} = –150 sin 45° = – 106.1 N

F_{1y} = 150 cos 45° = 106.1 N

Now we can write :

$\boldsymbol{F_1}$ = {– 106.1 $\boldsymbol{i}$ + 106.1 $\boldsymbol{j}$ + 259.8 $\boldsymbol{k}$ } N



The force $\boldsymbol{F_2}$ can be represented in the Cartesian vector form as:

$$\boldsymbol{F_2} = 500\{ \cos 60° \, \boldsymbol{i} + \cos 45° \, \boldsymbol{j} + \cos 120° \, \boldsymbol{k} \} \text{ N}$$

$$= \{ 250 \, \boldsymbol{i} + 353.6 \, \boldsymbol{j} - 250 \, \boldsymbol{k} \} \text{ N}$$

$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

$$= \{ 143.9 \, \boldsymbol{i} + 459.6 \, \boldsymbol{j} + 9.81 \, \boldsymbol{k} \} \text{ N}$$

$$F_R = (143.9^2 + 459.6^2 + 9.81^2)^{½} = 481.7 = 482 \text{ N}$$

$$\alpha = \cos^{-1} (F_{Rx} / F_R) = \cos^{-1} (143.9/481.7) = 72.6°$$

$$\beta = \cos^{-1} (F_{Ry} / F_R) = \cos^{-1} (459.6/481.7) = 17.4°$$

$$\gamma = \cos^{-1} (F_{Rz} / F_R) = \cos^{-1} (9.81/481.7) = 88.8°$$