## WEDGES

## Today's Objectives:

Students will be able to:
a) Determine the forces on a wedge.


## APPLICATIONS



Wedges are used to adjust the elevation or provide stability for heavy objects such as this large steel vessel.

How can we determine the force required to pull the wedge out?

When there are no applied forces on the wedge, will it stay in place (i.e., be selflocking) or will it come out on its own? Under what physical conditions will it come out?


## ANALYSIS OF A WEDGE



A wedge is a simple machine in which a small force P is used to lift a large weight W .

To determine the force required to push the wedge in or out, it is necessary to draw FBDs of the wedge and the object on top of it. It is easier to start with a FBD of the wedge since you know the direction of its motion.

Note that:
a) the friction forces are always in the direction opposite to the motion, or impending motion, of the wedge;
b) the friction forces are along the contacting surfaces; and,
c) the normal forces are perpendicular to the contacting surfaces.

## ANALYSIS OF A WEDGE (continued)



Next, a FBD of the object on top of the wedge is drawn. Please note that:
a) at the contacting surfaces between the wedge and the object the forces are equal in magnitude and opposite in direction to those on the wedge; and, b) all other forces acting on the object should be shown.

To determine the unknowns, we must apply EofE, $\sum \mathrm{F}_{\mathrm{x}}=0$ and $\sum \mathrm{F}_{\mathrm{y}}=0$, to the wedge and the object as well as the impending motion frictional equation, $\mathrm{F}=\mu_{\mathrm{S}} \mathrm{N}$.

Now of the two FBDs, which one should we start analyzing first?
We should start analyzing the FBD in which the number of unknowns are less than or equal to the number of equations.

## ANALYSIS OF A WEDGE (continued)



Impending
motion

If the object is to be lowered, then the wedge needs to be pulled out. If the value of the force P needed to remove the wedge is positive, then the wedge is self-locking, i.e., it will not come out on its own.

However, if the value of P is negative, or zero, then the wedge will come out on its own unless a force is applied to keep the wedge in place. This can happen if the coefficient of friction is small or the wedge angle $\theta$ is large.

## Example



The top block weighs 1500 N . The angle $\theta$ is $15^{\circ}$. The coefficient of static friction at all surfaces is $\mu=0.4$. What is the smallest value of $P$ to remove wedge Ignore the weight of the wedge.

## Plan

1. Assume impending slippage at all surfaces.
2. Draw FBDs.
3. Solve EofE for $P$

$$
\begin{align*}
& \uparrow+\mathrm{N}_{\mathrm{F}}-\mathrm{N} \cos \theta-\mathrm{f}_{\mathrm{S}} \sin \theta=0(1) \\
& \rightarrow+\mathrm{f}_{\mathrm{F}}+\mathrm{f}_{\mathrm{s}} \cos \theta-\mathrm{N} \sin \theta-\mathrm{P}=0  \tag{2}\\
& \uparrow+\mathrm{N} \cos \theta+\mathrm{f}_{\mathrm{s}} \sin \theta+\mathrm{f}_{\mathrm{W}}-\mathrm{W}=0 \\
& \rightarrow+\mathrm{N} \sin \theta-\mathrm{f}_{\mathrm{s}} \cos \theta-\mathrm{N}_{\mathrm{W}}=0  \tag{4}\\
& \mathrm{f}_{\mathrm{s}}=\mu \mathrm{N}, \mathrm{f}_{\mathrm{F}}=\mu \mathrm{N}_{\mathrm{F}} \text {, and } \mathrm{f}_{\mathrm{W}}=\mu \mathrm{N}_{\mathrm{W}}
\end{align*}
$$

$$
\begin{aligned}
& N_{F}=N(\cos \theta+\mu \sin \theta) \\
& \mu \mathrm{N}_{\mathrm{F}}-\mathrm{N}(\sin \theta-\mu \cos \theta)=\mathrm{P} \\
& \mathrm{~N}(\cos \theta+\mu \sin \theta)=\mathrm{W}-\mu \mathrm{N}_{\mathrm{W}} \\
& \mathrm{~N}(\sin \theta-\mu \cos \theta)=\mathrm{N}_{\mathrm{W}}
\end{aligned}
$$

$$
\mathrm{N}[\mu(\cos \theta+\mu \sin \theta)-(\sin \theta-\mu \cos \theta)]=\mathrm{P}
$$

$$
\mathrm{N}[(\cos \theta+\mu \sin \theta)+\mu(\sin \theta-\mu \cos \theta)]=\mathrm{W}
$$

$\mathrm{N}=1473 \mathrm{~N}$ and $\mathrm{P}=818 \mathrm{~N}$. (Self locking)

## CONCEPT QUIZ

1. Determine the direction of the friction force on object $B$ at the contact point between A and B .
A) $\rightarrow$
B) $\leftarrow$
C) $\longrightarrow$
D) $<$

2. The boy (hanging) in the picture weighs 100 lb and the woman weighs 150 lb . The coefficient of static friction between her shoes and the ground is $\underline{0.6}$. The boy will ___ ?

A) be lifted up.
B) slide down.
C) not be lifted up. D) not slide down.

## GROUP PROBLEM SOLVING



Given: Blocks A and B weigh 50 lb and 30 lb , respectively.

Find: The smallest weight of cylinder D which will cause the loss of static equilibrium.

## GROUP PROBLEM SOLVING (continued)



## Plan:

1. Consider two cases: a) both blocks slide together, and, b) block B slides over the block A.
2. For each case, draw a FBD of the block(s).
3. For each case, apply the EofE to find the force needed to cause sliding.
4. Choose the smaller P value from the two cases.

## GROUP PROBLEM SOLVING

(continued)
Case a:

$$
\begin{aligned}
\uparrow+\sum \mathrm{F}_{\mathrm{Y}} & =\mathrm{N}-80=0 \\
\mathrm{~N} & =80 \mathrm{lb} \\
\rightarrow+\sum \mathrm{F}_{\mathrm{X}} & =0.4(80)-\mathrm{P}=0 \\
\mathrm{P} & =32 \mathrm{lb}
\end{aligned}
$$




## GROUP PROBLEM SOLVING

Case b:
(continued)

$\begin{aligned} \rightarrow+\sum \mathrm{F}_{\mathrm{x}} & =-\mathrm{P}+0.6(26.2) \cos 20^{\circ}-26.2 \sin 20^{\circ}=0 \\ \mathrm{P} & =5.812 \mathrm{lb}\end{aligned}$
Case b has the lowest P and will occur first. A Block D weighing 5.8 lb will cause the block B to slide over the block A.

## ATTENTION QUIZ

1. When determining the force $P$ needed to lift the block of weight W , it is easier to draw a FBD of ___ first.
A) the wedge
C) the horizontal ground
B) the block
D) the vertical wall


Impending motion

