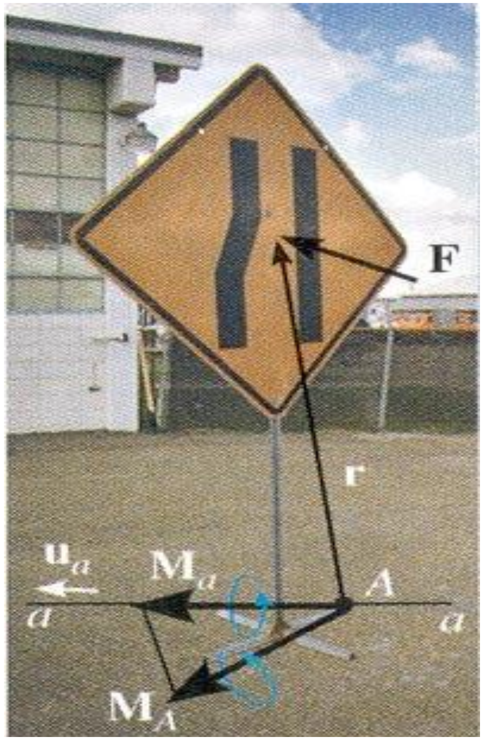
MOMENT ABOUT AN AXIS

Today's Objectives:

Students will be able to determine the moment of a force about an axis using

a) scalar analysis, and

b) vector analysis.



QUESTION

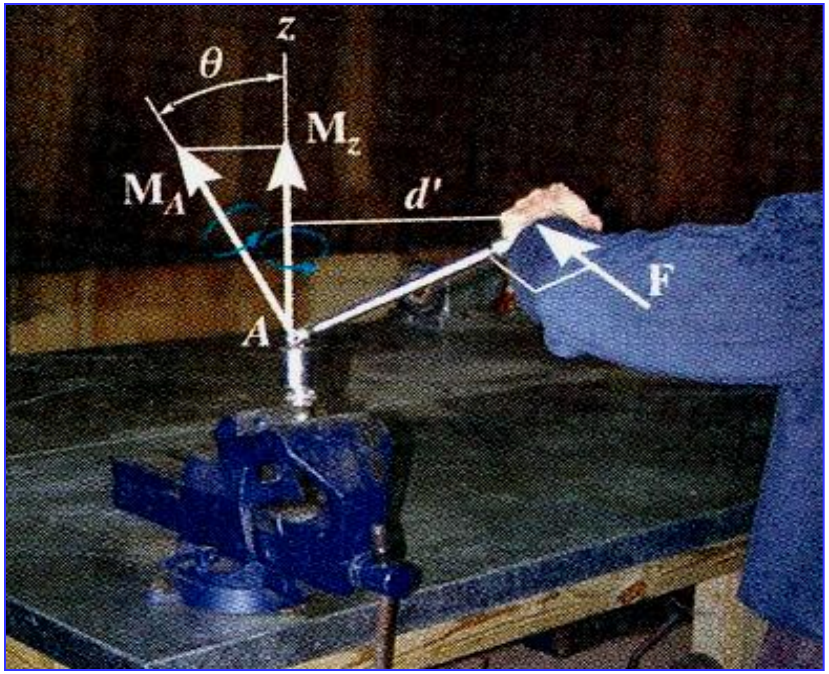
The triple scalar product $\boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$ results in

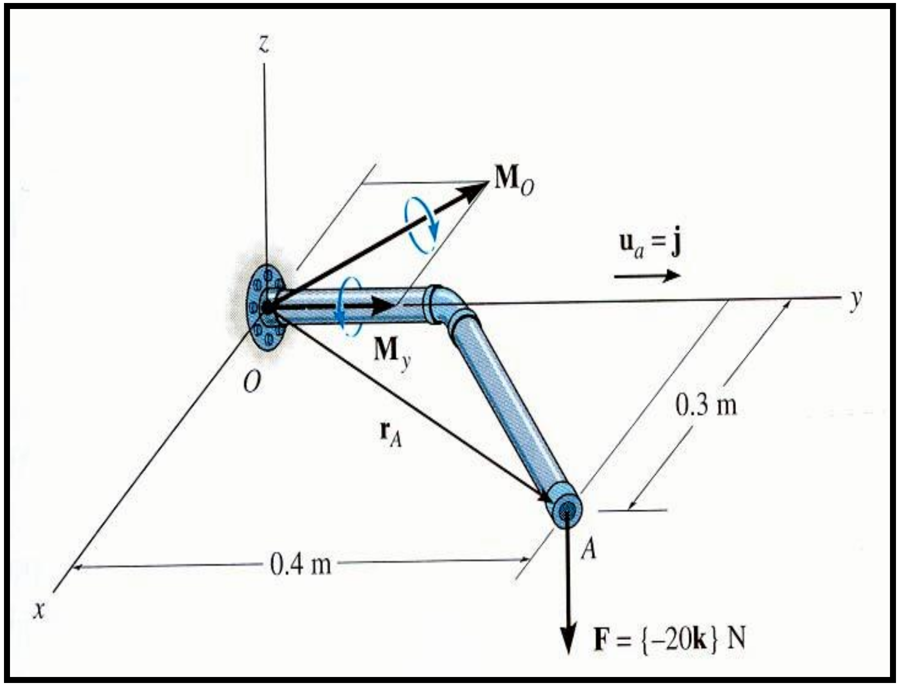
A) a scalar quantity (+ or -). B) a vector quantity.

C) zero. D) a unit vector.

E) an imaginary number.

APPLICATIONS



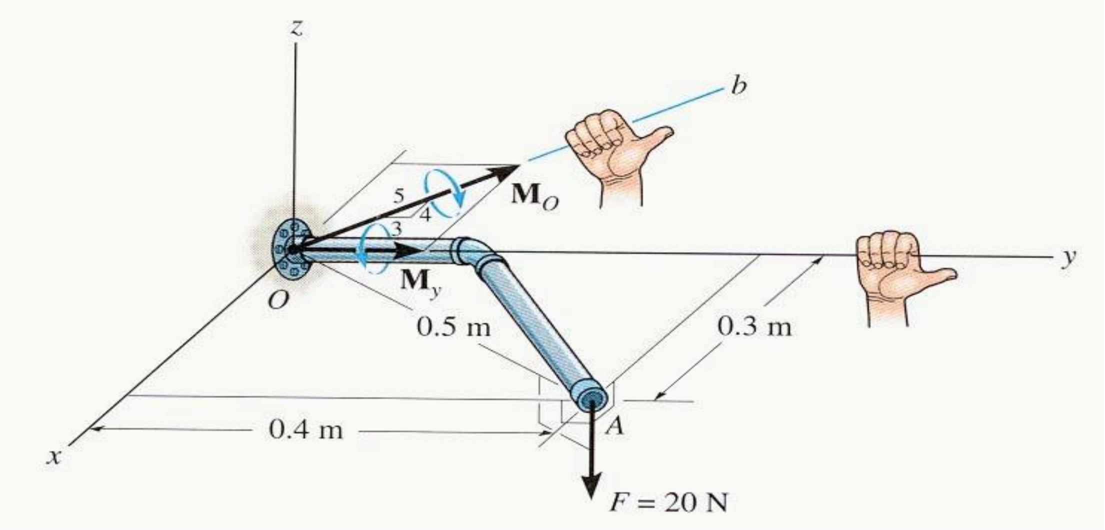


With the force ***F***, a person is creating the moment $\boldsymbol{M_A}$. What portion of $\boldsymbol{M_A}$ is used in turning the socket?

The force ***F*** is creating the moment $\boldsymbol{M_O}$. How much of $\boldsymbol{M_O}$ acts to unscrew the pipe?

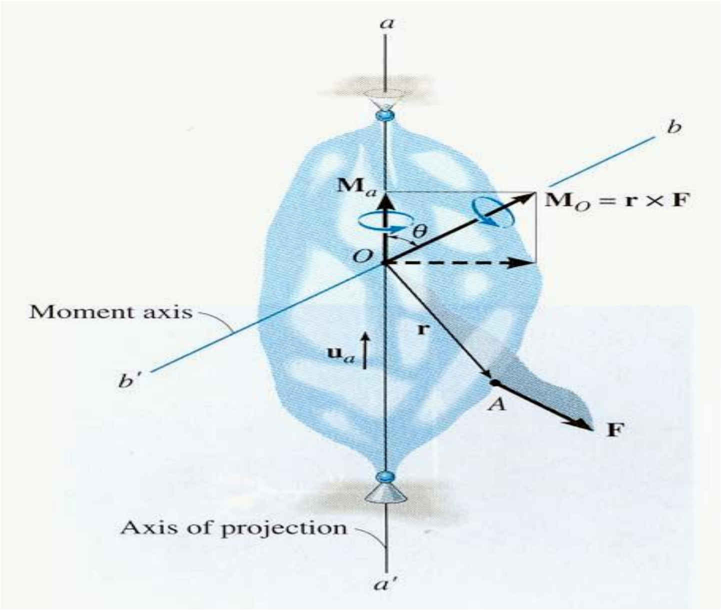
SCALAR ANALYSIS

Recall that the moment of a force about any point A is $M_A = F\, d_A$ where d_A is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.



In the figure above, the moment about the y-axis would be $M_y = 20\,(0.3) = 6$ N·m. However, this calculation is not always trivial and vector analysis may be preferable.

VECTOR ANALYSIS



Our goal is to find the moment of ***F*** (the tendency to rotate the body) about the axis a'-a.

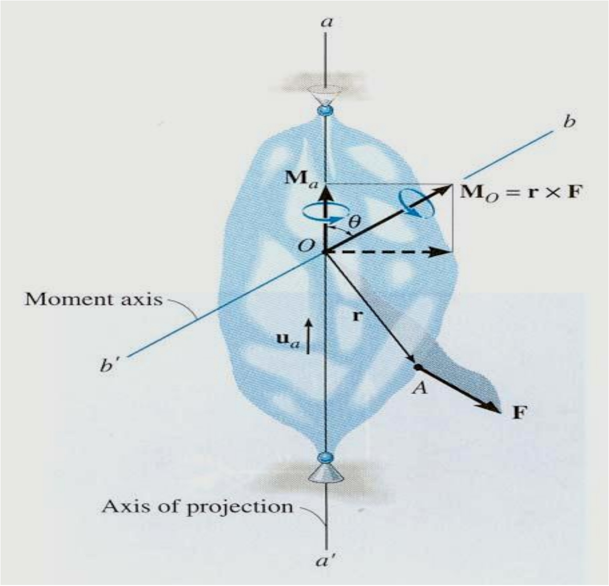
First compute the moment of ***F*** about any arbitrary point O that lies on the a'a axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

Now, find the component of $\boldsymbol{M_O}$ along the axis a'-a using the dot product. This is the projection of $\boldsymbol{M_O}$ along a'-a.

$$M_a = \boldsymbol{u_a} \cdot \boldsymbol{M_O}$$

VECTOR ANALYSIS (continued)



M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the <u>triple scalar product.</u>

In the this equation,

u*$_a$* represents the unit vector along the axis a'-a axis,

r is the position vector from *any point on the a'-a axis to any point A on the line of action of the force*, and

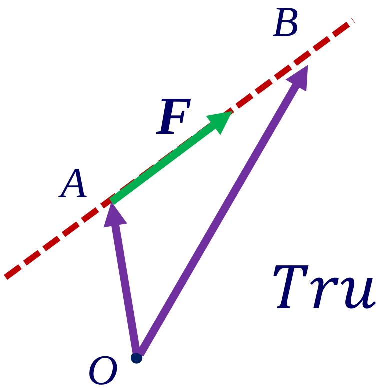
F is the force vector.

Note

r is the position vector from *any point on the a'-a axis to any point A on the line of action of the force.*

We get to pick the origin of ***r*** on the *a'-a* axis. We get to pick the termination of ***r*** on the line of action. Our job is to find easiest to see and use origin/terminal points for ***r***.

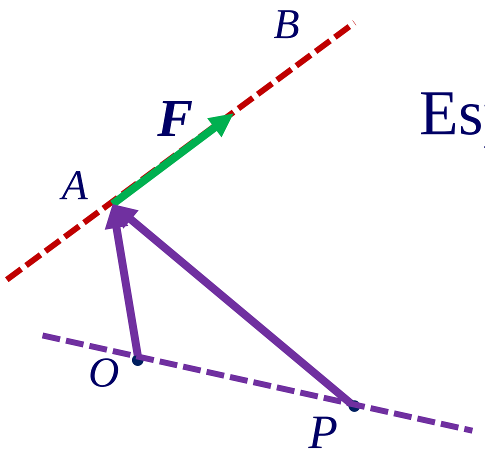
In previous section we saw why we could choose any point on line of action to determine a moment about a point.



$$\vec{r}_{OA} \times \vec{F} = \vec{r}_{OB} \times \vec{F}$$

$$True\ b/c\ \vec{r}_{OA} = \vec{r}_{OB} - \vec{r}_{AB}\ \&\ \vec{r}_{AB} \times \vec{F} = 0$$

But why can we choose any point on the axis of interest?



Especially since, in general, $\vec{r}_{OA} \times \vec{F} \neq \vec{r}_{PA} \times \vec{F}$

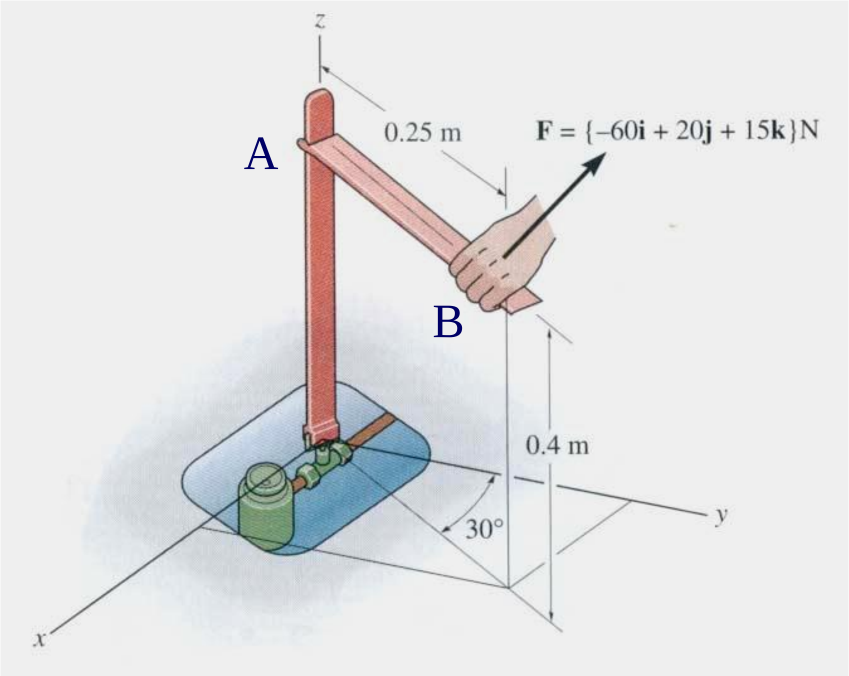
Although their moments are not the same, their projections along OP are the same!

First note $\vec{r}_{OA} = \vec{r}_{OP} + \vec{r}_{PA}$. So

$$M_{OP} = \hat{u}_{OP} \cdot \left(\vec{r}_{OA} \times \vec{F}\right) = \hat{u}_{OP} \cdot \left(\left(\vec{r}_{OP} + \vec{r}_{PA}\right) \times \vec{F}\right)$$
$$= \hat{u}_{OP} \cdot \left(\vec{r}_{OP} \times \vec{F}\right) + \hat{u}_{OP} \cdot \left(\vec{r}_{PA} \times \vec{F}\right)$$

Now $\vec{r}_{OP} \times \vec{F}$ is $\perp$ to $\vec{r}_{OP}$. So first term vanishes.

EXAMPLE



Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

Plan:

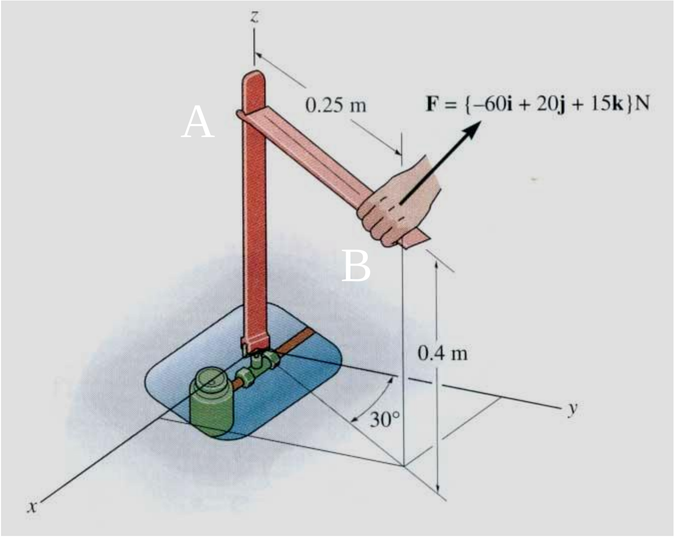
1) We need to use $\boldsymbol{M_z} = \boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$.

2) Note that $\boldsymbol{u} = 1\,\boldsymbol{k}$.

3) Choose point on z axis and on line of action. These can only be A and B here. The vector $\boldsymbol{r}$ is the position vector from A to B.

4) Force $\boldsymbol{F}$ is already given in Cartesian vector form.

EXAMPLE (continued)



$\boldsymbol{u} = 1\,\boldsymbol{k}$

$\boldsymbol{r_{AB}} = \{0.25 \sin 30°\, \boldsymbol{i} + 0.25 \cos 30°\, \boldsymbol{j}\}$ m

$= \{0.125\, \boldsymbol{i} + 0.2165\, \boldsymbol{j}\}$ m

$\boldsymbol{F} = \{-60\, \boldsymbol{i} + 20\, \boldsymbol{j} + 15\, \boldsymbol{k}\}$ N

$M_z = \boldsymbol{u} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.2165 & 0 \\ -60 & 20 & 15 \end{vmatrix}$$

$= 1\{0.125(20) - 0.2165(-60)\}$ N·m

$= 15.5$ N·m

CONCEPT QUIZ

1. The vector operation $(\mathbf{P} \times \mathbf{Q}) \bullet \mathbf{R}$ equals

   A) $\mathbf{P} \times (\mathbf{Q} \bullet \mathbf{R})$.

   B) $\mathbf{R} \bullet (\mathbf{P} \times \mathbf{Q})$.

   C) $(\mathbf{P} \bullet \mathbf{R}) \times (\mathbf{Q} \bullet \mathbf{R})$.

   D) $(\mathbf{P} \times \mathbf{R}) \bullet (\mathbf{Q} \times \mathbf{R})$.

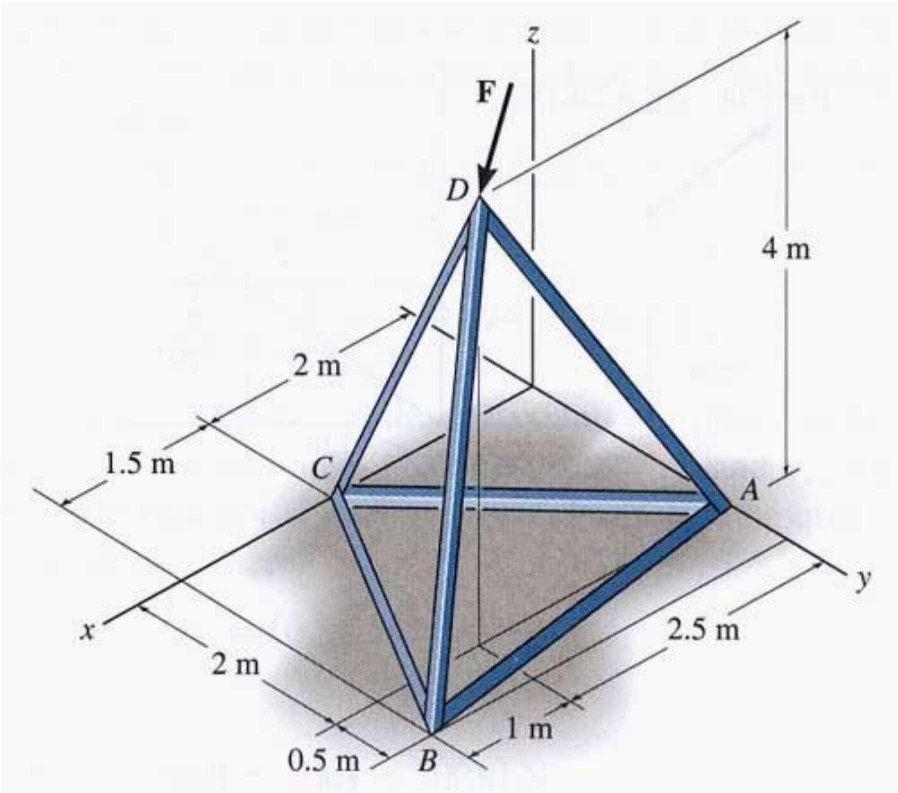
CONCEPT QUIZ

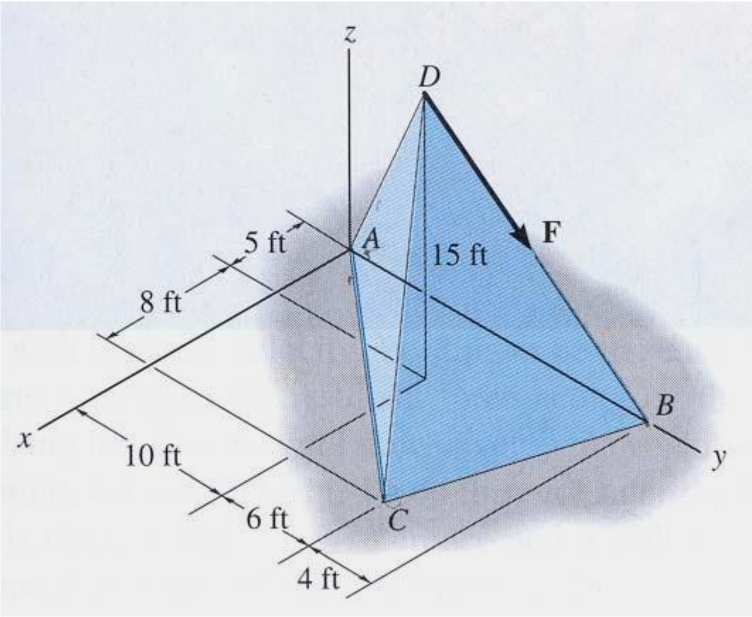
2. The force ***F*** is acting along DC. Using the triple product to determine the moment of ***F*** about the bar BA, you could use any of the following position vectors except ______.

   A) $\boldsymbol{r}_{BC}$ B) $\boldsymbol{r}_{AD}$

   C) $\boldsymbol{r}_{AC}$ D) $\boldsymbol{r}_{DB}$

   E) $\boldsymbol{r}_{BD}$





Example

Given: A force of 80 lb acts along the edge DB.

Find: The magnitude of the moment of this force about the axis AC.

Plan:

1) We need to use $M_{AC} = \boldsymbol{u_{AC}} \bullet (\boldsymbol{r_{??}} \times \boldsymbol{F_{DB}})$

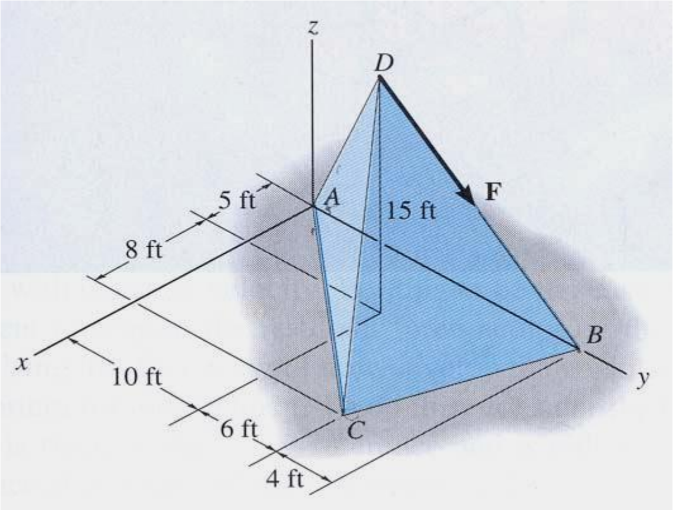
2) Can choose A or C and D or B. $\boldsymbol{r}_{AB}$ easy to find.

3) Find $\boldsymbol{u_{AC}} = \boldsymbol{r_{AC}} / r_{AC}$

4) Find $\boldsymbol{F_{DB}} = 80 \text{ lb } \boldsymbol{u_{DB}} = 80 \text{ lb } (\boldsymbol{r_{DB}} / r_{DB})$

5) Complete the triple scalar product..

SOLUTION



$\boldsymbol{r}_{AB} = \{\ 20\ \boldsymbol{j}\ \}$ ft

$\boldsymbol{r}_{AC} = \{\ 13\ \boldsymbol{i}\ +16\ \boldsymbol{j}\ \}$ ft

$\boldsymbol{r}_{DB} = \{\ -5\ \boldsymbol{i}\ +10\ \boldsymbol{j} - 15\ \boldsymbol{k}\ \}$ ft

$$\boldsymbol{u}_{AC} = (\ 13\ \boldsymbol{i}\ +16\ \boldsymbol{j}\ )\ \text{ft} / (13^{2} + 16^{2})^{½}\ \text{ft}$$

$$= 0.6306\ \boldsymbol{i}\ +\ 0.7761\ \boldsymbol{j}$$

$$\boldsymbol{F}_{DB} = 80\ \{\boldsymbol{r}_{DB} / (5^{2} + 10^{2} + 15^{2})^{½}\}\ \text{lb}$$

$$= \{-21.38\ \boldsymbol{i}\ +\ 42.76\ \boldsymbol{j}\ -\ 64.14\ \boldsymbol{k}\ \}\ \text{lb}$$

Solution (continued)

Now find the triple product, $M_{AC} = \boldsymbol{u_{AC}} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F_{DB}})$

$$M_{AC} = \begin{vmatrix} 0.6306 & 0.7706 & 0 \\ 0 & 20 & 0 \\ -21.38 & 42.76 & -64.14 \end{vmatrix} \begin{matrix} \\ \text{ft} \\ \text{lb} \end{matrix}$$

$$M_{AC} = 0.6306\ \{20\ (-64.14) - 0\ - 0.7706\ (0 - 0)\}\ \text{lb}\cdot\text{ft}$$

$$= -809\ \text{lb}\cdot\text{ft}$$

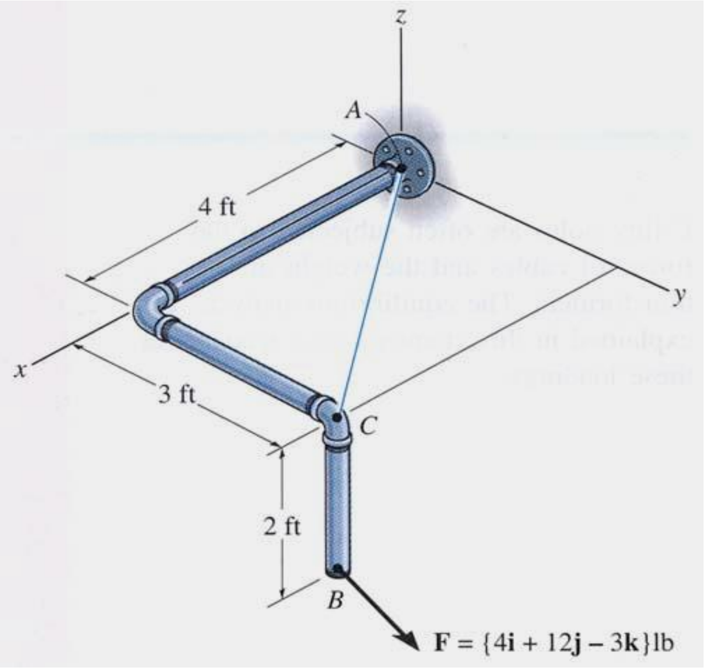
The negative sign indicates that the sense of $\boldsymbol{M_{AC}}$ is opposite to that of $\boldsymbol{u_{AC}}$

ATTENTION QUIZ

1. For finding the moment of the force ***F*** about the x-axis, the position vector in the triple scalar product should be ___ .

A) $\boldsymbol{r}_{AC}$ B) $\boldsymbol{r}_{BA}$

C) $\boldsymbol{r}_{AB}$ D) $\boldsymbol{r}_{BC}$



2. If $\boldsymbol{r} = \{1\,\boldsymbol{i} + 2\,\boldsymbol{j}\}$ m and $\boldsymbol{F} = \{10\,\boldsymbol{i} + 20\,\boldsymbol{j} + 30\,\boldsymbol{k}\}$ N, then the moment of ***F*** about the y-axis is ____ N·m.

A) 10 B) -30

C) -40 D) None of the above.