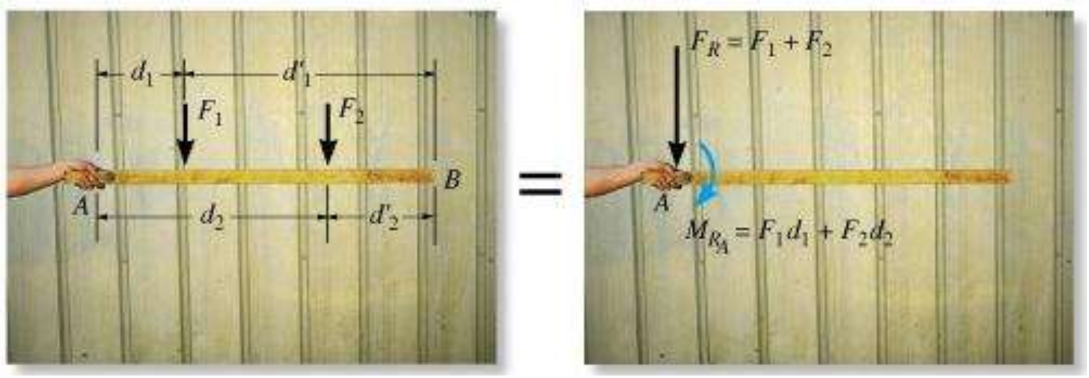
EQUIVALENT SYSTEMS, RESULTANTS OF FORCE AND COUPLE SYSTEM, & FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM

Today's Objectives:

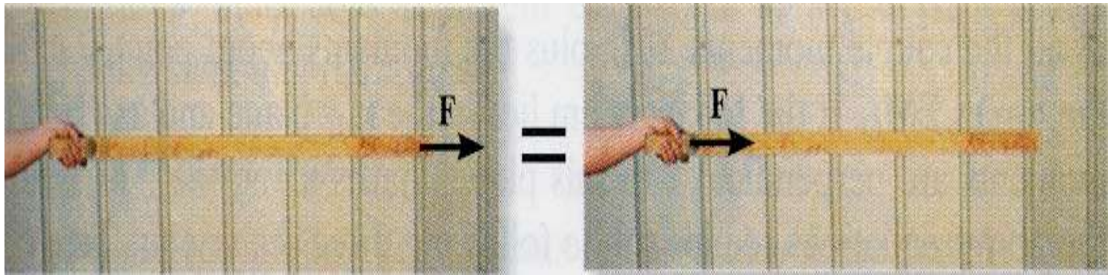
Students will be able to:

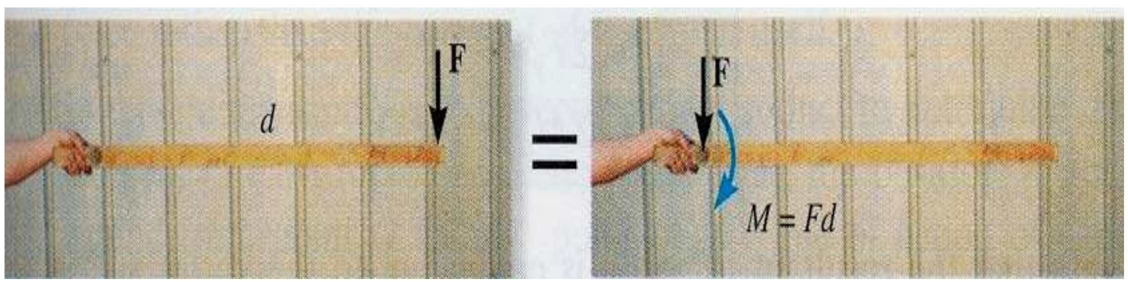
a) Determine the effect of moving a force.

b) Find an equivalent force-couple system for a system of forces and couples.



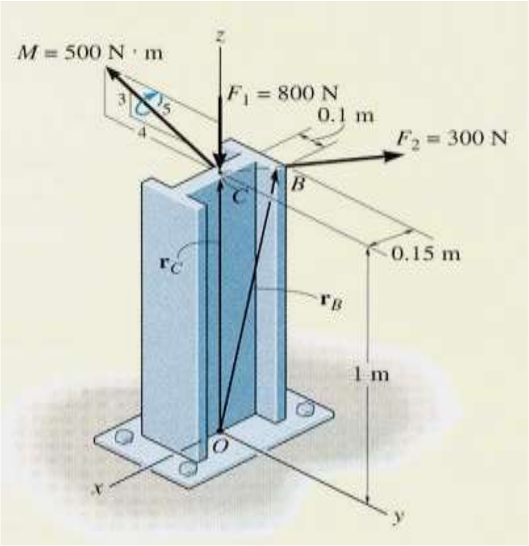
APPLICATIONS



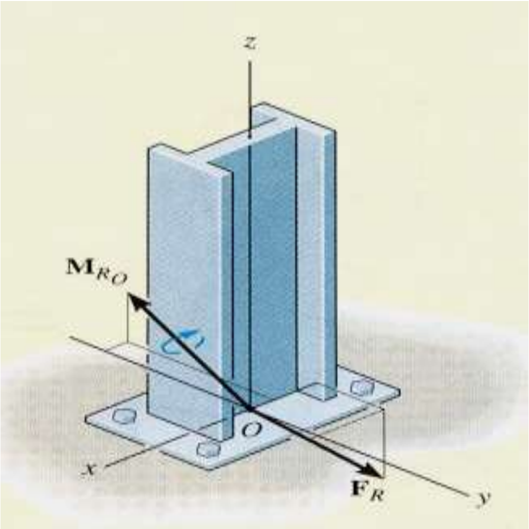


What is the resultant effect on the person’s hand when the force is applied in four different ways ?

APPLICATIONS (continued)



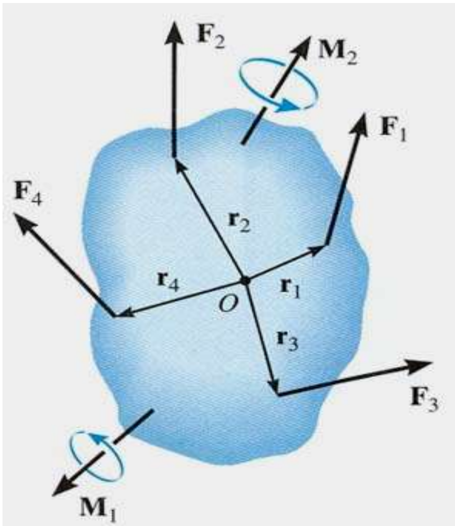
|| ??



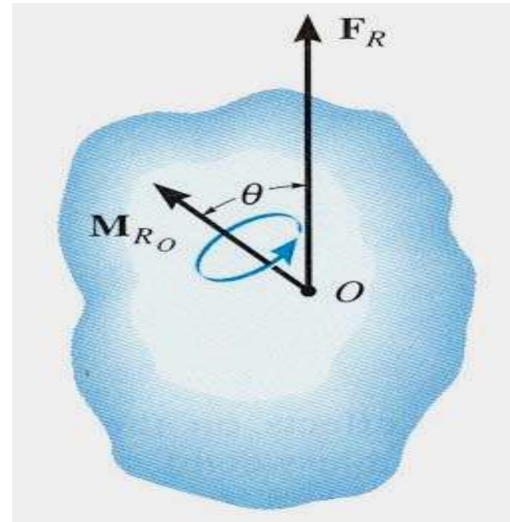
Several forces and a couple moment are acting on this vertical section of an I-beam.

Can you replace them with just one force and one couple moment at point O that will have the same external effect? If yes, how will you do that?

AN EQUIVALENT SYSTEM
(Section 4.7)



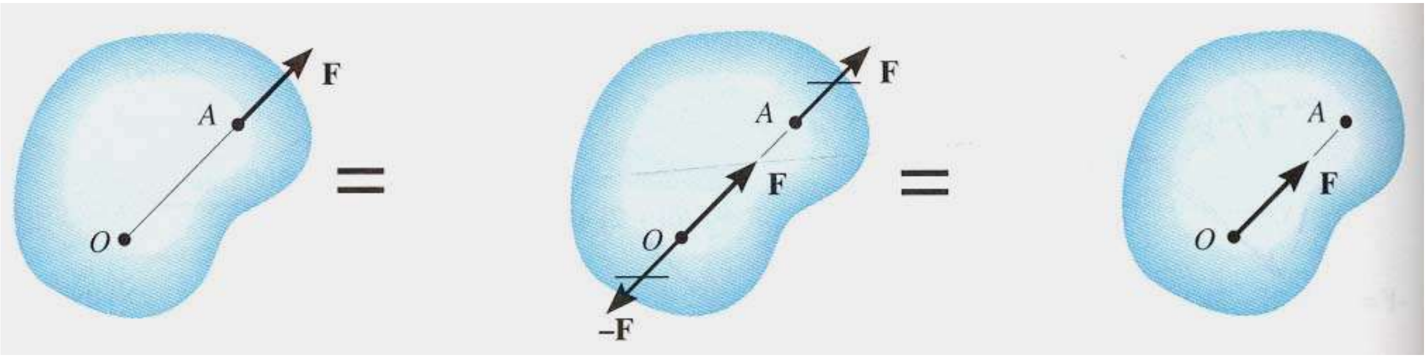
=



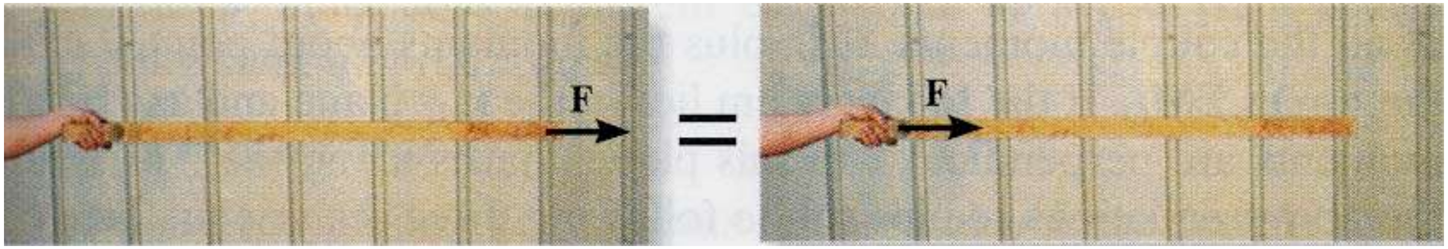
When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

The two force and couple systems are called equivalent systems since they have the same external effect on the body.

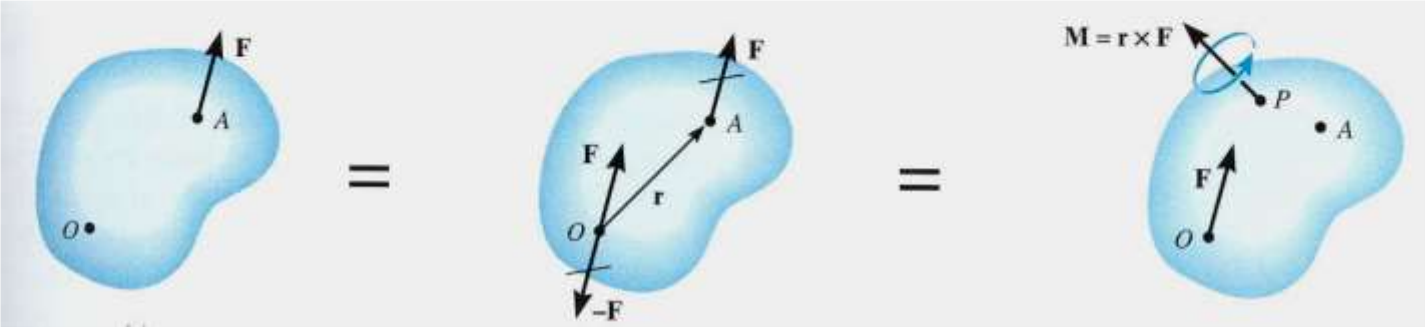
MOVING A FORCE ON ITS LINE OF ACTION



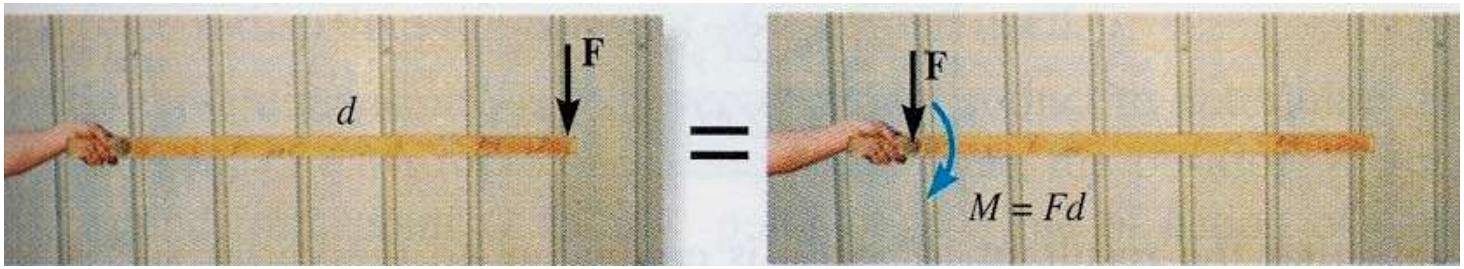
Moving a force from A to O, when both points are on the vectors' line of action, does not change the external effect. Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

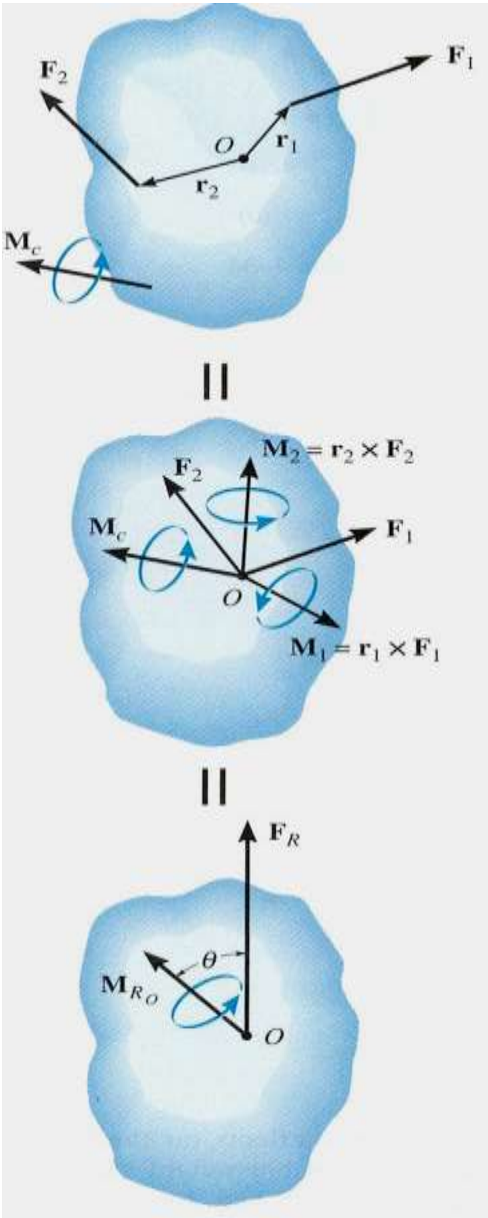


MOVING A FORCE OFF OF ITS LINE OF ACTION



Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a "free" vector, it can be applied at any point P on the body.





RESULTANTS OF A FORCE AND COUPLE SYSTEM (Section 4.8)

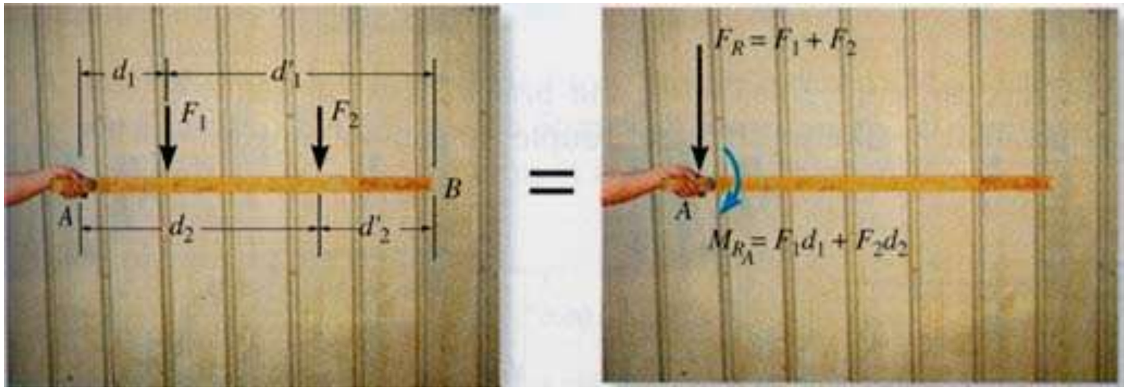
When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

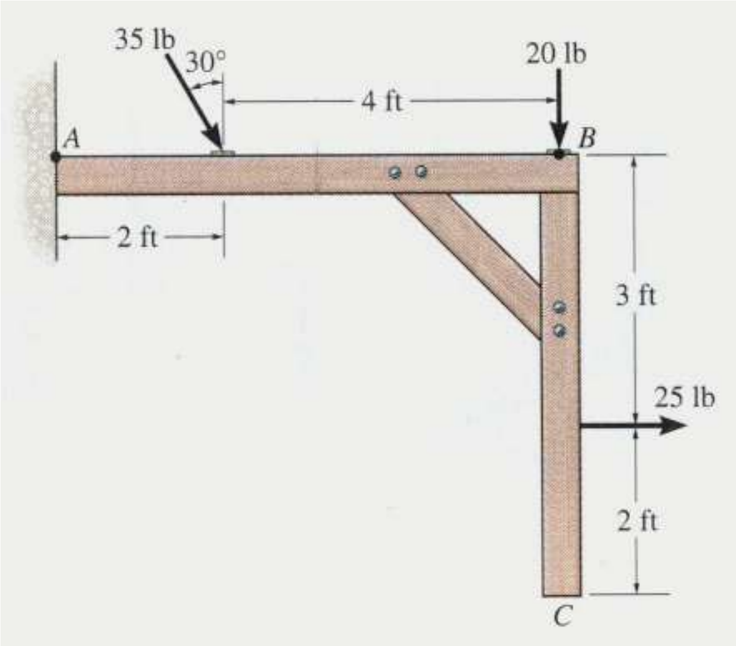
RESULTANT OF A FORCE AND COUPLE SYSTEM (continued)



If the force system lies in the x-y plane (the 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

EXAMPLE #1



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A.

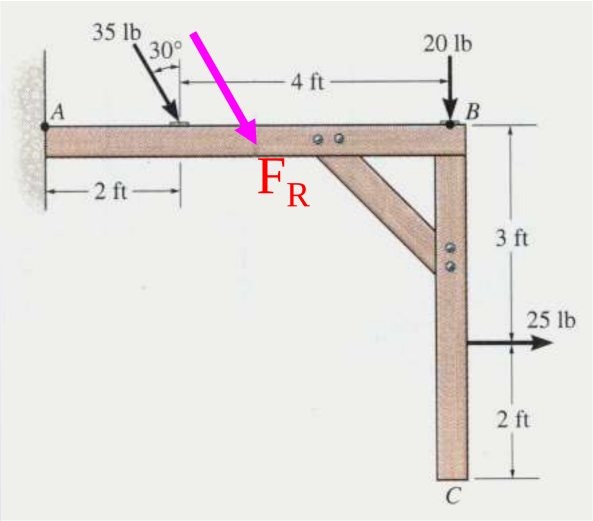
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A.

EXAMPLE #1

(continued)



$+ \rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30° = 42.5 \text{ lb}$

$+ \downarrow \Sigma F_{Ry} = 20 + 35 \cos 30° = 50.31 \text{ lb}$

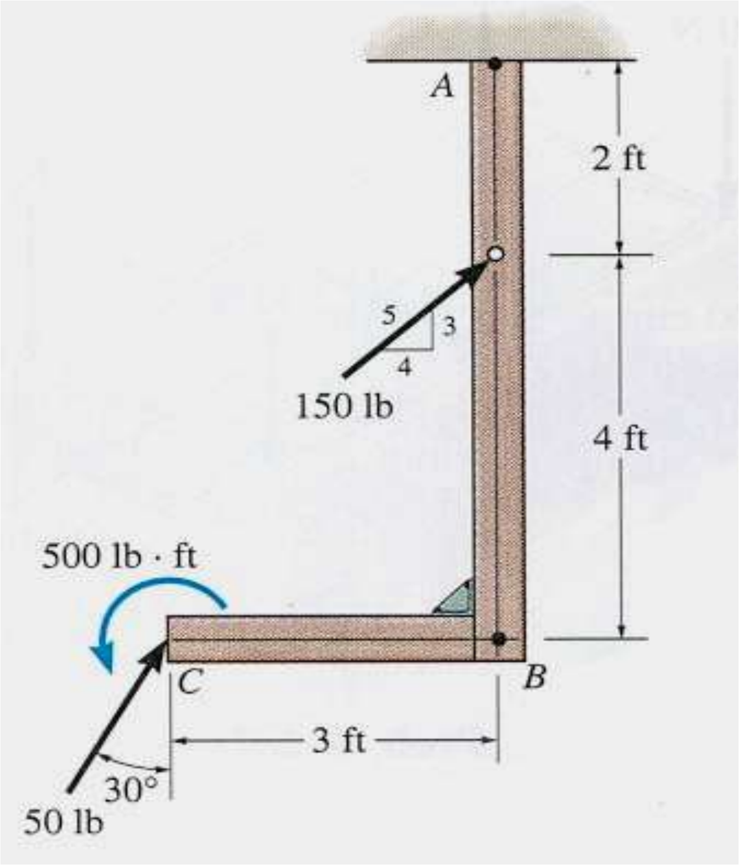
$+ \circlearrowleft M_{RA} = 35 \cos 30° (2) + 20(6) - 25(3)$

$= 105.6 \text{ lb} \cdot \text{ft}$

$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9 \text{ lb}$

$\searrow \theta = \tan^{-1} (50.31/42.5) = 49.8°$

EXAMPLE #2



Given: A 2-D force and couple system as shown.

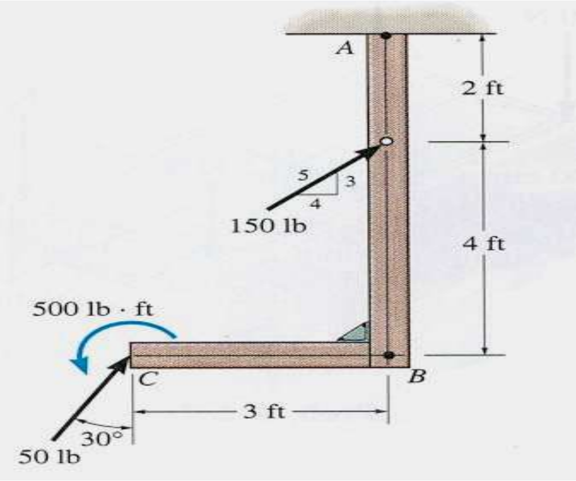
Find: The equivalent resultant force and couple moment acting at A.

Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A and add them to the 500 lb - ft free moment to find the resultant M_{RA} .

EXAMPLE #2 (continued)



Summing the force components:

$+\rightarrow \Sigma F_x = (4/5)\,150 \text{ lb} + 50 \text{ lb} \sin 30° = 145 \text{ lb}$

$+\uparrow \Sigma F_y = (3/5)\,150 \text{ lb} + 50 \text{ lb} \cos 30° = 133.3 \text{ lb}$

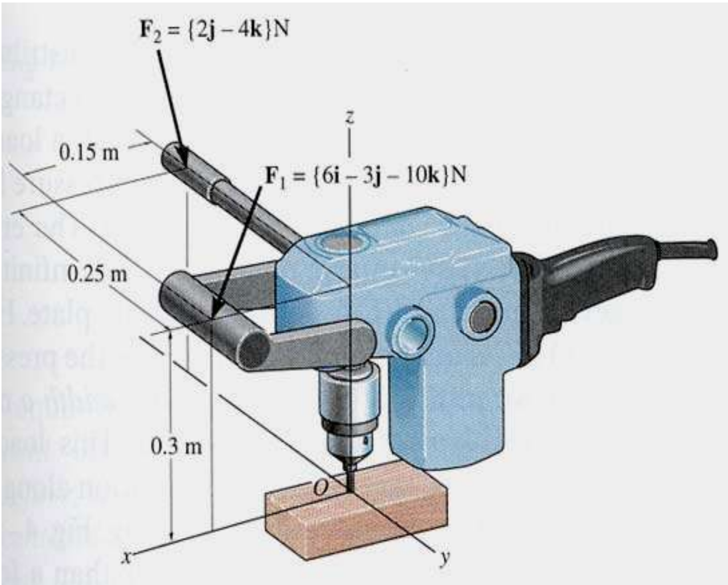
Now find the magnitude and direction of the resultant.

$F_{RA} = (145^2 + 133.3^2)^{1/2} = 197 \text{ lb}$ and $\theta = \tan^{-1}(133.3/145) = 42.6°$

$+\circlearrowright M_{RA} = \{ (4/5)(150)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500 \}$

$= 760 \text{ lb}\cdot\text{ft}$

Example #3



Given: Handle forces F_1 and F_2 are applied to the electric drill.

Find: An equivalent resultant force and couple moment at point O.

<u>Plan</u>:

a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i}$

b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\, (\, \boldsymbol{r_i} \times \boldsymbol{F_i}\,)$

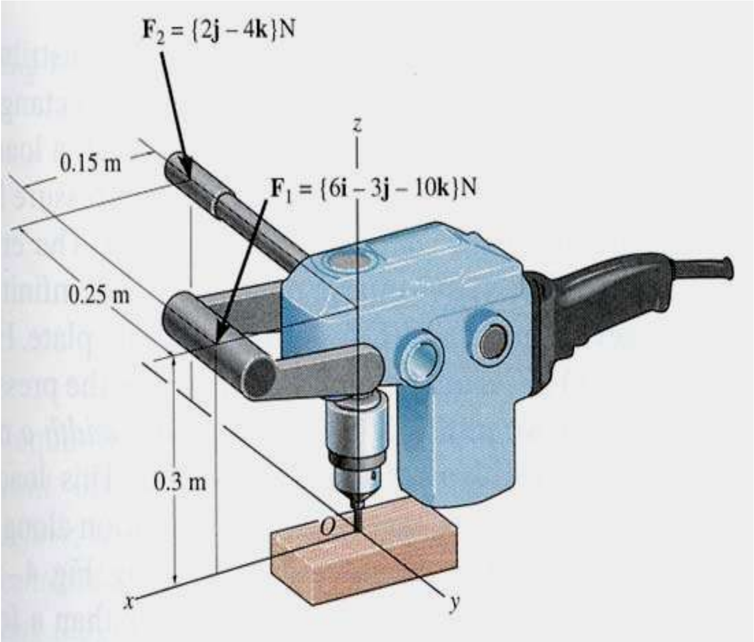
Where,

$\boldsymbol{F_i}$ are the individual forces in Cartesian vector notation (CVN).

$\boldsymbol{M_C}$ are any free couple moments in CVN (none in this example).

$\boldsymbol{R_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

SOLUTION



$$F_1 = \{6\,\boldsymbol{i} - 3\,\boldsymbol{j} - 10\,\boldsymbol{k}\}\ \text{N}$$

$$F_2 = \{0\,\boldsymbol{i} + 2\,\boldsymbol{j} - 4\,\boldsymbol{k}\}\ \text{N}$$

$$F_{RO} = \{6\,\boldsymbol{i} - 1\,\boldsymbol{j} - 14\,\boldsymbol{k}\}\ \text{N}$$

$$r_1 = \{0.15\,\boldsymbol{i} + 0.3\,\boldsymbol{k}\}\ \text{m}$$

$$r_2 = \{-0.25\,\boldsymbol{j} + 0.3\,\boldsymbol{k}\}\ \text{m}$$

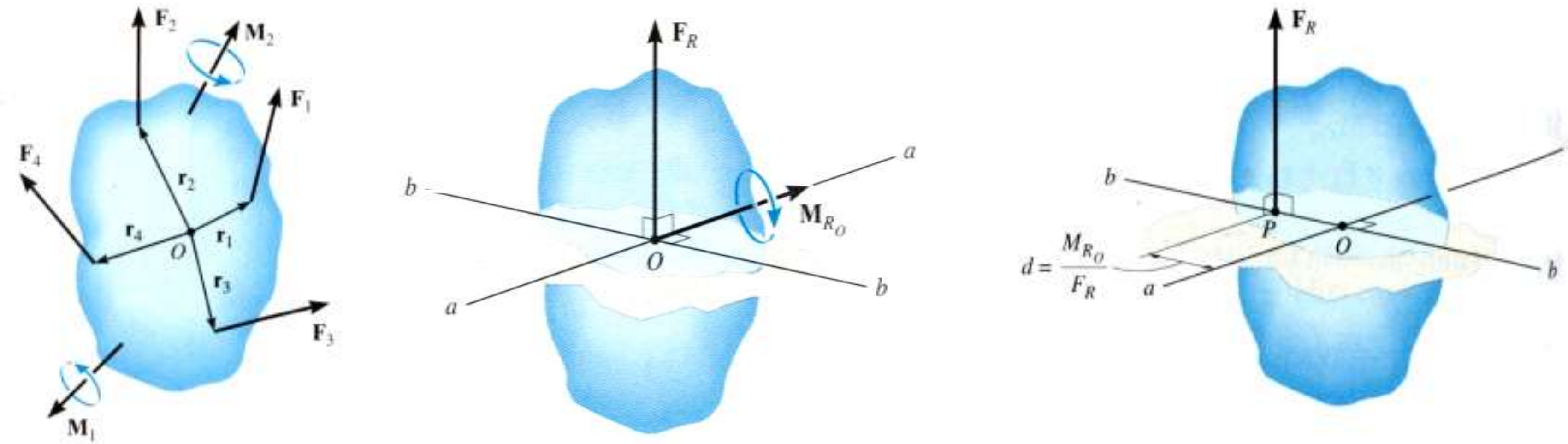
$$M_{RO} = r_1 \times F_1 + r_2 \times F_2$$

$$M_{RO} = \left\{ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix} \right\}\ \text{N·m}$$

$$= \{0.9\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k} + 0.4\,\boldsymbol{i} + 0\,\boldsymbol{j} + 0\,\boldsymbol{k}\}\ \text{N·m}$$

$$= \{1.3\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k}\}\ \text{N·m}$$

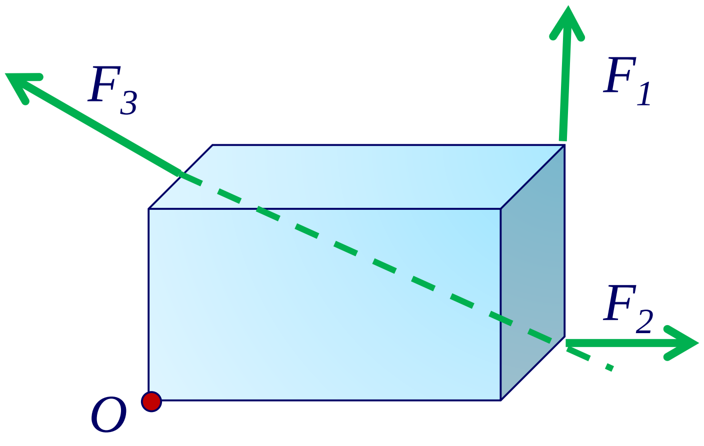
FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM
(Section 4.9)

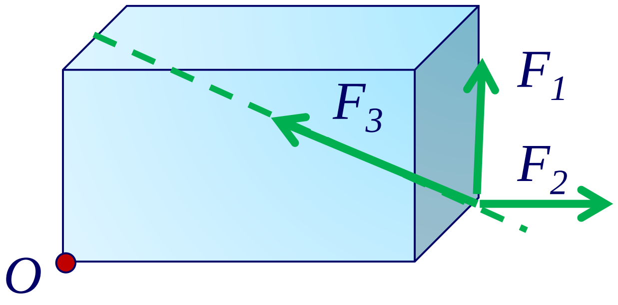


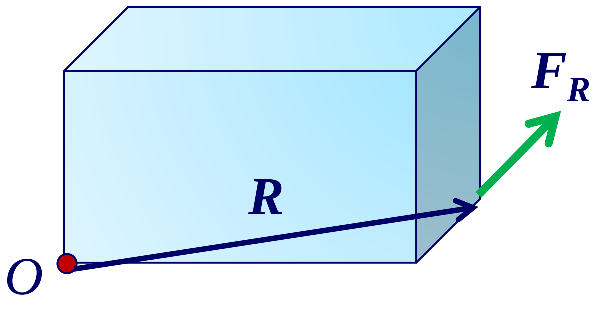
If $\boldsymbol{F_R}$ and $\boldsymbol{M_{RO}}$ are perpendicular to each other, then the system can be further reduced to a single force, $\boldsymbol{F_R}$, by simply moving $\boldsymbol{F_R}$ from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force as long as $\boldsymbol{F_R} \neq 0$.

Concurrent means lines of action of all the forces pass through the same point.

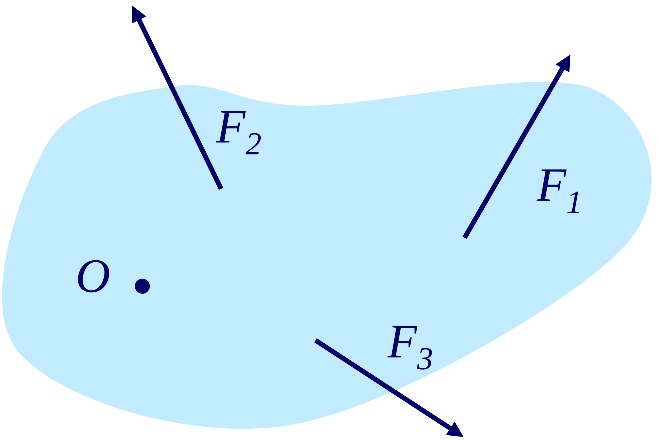






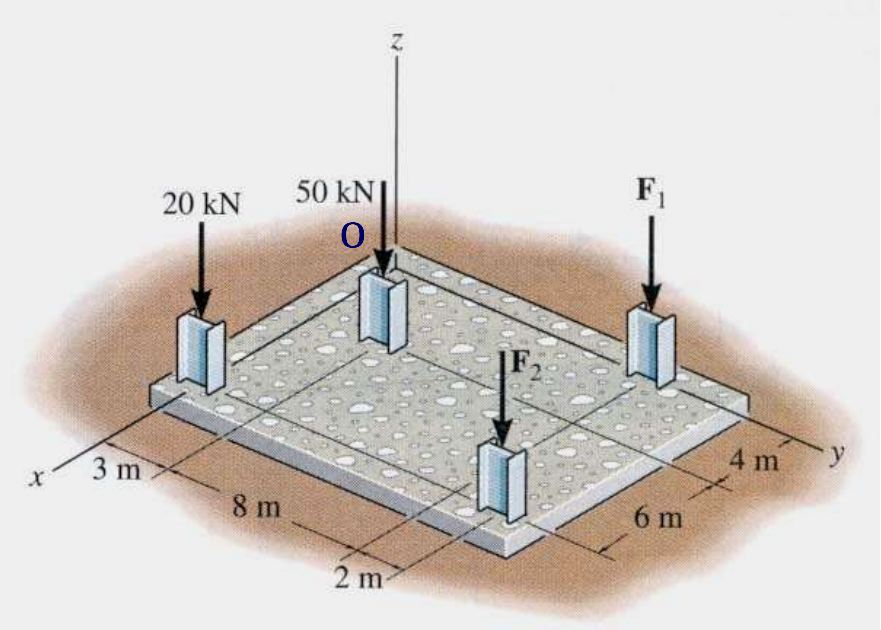
By definition, $\boldsymbol{M} = \boldsymbol{R} \times \boldsymbol{F_R}$ will be perpendicular to $\boldsymbol{F_R}$

Coplanar – all forces in same plane
– effectively a 2D problem
– $\boldsymbol{F_R}$ also in plane



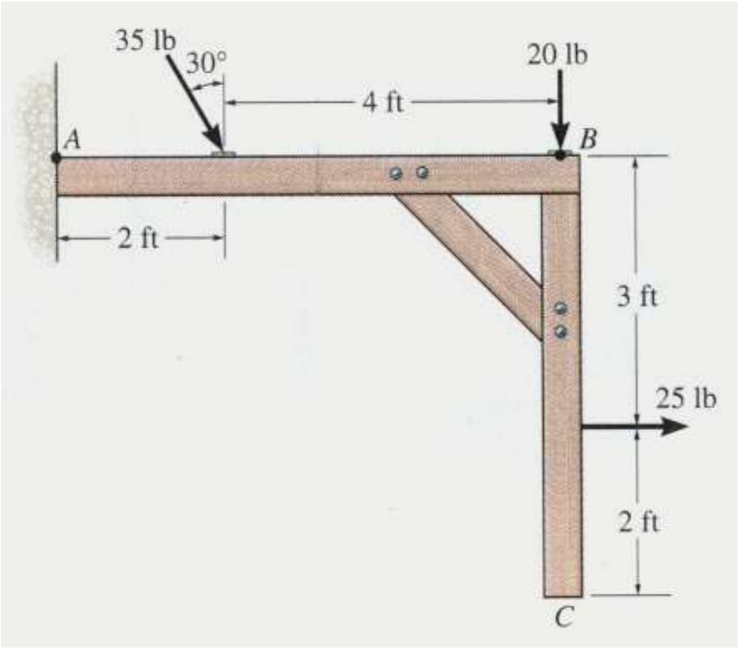
For a 2D problem, all moments are out of or into page, so $\boldsymbol{M_O} \perp \boldsymbol{F_R}$

Parallel forces can create no moments in their own direction



Hence the only moments are perpendicular to the forces as is required.

EXAMPLE #4 - Coplanar



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location along the beam AB.

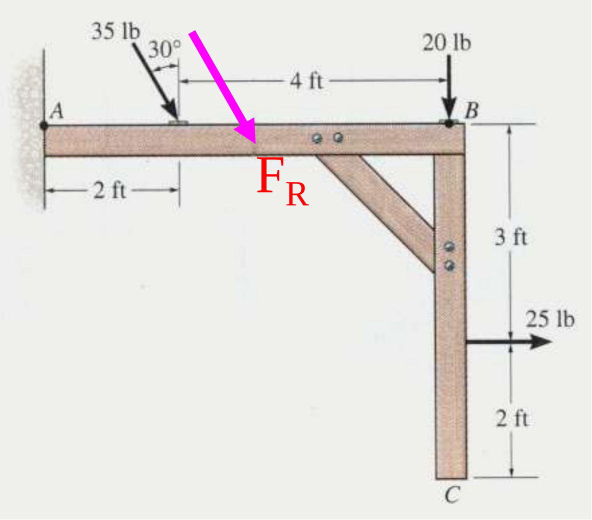
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A.

3) Shift the F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE #4
(continued)



$$+\rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30° = 42.5 \text{ lb}$$

$$+\downarrow \Sigma F_{Ry} = 20 + 35 \cos 30° = 50.31 \text{ lb}$$

$$+\circlearrowleft M_{RA} = 35 \cos 30° (2) + 20(6) - 25(3)$$

$$= 105.6 \text{ lb}\cdot\text{ft}$$

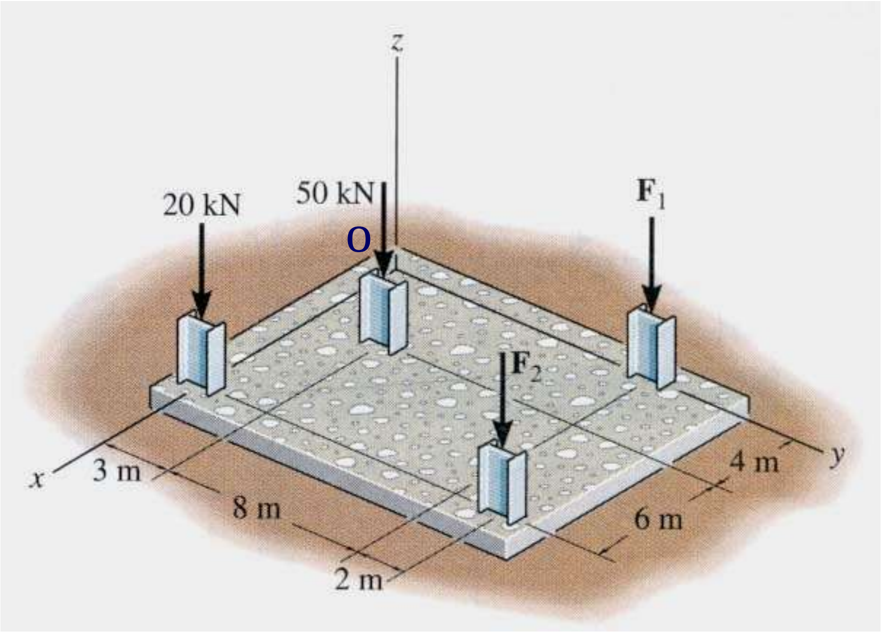
$$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9 \text{ lb}$$

$$\theta = \tan^{-1}(50.31/42.5) = 49.8°$$

The equivalent single force F_R can be located on the beam AB at a distance d measured from A.

$$d = M_{RA}/F_{Ry} = 105.6/50.31 = 2.10 \text{ ft.}$$

EXAMPLE #5 Parallel



Given: The building slab has four columns. F_1 and $F_2 = 0$.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x,y) of the single equivalent resultant force.

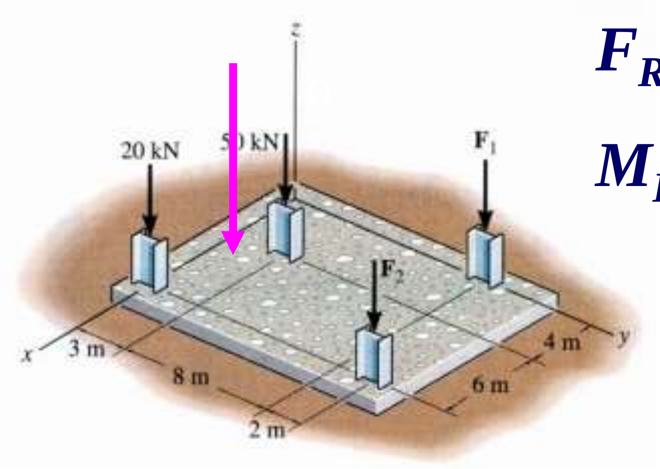
Plan:

1) Find $\boldsymbol{F_{RO}} = \sum \boldsymbol{F_i} = F_{Rzo}\, \boldsymbol{k}$

2) Find $\boldsymbol{M_{RO}} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i}) = M_{RxO}\, \boldsymbol{i} + M_{RyO}\, \boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

EXAMPLE #5
(continued)



$F_{RO} = \{-50\,\boldsymbol{k} - 20\,\boldsymbol{k}\} = \{-70\,\boldsymbol{k}\}$ kN

$M_{RO} = (10\,\boldsymbol{i})\times(-20\,\boldsymbol{k}) + (4\,\boldsymbol{i} + 3\,\boldsymbol{j})\times(-50\,\boldsymbol{k})$

$= \{200\,\boldsymbol{j} + 200\,\boldsymbol{j} - 150\,\boldsymbol{i}\}$ kN·m

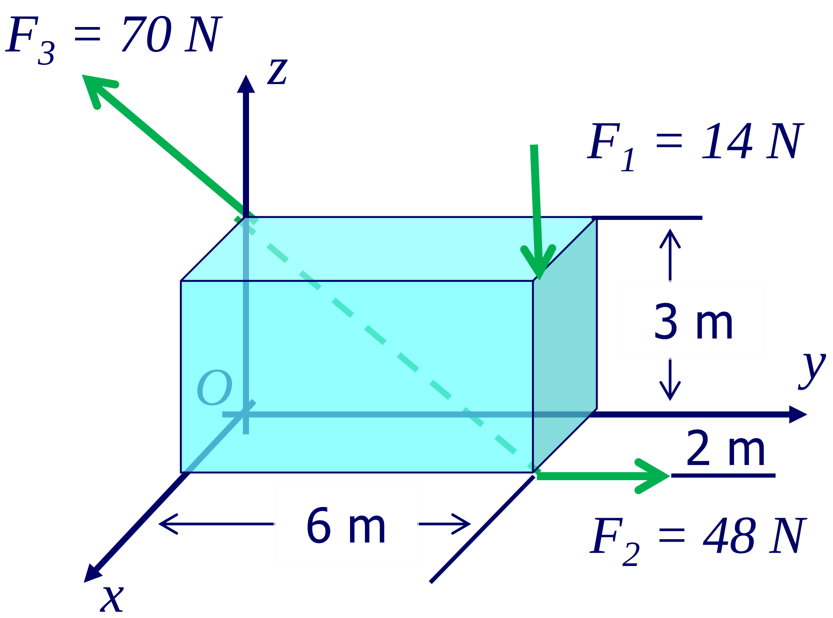
$= \{-150\,\boldsymbol{i} + 400\,\boldsymbol{j}\}$ kN·m

The location of the single equivalent resultant force is given as,

$x = -M_{Ryo}/F_{Rzo} = -400/(-70) = 5.71$ m

$y = M_{Rxo}/F_{Rzo} = (-150)/(-70) = 2.14$ m

EXAMPLE #6 Concurrent



Given: The block is acted upon by three forces.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (0, *y*, *z*) of the single equivalent resultant force.

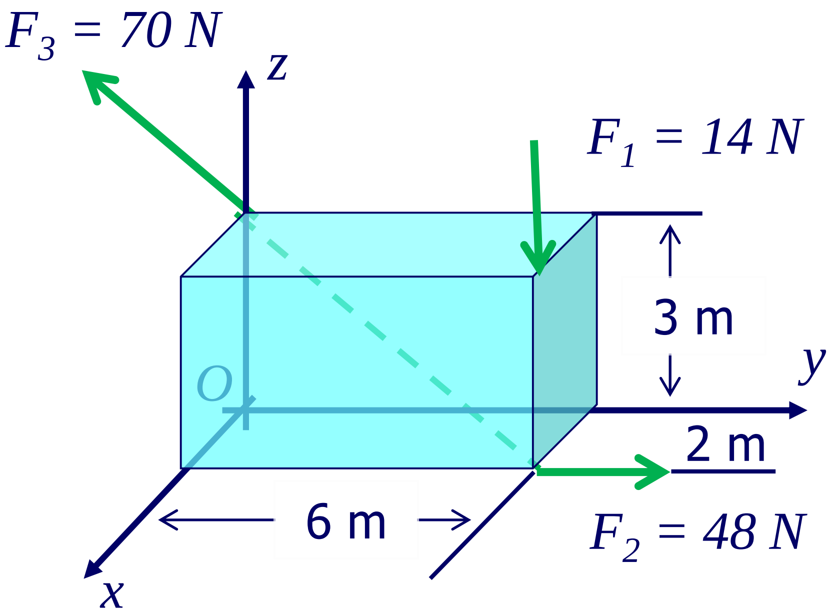
<u>Plan</u>:

1) Find $\boldsymbol{F_{RO}} = \sum \boldsymbol{F_i}$

2) Find $\boldsymbol{M_{RO}} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i}) = \boldsymbol{r} \times \boldsymbol{F_{RO}}$

3) The location of the single equivalent resultant force is given as $x = M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

EXAMPLE #6 Continued



$\boldsymbol{F}_1 = -14\,\boldsymbol{k}$ N

$\boldsymbol{F}_2 = 48\,\boldsymbol{j}$ N

$\boldsymbol{F}_3 = (70\text{ N})(-2\boldsymbol{i} - 6\boldsymbol{j} + 3\boldsymbol{k})/7$

$= (-20\boldsymbol{i} - 60\boldsymbol{j} + 30\boldsymbol{k})$ N

$\boldsymbol{F}_R = (-20\boldsymbol{i} - 12\boldsymbol{j} + 16\boldsymbol{k})$ N

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}_R = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 6 & 0 \\ -20 & -12 & 16 \end{vmatrix} m\cdot N$$

$$= (6*16)\boldsymbol{i} - \boldsymbol{j}(2*16) + \boldsymbol{k}(-24+120)\ m\cdot N$$

$$= \{96\boldsymbol{i} - 32\boldsymbol{j} + 96\boldsymbol{k}\}\ m\cdot N$$

EXAMPLE #6 Continued

Want

$$\boldsymbol{M}_O = \boldsymbol{r}' \times \boldsymbol{F}_{\boldsymbol{R}} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & y & z \\ -20 & -12 & 16 \end{vmatrix} m \cdot N = \{96\boldsymbol{i} - 32\boldsymbol{j} + 96\boldsymbol{k}\}\ m \cdot N$$

Or

$$(16y+20z)\boldsymbol{i} - \boldsymbol{j}(20z) + \boldsymbol{k}(20y) = 96\boldsymbol{i} - 32\boldsymbol{j} + 96\boldsymbol{k}$$

Get $z = 32/20 = 1.6$, $y = 96/20 = 4.8$

Check: 16*4.8+20*(1.6) = 96 ☑

If y and z were not on the face, solution would be unphysical.

Wrench or Screw

In general, $\boldsymbol{F_R}$ is not $\perp$ $\boldsymbol{M_O}$ but can still simplify somewhat.

$$\boldsymbol{M_O} = \boldsymbol{M_{\|}} + \boldsymbol{M_{\perp}}$$

Can find a location, such that $\boldsymbol{F_R}$ and $\boldsymbol{M_{\perp}}$ have the same external effect.

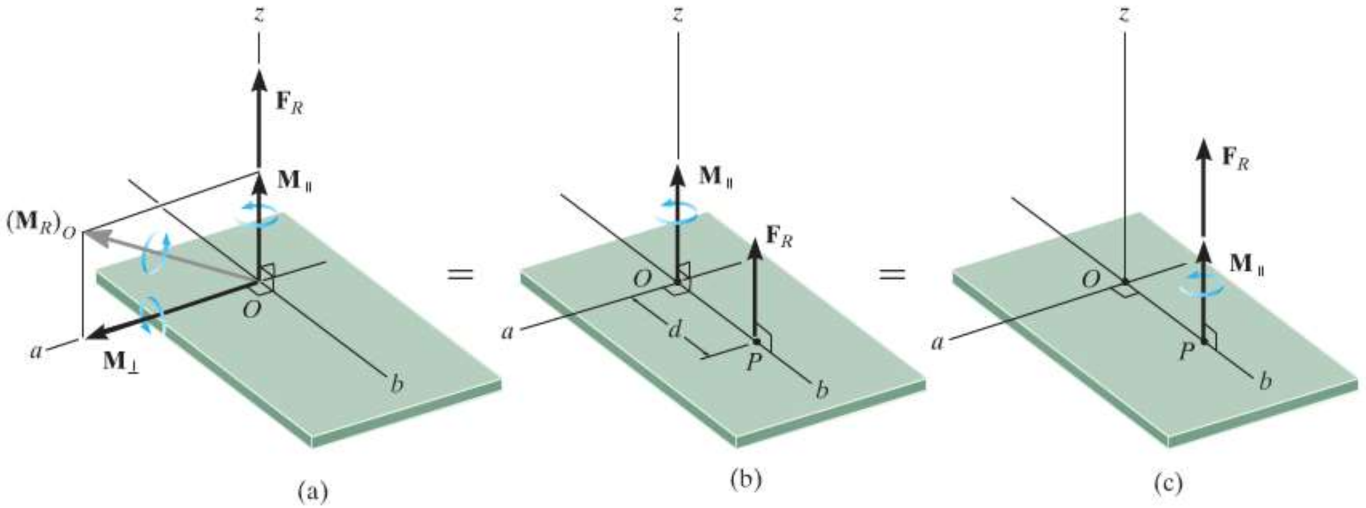
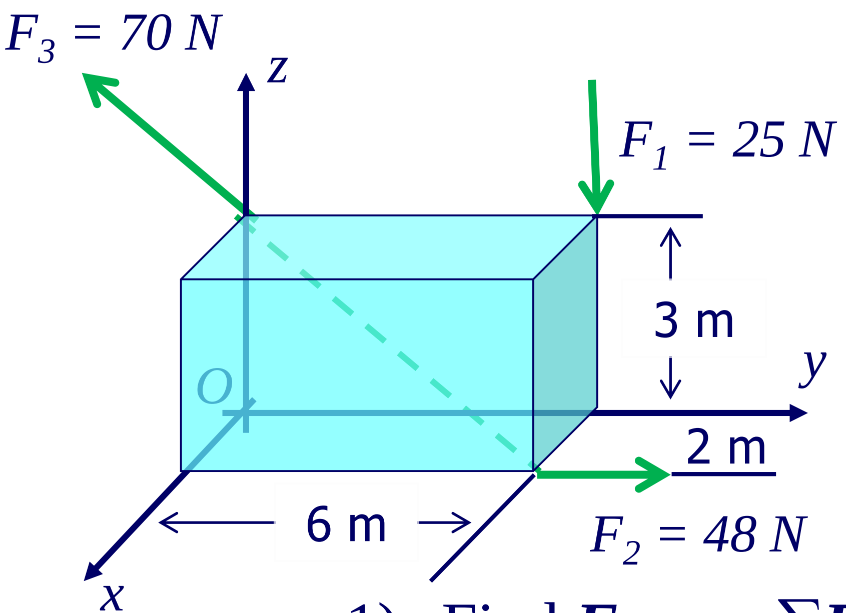


Fig. 4–43

Approach

- Find $\boldsymbol{F_R}$ and $\boldsymbol{M_O}$ as usual
- Use dot product to find $\boldsymbol{M}_{\|}$
- Recall $\boldsymbol{M}_{\|} = (\boldsymbol{M_O} \cdot \boldsymbol{u_F})\, \boldsymbol{u_F}$
- Next $\boldsymbol{M}_{\perp} = \boldsymbol{M_O} - \boldsymbol{M}_{\|}$
- Find $\boldsymbol{r}$ such that $\boldsymbol{r} \times \boldsymbol{F_R} = \boldsymbol{M}_{\perp}$

EXAMPLE #7



Given: The block is acted upon by three forces.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (0, *y*, *z*) of the wrench.

Plan:

1) Find $\boldsymbol{F_{RO}} = \sum \boldsymbol{F_i}$

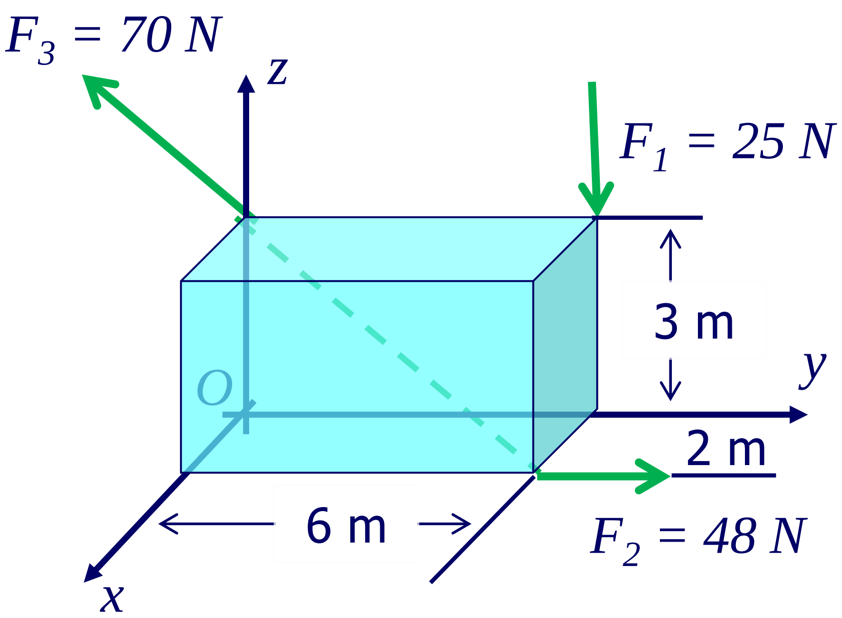
2) Find $\boldsymbol{M_{RO}} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i})$

3) Break $\boldsymbol{M_{RO}}$ into components

4) Find $\boldsymbol{r} \times \boldsymbol{F_{RO}} = \boldsymbol{M_{\perp}}$

5) Wrench is $\boldsymbol{F_{RO}}$, $\boldsymbol{M_{\parallel}}$, and location $\boldsymbol{r}$

EXAMPLE #7 (continued)



$$\boldsymbol{F}_1 = -25\,\boldsymbol{k}\ \text{N}$$

$$\boldsymbol{F}_2 = 48\,\boldsymbol{j}\ \text{N}$$

$$\boldsymbol{F}_3 = (70\ \text{N})(-2\boldsymbol{i} - 6\boldsymbol{j} + 3\boldsymbol{k})/7$$
$$= (-20\boldsymbol{i} - 60\boldsymbol{j} + 30\boldsymbol{k})\ \text{N}$$

$$\boldsymbol{F}_R = (-20\boldsymbol{i} - 12\boldsymbol{j} + 5\boldsymbol{k})\ \text{N}$$

$$\boldsymbol{M}_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 6 & 0 \\ 0 & 0 & -25 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 0 & 0 \\ 0 & 48 & 0 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0 & 3 \\ -20 & -60 & 30 \end{vmatrix}\ m{\cdot}N$$

$$= (-150)\boldsymbol{i} + (96)\boldsymbol{k} + (180\boldsymbol{i} - 60\boldsymbol{j})\ m{\cdot}N$$

$$= \{30\boldsymbol{i} - 60\boldsymbol{j} + 96\boldsymbol{k}\}\ m{\cdot}N$$

EXAMPLE #7 (continued)

Since $\boldsymbol{F_R} = (-20\boldsymbol{i} - 12\boldsymbol{j} + 5\boldsymbol{k})$ N, $\boldsymbol{u_F} = (-20\boldsymbol{i} - 12\boldsymbol{j} + 5\boldsymbol{k})/\sqrt{569}$

$$\boldsymbol{M}_{||} = (\boldsymbol{M_O} \bullet \boldsymbol{u_F})\, \boldsymbol{u_F}$$

$$= [\{30\boldsymbol{i} - 60\boldsymbol{j} + 96\boldsymbol{k}\} \bullet (-20\boldsymbol{i} - 12\boldsymbol{j} + 5\boldsymbol{k})/\sqrt{569}]$$

$$* (-20\boldsymbol{i} - 12\boldsymbol{j} + 5\boldsymbol{k})/\sqrt{569} \quad m{\cdot}N$$

$$= (-600 + 720 + 480)\,(-20\boldsymbol{i} - 12\boldsymbol{j} + 5\boldsymbol{k})/569 \quad m{\cdot}N$$

$$= 600\,(-20\boldsymbol{i} - 12\boldsymbol{j} + 5\boldsymbol{k})/569 \quad m{\cdot}N$$

$$= \boldsymbol{i}(-21.0896) + \boldsymbol{j}\,(-12.6538) + \boldsymbol{k}\,(5.2724) \quad m{\cdot}N$$

$$\boldsymbol{M}_{\perp} = \boldsymbol{M_O} - \boldsymbol{M}_{||}$$

$$= \{30\boldsymbol{i} - 60\boldsymbol{j} + 96\boldsymbol{k}\} - \{-21.0896\boldsymbol{i} - 12.6538\boldsymbol{j} + 5.2724\boldsymbol{k}\}$$

$$= \{51.0896\,\boldsymbol{i} - 47.3462\,\boldsymbol{j} + 90.7276\,\boldsymbol{k}\}\ m{\cdot}N$$

EXAMPLE #7 Continued

Want $\boldsymbol{r} \times \boldsymbol{F_R} = \boldsymbol{M}_{\perp}$

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & y & z \\ -20 & -12 & 5 \end{vmatrix} = \{51.0896\, \boldsymbol{i} - 47.3462\, \boldsymbol{j} + 90.7276\, \boldsymbol{k}\}$$

Or

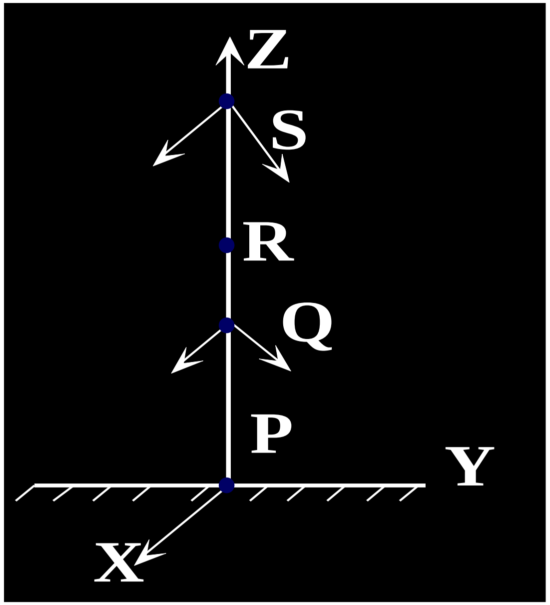
$$(5y+12z)\boldsymbol{i} - \boldsymbol{j}(20z) + \boldsymbol{k}(20y) = 51.0896\, \boldsymbol{i} - 47.3462\, \boldsymbol{j} + 90.7276\, \boldsymbol{k}$$

Get $z = 47.3462/20 = 2.3673$, $y = 90.7276/20 = 4.5364$

Check: 5*4.5364 + 12*(2.3673) = 51.0896 ☑

If y and z were not on the face, solution would be unphysical.

CONCEPT QUIZ



1. The forces on the pole can be reduced to a single force and a single moment at point ____ .

   1) P 2) Q 3) R

   4) S 5) Any of these points.

2. Consider two couples acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

   1) one force and one couple moment.

   2) one force.

   3) one couple moment.

   4) two couple moments.

ATTENTION QUIZ

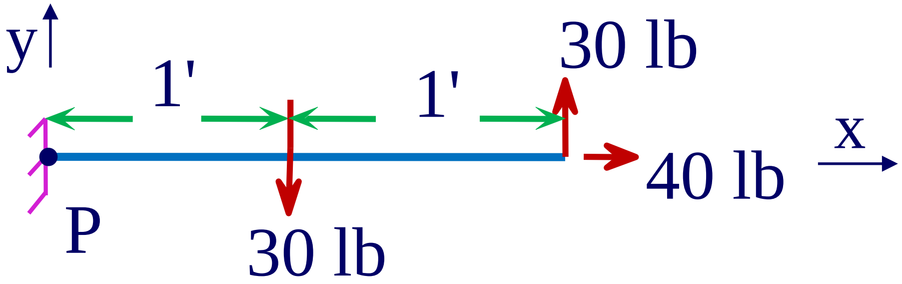
1. For this force system, the equivalent system at P is ____________ .

A) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +60 ft ·lb

B) F_{RP} = 0 lb and M_{RP} = +30 ft · lb

C) F_{RP} = 30 lb (along +y-dir.) and M_{RP} = -30 ft ·lb

D) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +30 ft ·lb



ATTENTION QUIZ

2. Consider three couples acting on a body. Equivalent systems will be ________ at different points on the body.

A) different when located

B) the same even when located

C) zero when located

D) None of the above.