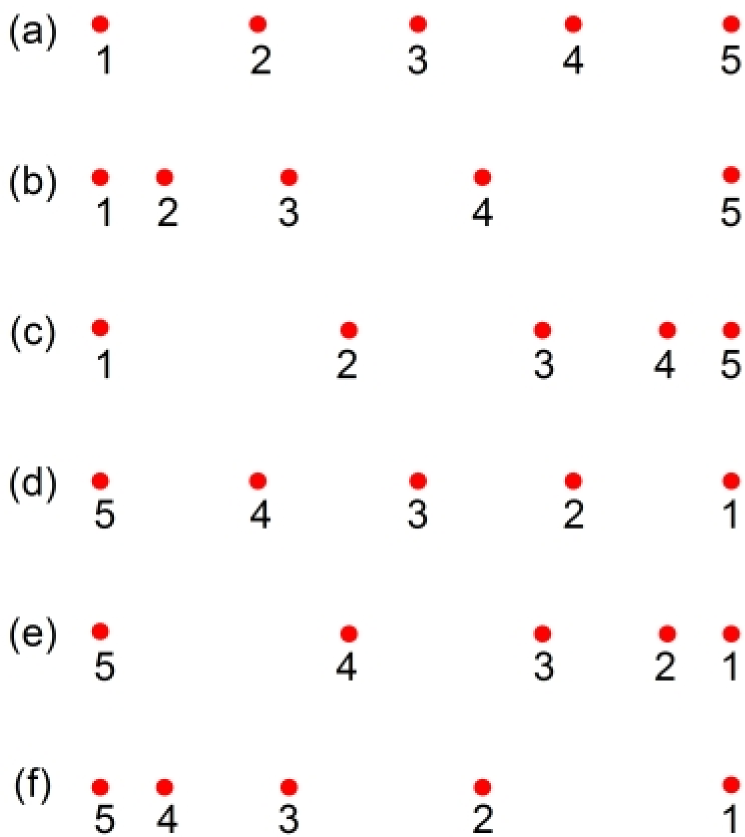
Physics 1100: 1D Kinematics Solutions

1. Neatly sketch the following dot-motion diagrams:
   (a) A particle moving right at constant speed.
   (b) A particle moving right and speeding up.
   (c) A particle moving right and slowing down.
   (d) A particle moving left at constant speed.
   (e) A particle moving left and speeding up.
   (f) A particle moving left and slowing down.



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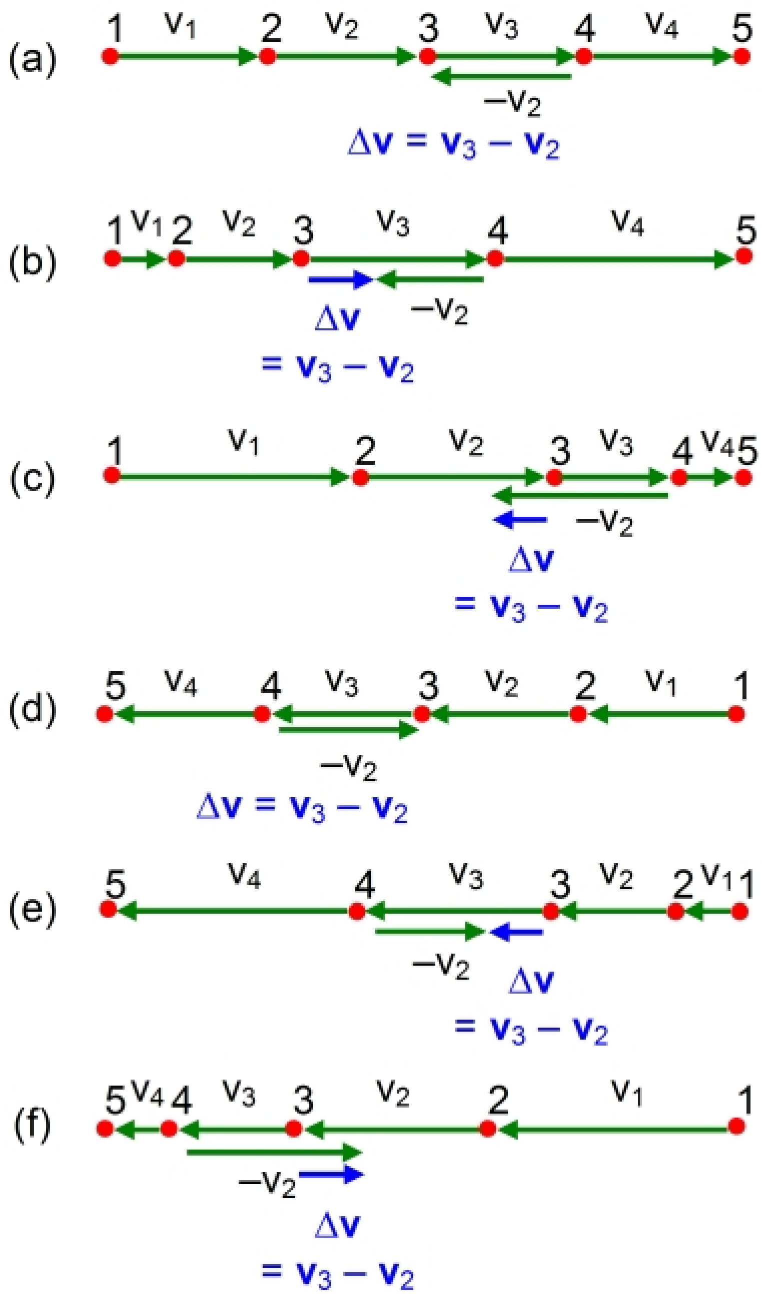
2. Use the definition of acceleration $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ to draw the direction of the acceleration at a midpoint of the sketch from Question 1.



Note that the direction of the acceleration, as indicated by $\Delta \mathbf{v}$, and the direction of motion at a point are both needed to determine if an object is speeding up or slowing down. If **a** and **v** are in the same direction, the object is speeding up. If **a** and **v** are in the opposite direction, the object is slowing down.

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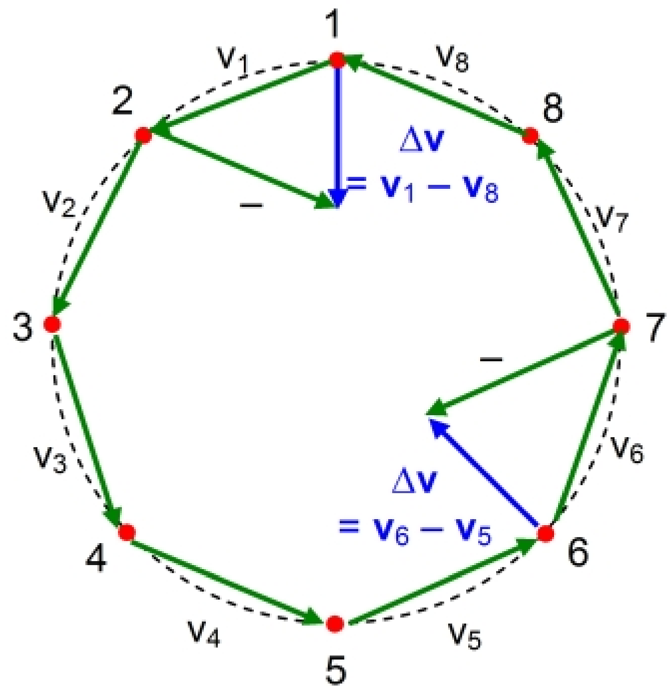
3. Draw a dot-motion diagram for an object moving in a circle at constant speed. Determine the direction of acceleration at several points.



Note that the change in velocity, and thus the acceleration, is the same magnitude at points 1 and 6 and at all other points around the circle. Moreover, the direction of the acceleration is towards the centre. This acceleration is usually called the centripetal acceleration.

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4. Describe in simple terms the motion of the particles in the dot-motion diagrams below.

(a)

(b)

(c) Which point in each dot-motion diagram has the greatest acceleration? Which way does it point?

a. The particle is moving to the left. For points 1 to 5 the speed is constant as the gaps between the points are constant. For points 5 to 8, the gaps are increasing so the particle is speeding up. Finally, between points 8 and 11 the gaps are decreasing and thus the particle is slowing down.

b. The particle is moving to the right. For points 1 to 4 the speed is increasing as the gaps between the points are increasing. For points 4 to 7, the gaps are constant so the particle is travelling at constant speed. Finally, between points 7 and 11 the gaps are constant again indicating constant albeit slower speed. Note the abrupt change at point 7.

c. The biggest change in velocity in (a) is at point 8. The particle is moving left but slowing down, so the acceleration must be opposite the velocity or to the right. The biggest change in velocity in (b) is at point 7. The particle is moving right but slows down between points 6 and 8, so the acceleration must be opposite the velocity or to the left.

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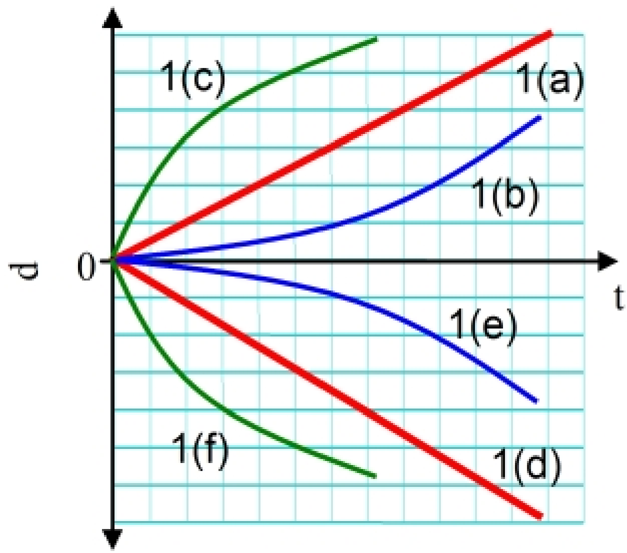
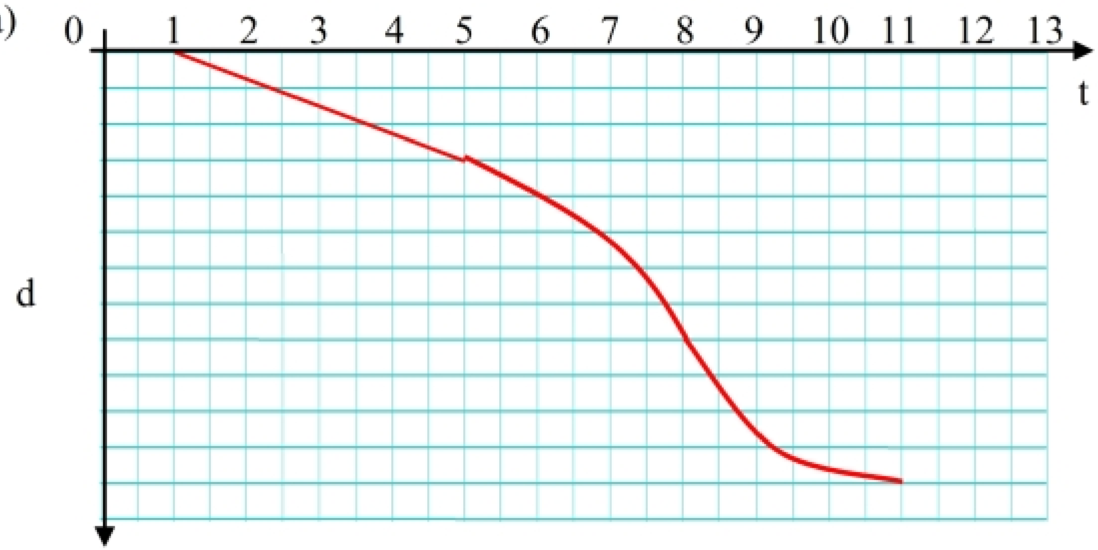
5. (a) Sketch x-t graphs for Question 1.



(b) Sketch x-t graphs for Question 4(a) and 4(b).

4(a)



4(b)

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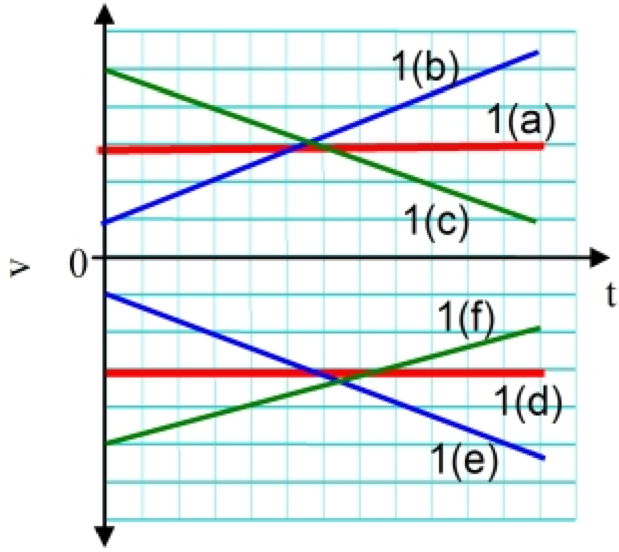
6. (a) Sketch v-t graphs for Question 1



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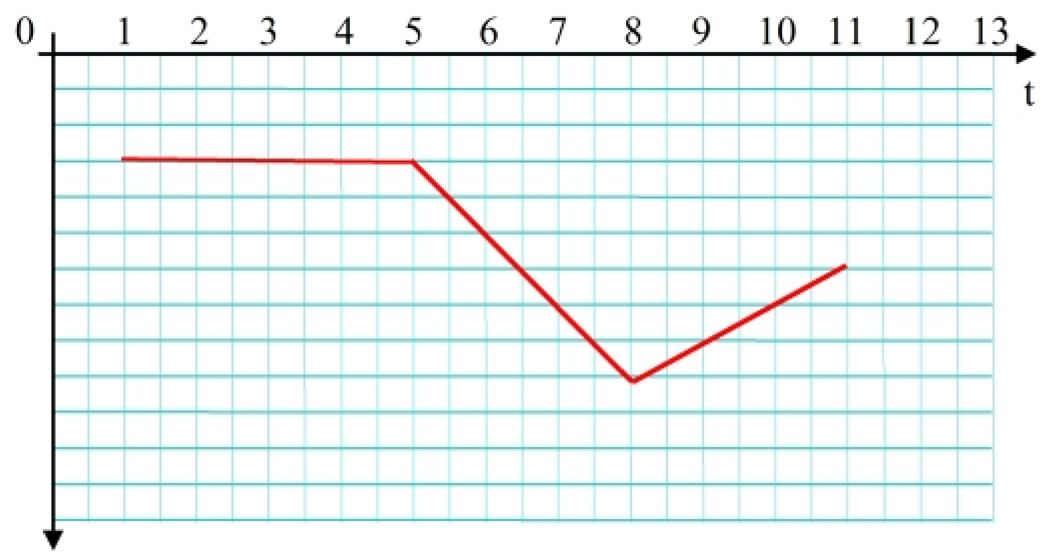
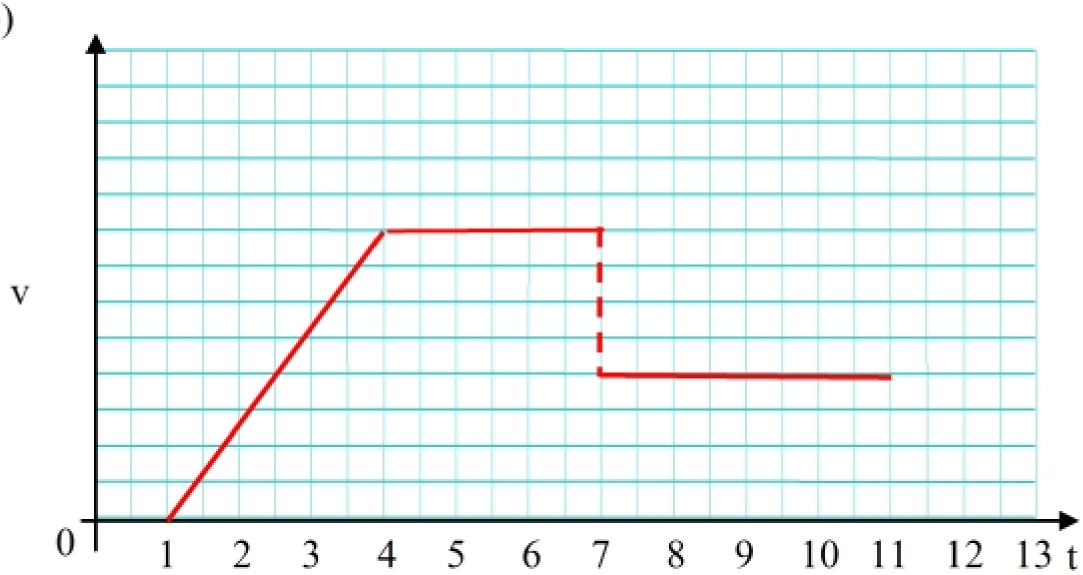
(b) Sketch v-t graphs for Question 4(a) and 4(b).

4(a)



4(b)



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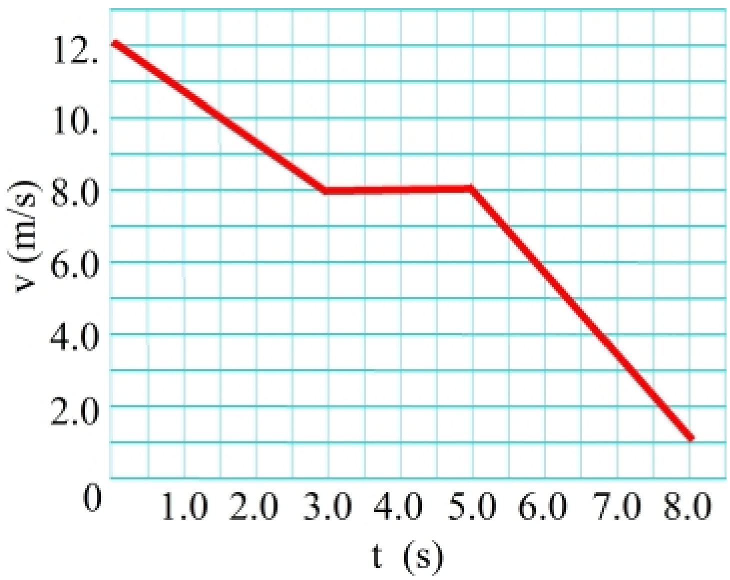
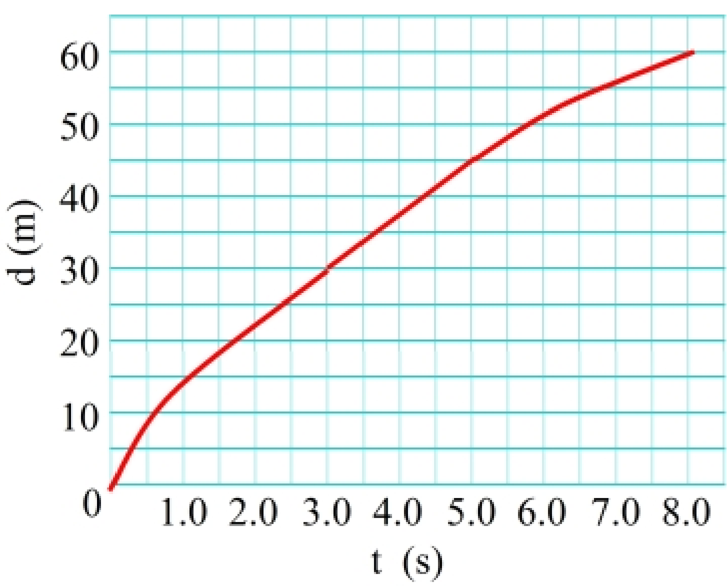
7. Draw a dot-motion diagram and an x-t graph from the following v-t graph.



First note that all the velocities are positive so the object is always moving to the right. Next, note that there are three separate sections to the motion. First the object is moving right but slowing. Second it travels at constant speed. Third is slows again but faster than in the first segment. A possible dot-motion diagram would look like:

To accurately sketch the x-t graph note that the object travels 30 m in the first section, 16 m in the second, and 14.25 m in the third. We can either look at the area under the curve or use average velocity for each section.



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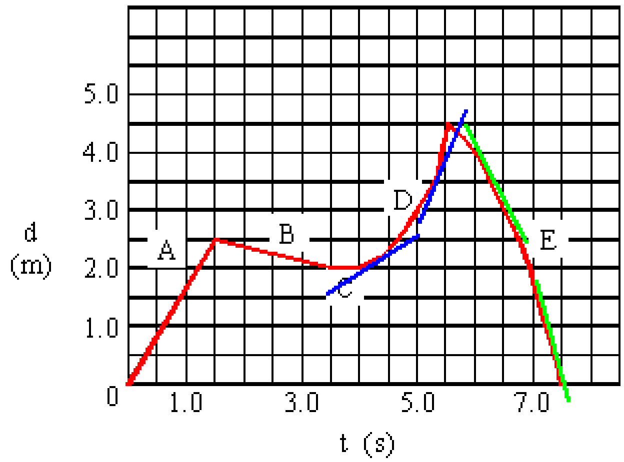
8. From the position versus time graph given below, determine the average velocity in segments A to E. Is the acceleration in each segment zero, negative, or positive? What is the average velocity over the entire time interval?



We calculate average velocity from $v_{average} = (x_f - x_0)/(t_f - t_0)$, and read the values of x and t off the graph.

Determining the sign of the acceleration is more complicated. Since the acceleration is defined $a = \Delta v/t$, the sign of a is the same as the sign of v. For segment A, B, and C velocity is constant so $v = 0$ and thus $a = 0$. For segment D, the slope of the tangent lines is increasing, thus $v > 0$ and $a > 0$. For segment E, the slope of the tangent lines is getting more negative, thus $v < 0$ and $a < 0$.

Segment	$v_{average}$ *(m/s)*	***a***	
A	= (2.5 – 0)/(1.5 – 0s) = 1.67	0	Straight line d vs t, means constant velocity or zero acceleration
B	= (2.0 – 2.5)/(3.5 – 1.5) = –0.25	0	Straight line d vs t, means constant velocity or zero acceleration
C	= (2.0 – 2.0)/(4.0 – 3.5) = 0	0	Straight line d vs t, means constant velocity or zero acceleration
D	= (4.5 – 2.0)/(5.5 – 4.0) = 1.67	> 0	Curved upward d vs t line means increasing v
E	= (0.0 – 4.5)/(7.5 – 5.5) = –2.25	< 0	Curved downward d vs t line means v is becoming more negative.

The displacement over the entire time interval is zero as the object returns to d = 0, so the average velocity is zero as well.

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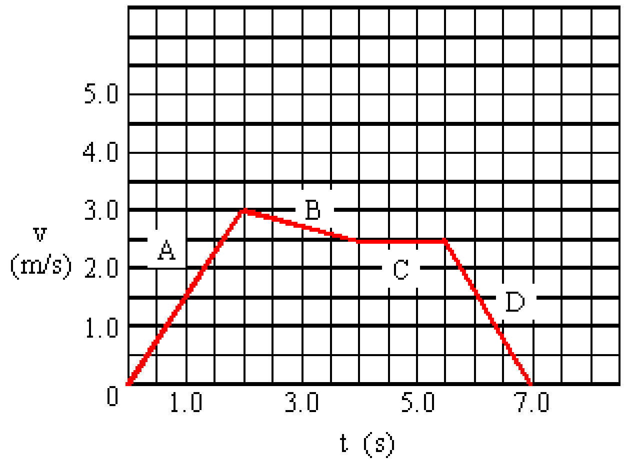
9. From the velocity versus time graph below, determine the acceleration in all segments. What is the average velocity in each segment? What is the displacement in each segment? What is the average velocity over the entire time interval?



We may find the average velocity from $v_{average} = ½(v_f + v_0)$. The acceleration is defined by $a = (v_f - v_0)/(t_f - t_0)$. We may find the displacement from the area under each segment or use $\Delta x = v_{average} t$.

Segment	$v_{average}$ *(m/s)*	***a*** *(m/s²)*	**Δx** *(m)*
A	= (3 + 0)/2 = 1.5	= (3 – 0) / (2 – 0) = 1.5	= 1.5 / 2 = 3.0
B	= (2.5 + 3)/2 = 2.75	= (2.5 – 3) / (4 – 2) = –0.25	= 2.75 × 2 = 5.5
C	= 2.5	= 0	= 2.5 × 1.5 = 3.75
D	= (0 + 2.5)/2 = 1.25	= (0–2.5) / (7–5.5) = –1.67	= 1.25 × 1.5 = 1.88

Since the velocity is not a straight line for the entire time interval, we cannot use $v_{average} = ½(v_f + v_0)$ to find the average velocity. Instead we must use the definition $v_{average} = \Delta x/\Delta t$. For the entire time interval, $\Delta x_{total} = \Delta x_A + \Delta x_B + \Delta x_C + \Delta x_D = 3.0\ m + 5.5 + 3.75\ m + 1.88\ m = 14.13\ m$. Thus $v_{average} = (14.13\ m)/(7.0\ s) = 2.02\ m/s$.

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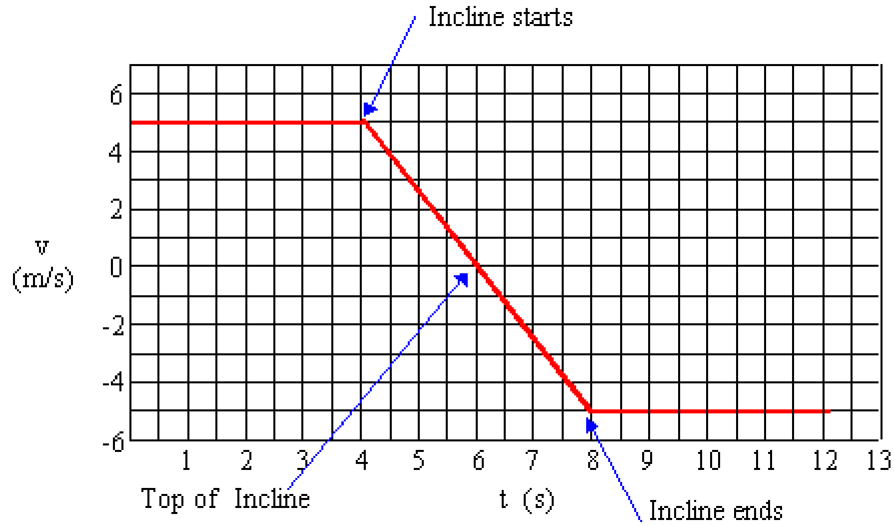
10. The velocity versus time sketch shown below is for a ball rolling across a horizontal floor, rolling up then down an incline, and finally rolling back across the floor.



(a) At what time does the ball encounter the incline?

When the ball encounters the incline, it slows down. Examining the graph this occurs at t = 4.0 s.

(b) When is it at the top of the incline? How do you know this?

At the top of the incline, the ball is turning around. A turning object has a velocity v = 0. Examining the graph, this occurs at t = 6 s.

(c) When does it leave the incline?

When the ball leaves the incline, it will have a constant velocity again. This starts at t = 8 s on the graph.

(d) What was the acceleration of the ball up the incline?

Acceleration is given by $a = (v_f - v_0)/(t_f - t_0) = (0 - 5)/(6 - 4) = -2.5\ m/s^2$.

(e) What was the acceleration of the ball down the incline?

Acceleration is given by $a = (v_f - v_0)/(t_f - t_0) = (-5 - 0)/(8 - 6) = -2.5\ m/s^2$. Since we a dealing with a straight line segment, the value is the same as part (f)

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11. A skytrain starts from rest at a station and accelerates at a rate of 1.60 m/s^2 for 8.00 s. It then runs at a constant velocity for 12.0 s, and slows down at 2.50 m/s^2 until it stops at the next station. Sketch the d versus t graph. Find the time for the last segment. Find the average velocity in each segment. Find the total distance between these two stations. Assume the track is straight between the two stations.

An accurate sketch requires that we know some distances and times.

A $\quad v_0 = 0$ m/s, a = 1.60 m/s^2, t = 8.0 s

$v_f = v_0 + at = 12.8$ m/s

$v_{ave} = ½(v_0 + v_f) = 6.4$ m/s

$\Delta x_A = v_{ave}t = 51.2$ m

B $v_0 = 12.8$ m/s, $a = 0$, $t = 12.0$ s

$v_f = v_0 = 12.8$ m/s

$v_{ave} = ½(v_0 + v_f) = 12.8$ m/s

$\Delta x_A = v_{ave}t = 153.6$ m

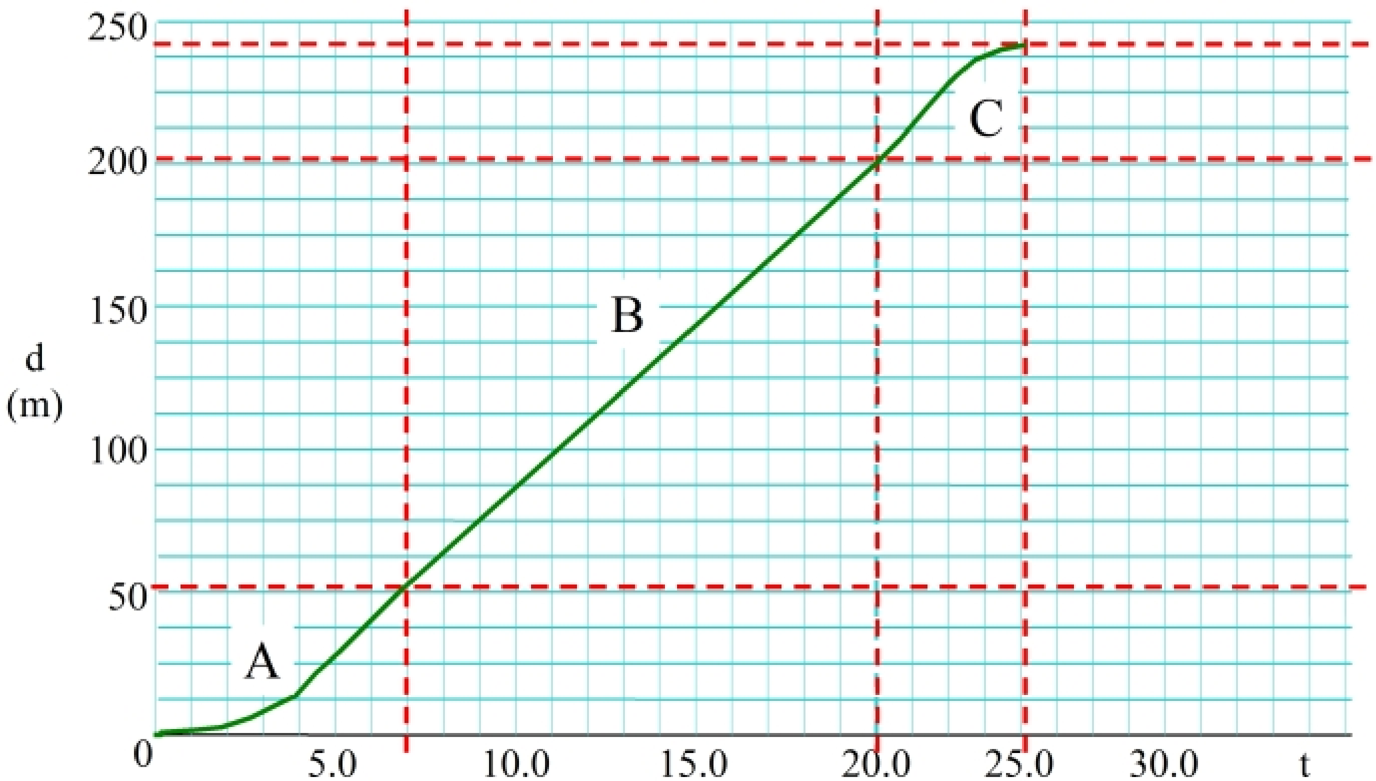
C $v_0 = 12.8$ m/s, $v_f = 0$, $a = -2.50$ m/s^2

$t = (v_f - v_0)/a = 5.12$ s

$v_{ave} = ½(v_0 + v_f) = 6.4$ m/s

$x_A = v_{ave}t = 32.8$ m

In segment A, there is an acceleration so the line curves upwards as velocity increases. In B the velocity is constant, so the line is straight. In C the train decelerates, so the line flattens out as velocity decreases.



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12. The record for the 100 m dash is 9.90 s. Calculate the average speed over this distance.

Average speed is defined as distance travelled divided by travel time,

$$s = d/t = 100\ m/ 9.90\ s = 10.1\ m/s\ .$$

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13. The moon moves around the earth in a circle of radius 3.84×10^8 m. The time for one complete trip of the moon around the earth is 27.3 days. Calculate the moon's average speed in m/s and km/h. Recall that the circumference of a circle is C = 2πr; a fact you should remember for this course.

Average speed is defined as distance travelled divided by travel time,

$$s = d/t = 2\pi r / t = 2(3.84 \times 10^8 \ m) / (27.3 \ d \ 24h/d \ 3600 \ s/h) = 1.02 \times 10^3 \ m/s .$$

Converting to km/h,

$$1.02 \times 10^3 \ m/s = 1.02 \times 10^3 \ m/s \ (1000 \ m/km)(3600 \ s/h) = 3.68 \times 10^3 \ km/h$$

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14. A jogger runs four times around an oval track of circumference 400 m and finishes where she started. If it takes her 10 minutes to complete her run, what was her average speed? Her average velocity? Why are they different?

Average speed is defined by distance traveled over time, $s = d/t$. For this problem

$$s = 4 \times 400 \ m / (10 \times 60 \ s) = 2.67 \ m/s.$$

Average velocity is defined by displacement over time, $v = \Delta x/t$. For this problem

$$v = 0 \ m / (10 \times 60 \ s) = 0 \ m/s.$$

The difference reflects the fact that displacement ignores the details of the motion and is concerned only with where the final position is relative ti the initial position.

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15. A certain brand of car can accelerate from 0 to 60 km/h in 4.20 s. What is it's acceleration? What is it's average velocity? How far did it travel?

Acceleration is defined as the change in velocity with respect to time. Before we calculate this, we must convert the final velocity to **SI** units,

$$60 \text{ km/h} = 60 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 16.67 \text{ m/s}.$$

Thus the acceleration is

$$a = (v_f - v_0)/t = (16.67 \ m/s - 0)/(4.20 \ s) = 3.97 \ m/s^2.$$

Since we have the initial and final velocities, we can calculate the average velocity from

$$v_{\text{average}} = ½(v_f + v_0) = ½(16.67 \ m/s + 0) = 8.33 \ m/s .$$

Since the motion is in a straight line, the distance travelled is the same as the displacement. For the given data and examining our kinematic equations, the displacement can be calculated several ways,

$$\Delta x = v_{average}t = (8.333\ m/s)(4.20\ s) = 35.0\ m,$$

or, equivalently

$$\Delta x = v_0 t + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}(3.97\ m/s^2)(4.20\ s)^2 = 35.0\ m.$$

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16. A bullet in a rifle accelerates uniformly from rest at a = 70000 m/s^2 barrel. If the muzzle velocity of the bullet is 500 m/s, how long is the rifle barrel? How long did it take for the bullet to travel the length of the barrel? What is the average velocity of the bullet?

To solve this problem, we list the list the given information and what we are looking for:

v_0 = 0.0 m/s	(since the bullet is initially at rest)
v_f = 500 m/s	(velocity of the bullet as it leaves the barrel)
a = 70,000 m/s^2	
Δx = ?	(the length of the barrel)
t = ?	(the time it takes to travel the barrel)
$v_{average}$ = ?	

To find the length of the barrel, we find the kinematics equation that contains x and the given quantities. Examining our equations we see that we can use $2a\Delta x = v_f^2 - v_0^2$. Rearranging this equation to find a yields

$$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{(500 m/s)^2 - (0.0 m/s)^2}{2 \times 70000 m/s^2} = 1.79 m.$$

To find the time it takes for the bullet to travel the length of barrel, we find the kinematics equation that contains t and the given quantities. Examining our equations we see that we can use $v_f = v_0 + at$. Rearranging this equation to find t yields

$$t = \frac{v_f - v_0}{a} = \frac{(500 m/s) - (0.0 m/s)}{70000 m/s^2} = 7.1 \times 10^{-3} s.$$

The average velocity is defined $v_{average} = \frac{v_0 + v_f}{2} = \frac{0 m/s + 500 m/s}{2} = 250 m/s.$

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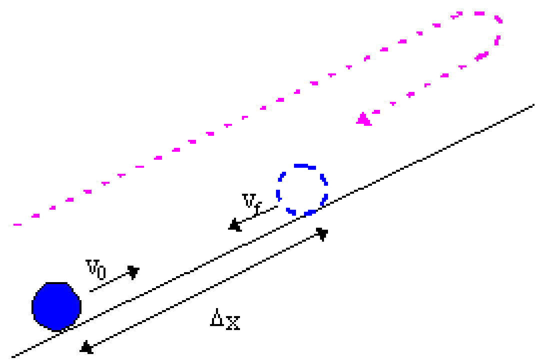
17. Initially, a ball has a speed of 5.0 m/s as it rolls up an incline. Some time later, at a distance of 5.5 m up the incline, the ball has a speed of 1.5 m/s DOWN the incline.
(a)What is the acceleration? What is the average velocity? How much time did this take?

(b)At some point the velocity of the ball had to have been zero. Where and when did this occur?

A well–labeled sketch usually helps make the problem clearer.



(a) Next, we list the list the given information and what we are looking for:

$v_0 = +5.0$ m/s

$v_f = -1.5$ m/s

$\Delta x = +5.5$ m

a = ?

$v_{average} = ?$

t = ?

Note that I have taken the direction up the incline as positive and that the signs are explicitly stated. It is a very common source of error to leave out or to not consider the signs of directions of all vector quantities.

To find the acceleration, we find the kinematics equation that contains a and the given quantities. Examining our equations we see that we can use $2a\Delta x = v_f^2 - v_0^2$. Rearranging this equation to find a yields

$$a = \frac{v_f^2 - v_0^2}{2\Delta x} = \frac{(-1.5m/s)^2 - (5.0m/s)^2}{2 \times 5.5m} = -2.068m/s^2.$$

Notice that the acceleration is negative. This means that the acceleration points down the incline. It means that an object traveling up an incline will slow, turn around, and roll down the incline.

The average velocity is defined $v_{average} = \frac{v_0 + v_f}{2} = 1.75m/s$.

To find the time, we find the kinematics equation that contains a and the given quantities. Examining our equations we see that we can use $\Delta x = \frac{v_0 + v_f}{2}t$. Rearranging this equation to find t yields $t = \frac{2\Delta x}{v_0 + v_f} = 3.143s$.

(b) When an object moving in 1D turns around we know that the object is instantaneously at rest and that its velocity at that point is $v_3 = 0$. The information that we know is thus:

$v_0 = +5.0$ m/s

$v_3 = 0$ m/s   This is our new final velocity

$a = -2.068$ m/s^2 From part (a)

$\Delta x = ?$

$v_{average} = ?$

$t = ?$

Notice that the acceleration is a constant of the motion; it has the same value in both parts of the problem.

To find the displacement from the initial position where the ball turns around, we find the kinematics equation that contains x and the given quantities. Examining our equations we see that we can use $2a\Delta x = v_f^2 - v_0^2$. Rearranging this equation to find x yields

$$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{(0\,m/s)^2 - (5.0\,m/s)^2}{2 \times (-2.068\,m/s^2)} = 6.04m.$$

Notice that this value is bigger than the original 5.5 m and is consistent with the sketch, i.e. the ball was farther up the incline when it turned around.

To find the time it takes for the ball to reach the point where it turns around, we find the kinematics equation that contains t and the given quantities. Examining our equations we see that we can use $v_f = v_0 + at$. Rearranging this equation to find t yields

$$t = \frac{v_f - v_0}{a} = \frac{(0\,m/s) - (5.0\,m/s)}{-2.068\,m/s^2} = 2.42s.$$

Notice that this value is smaller than the time in part (a) and is consistent with the sketch, i.e. the ball hasn't come back down the incline yet.

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18. A tortoise and a hare have a 25.0 m race. The tortoise moves at a slow but constant 0.101 m/s while the hare accelerates from rest at 0.500 m/s^2. The hare magnanimously gives the tortoise a 4.00–minute headstart. Who wins? Explain with calculations.

(Of course, in the modern telling of the story, the hare would be proclaimed the winner after a random drug test showed that the tortoise had been taking steroids!)

As with any kinematics problem, we list the list the given information.

tortoise	***hare***
$\Delta x = 25.0$ m	$\Delta x = 25.0$ m
$v_{0T} = 0.101$ m/s	$v_{0H} = 0$
$v_{fT} = v_{0T}$ (constant velocity)	$v_f = ?$
$a_T = 0$ (constant velocity)	$a_H = 0.5$ m/s^2
$t_T = ?$	$t_H = ?$

Examining each set of data, we can see that we have enough data to use our kinematics equations to calculate t. Whoever wins the race will have the smaller time.

Calculating t_T, we use the kinematics equation $\Delta x = v_{0T}t_T + \frac{1}{2}a_T(t_T)^2$. Since $a_T = 0$, rearranging the equation yields

$$t_T = \Delta x / v_{0T} = 25.0\ m / 0.101\ m/s = 247.5\ s.$$

Similarly, we use the kinematics equation $\Delta x = v_{0H}t_H + \frac{1}{2}a_H(t_H)^2$ to calculate t_H. Since $v_{0H} = 0$, we have

$$t_H = [2\Delta x/a_H]^{1/2} = [2(25.0\ m)/(0.5\ m/s^2)]^{1/2} = 10.0\ s.$$

We have to be careful, t_H is the time it takes the hare to run the 25 m but he gave the tortoise a 4–minute, or a 240–second, headstart. The hare's race time is thus 250 s, and the tortoise wins the race.

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19. Fifteen weeks later, having taken Physics 1100, a wiser hare asks for a rematch. This time the hare has done some calculations. What would be the maximum headstart that the hare can give the tortoise and still win the race?

Examining our results, we see that the tortoise won by 2.5 seconds, if the hare gave the tortoise of 3 minutes and 57 seconds or less, the hare should win.

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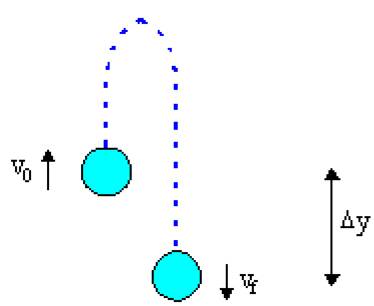
20. A ball is thrown up into the air. At one point it has an upward velocity of 10.0 m/s. Some time later, it has a downward velocity of 12.0 m/s. How much time passed between these two events? What is the displacement between the ball at those two heights? What total distance did the ball travel between those times?

A well–labeled sketch usually helps make the problem clearer.



Next, we list the list the given information and the unknowns:

$v_0 = +10.0$ m/s

$v_f = –12.0$ m/s

$\Delta y = ?$

$a = –g = –9.81$ m/s^2

$t = ?$

Note that I have taken the direction up as positive and that the signs are explicitly stated. It is a very common source of error to leave out or to not consider the signs of directions of all vector quantities.

We can use the equation $v_f = v_0 + at$ to find t. Rearranging yields

$$t = (v_f - v_0) / a = (-12 - 10) / -9.81 = 2.24\ s\ .$$

To find the displacement, we can use the formula $2a\Delta y = (v_f)^2 - (v_0)^2$. Rearranging yields

$$\Delta y = [(v_f)^2 - (v_0)^2]/2a = [(-12)^2 - (10)^2] / (2 \times -9.81) = -2.24\ m\ .$$

So the ball is 2.24 m lower at the end of the 2.24 s.

We are asked to calculate the total distance travelled by the ball, however distance is not a kinematic quantity that we can calculate. Fortunately, we can calculate the displacement up to where the ball turned and then the displacement down. We recall that v = 0 at that point. So we have two calculation to do and we first list the information

up	***down***
$v_0 = +10.0$ m/s	$v_0 = 0$
$v_f = 0$	$v_f = -12.0$ m/s
$\Delta y_1 = ?$	$\Delta y_2 = ?$
$a = -g = -9.81$ m/s^2	$a = -g = -9.81$ m/s^2
$t_1 = ?$	$t_2 = ?$

To find the displacement for each part, we can use the formula $2a\Delta y = (v_f)^2 - (v_0)^2$. Rearranging yields

$$\Delta y_1 = [(v_f)^2 - (v_0)^2]/2a = [(0)^2 - (10)^2] / (2 \times -9.81) = 5.097\ m\ \text{, and}$$

$$\Delta y_1 = [(v_f)^2 - (v_0)^2]/2a = [(-12)^2 - (0)^2] / (2 \times -9.81) = -7.339\ m$$

We have to be careful about signs since distance is a scalar quantity while displacement is a vector,

$$d = |\Delta y_1| + |\Delta y_2| = 5.097\ m + 7.339\ m = 12.4\ m\ .$$

The ball travelled a total distance of 12.4 m.

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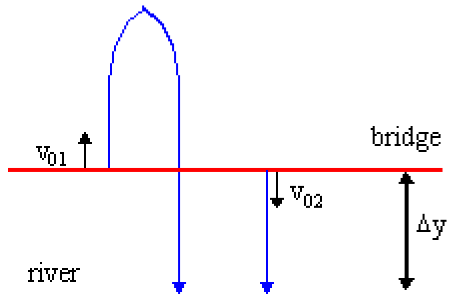
21. A woman standing on the edge of a bridge fires a pistol once straight up into the air and one straight down. Which bullet will hit the river below with the greatest velocity? Ignore the effects of air resistance and assume the opening of the barrel is at the same level in each case. If the gun is held 120 m above the river and has a muzzle velocity of 325 m/s, find the following in each case: the time in air and the average velocity. In the first case how far did the bullet rise? What total distance did it travel?

A well–labeled sketch usually helps make the problem clearer.



Next, we list the list the given information and the unknowns for each case:

up	***down***
$v_{01} = +325.0$ m/s	$v_{02} = -325$ m/s
$v_{f1} = ?$	$v_{f2} = ?$ <p><sup><sub><sub><sub
$\Delta y_1 = -120$ m	$\Delta y_2 = -120$ m
$a = -g = -9.81$ m/s^2	$a = -g = -9.81$ m/s^2
$t_1 = ?$	$t_2 = ?$

Note that I have taken the direction up as positive and that the signs are explicitly stated. It is a very common source of error to leave out or to not consider the signs of directions of all vector quantities. Note that the displacement of each bullet is exactly the same.

We can use the equation $\Delta y = v_0 t + ½at^2$ to find t. We have a quadratic in t. When we substitute in the numbers and rearrange into standard quadratic form, we have

up: $-4.905t^2 + 325t + 120 = 0$ (1)

down: $-4.905t^2 - 325t + 120 = 0$ (2)

The solutions to equation (1) are $t_1 = -0.367$ s and $t_1 = 66.6$ s. We are looking for the forward in time solution so $t_1 = 66.6$ s is the total time the first bullet is in the air – neglecting the effects of air resistance.

The solutions to equation (2) are $t_2 = -66.6$ s and $t_1 = 0.367$ s. We are looking for the forward in time solution so $t_2 = 0.367$ s is the total time the second bullet is in the air.

To calculate the average velocity we need to know the final velocity of each bullet. We can use the equation $2a\Delta y = (v_f)^2 - (v_0)^2$ for this.

up: $v_{f1} = [2a\Delta y_1 + (v_{01})^2]^{½} = [2(-9.81)(-120) + (+325)^2]^{½} = <\pm 328.6\ m/s$,

down: $v_{f2} = [2a\Delta y_2 + (v_{02})^2]^{½} = [2(-9.81)(-120) + (-325)^2]^{½} = <\pm 328.6\ m/s$.

The proper choice of sign in both case is minus because the bullets are moving downwards just before impact, so $v_{f1} = v_{f2} = -328.6$ m/s .

The average velocity for the each bullet is thus

up: $v_{1\ average} = \frac{1}{2}(v_{f1} + v_{01}) = \frac{1}{2}(-328.6 + 325) = -1.80\ m/s$,

down: $v_{2\ average} = \frac{1}{2}(v_{f2} + v_{02}) = \frac{1}{2}(-328.6 + -325) = 326.8\ m/s$.

This results are different because the first bullet, which is shot upwards, takes much longer to complete the same displacement than does the second bullet.

We could have also calculated the average velocity from

up: $v_{1\ average} = \Delta y/t_1 = -120\ m\ /\ 66.6\ s = -1.80\ m/s$,

down: $v_{2\ average} = \Delta y/t_2 = -120\ m\ /\ 0.367\ s = 326.8\ m/s$.

Both methods give exactly the same results!

To find how far the first bullet rose, we know that at the top $v_{top} = 0$ as it is turning around. We can use the formula $2a\Delta y = (v_f)^2 - (v_0)^2$. Rearranging yields

$$\Delta y_{up} = [(v_f)^2 - (v_0)^2]/2a = [(0)^2 - (+325)^2]\ /\ (2 \times -9.81) = 5384\ m\ .$$

So the first bullet rose 5384 m

The bullet will have to return down this 5384 m, to the total distance travelled by the first bullet is

$$d = 5384\ m + 5384\ m + 120\ m = 10887\ m = 1.09 \times 10^3\ m.$$

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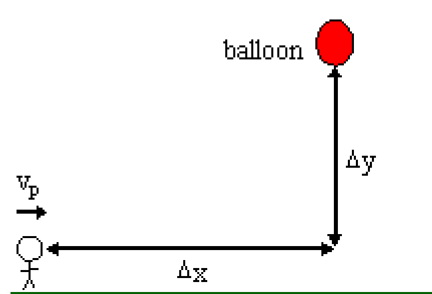
22. A person in a building is 12.0 m above a person walking below. She plans to drop a water balloon on him. He is currently walking directly towards a point under her at 2.75 m/s. How far away should he be when she drops the balloon if she is to hit him?

A well–labeled sketch usually helps make the problem clearer.



Next, we list the list the given information and the unknowns for the person and the water balloon:

person	***balloon***
$v_0 = v_p = 2.75$ m/s	$v_0 = 0$ (assuming it starts from rest)
$v_f = v_p$	$v_f = ?$
$\Delta x = ?$	$\Delta y = -12$ m

$a_x = 0$ $a_y = -g = -9.81\ m/s^2$

$t = ?$

Note that I have taken the directions up and to the right as positive and that the signs are explicitly stated. It is a very common source of error to leave out or to not consider the signs of directions of all vector quantities.

The common element to the balloon and the person is the time, t, each is in motion. Examining the balloon column we see we can use the equation $\Delta y = v_0 t + ½at^2$ to find t. When we substitute in the numbers and rearrange to find t, we have

$$t = [-2\Delta y/a]^{½} = [-2(-12)/(-9.81)]^{½} = 1.564\ s.$$

We choose the positive, forward in time, solution so the balloon take 1.564 s to fall. With this information, we use the equation $\Delta x = v_0 t + ½at^2$ to find out how far away the person has to be,

$$\Delta x = v_0 t + ½at^2 = 2.75\ m/s \times 1.564\ s = 4.30\ m\ .$$

The person should be 4.30 m away when the ball is dropped.

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Chase Problems

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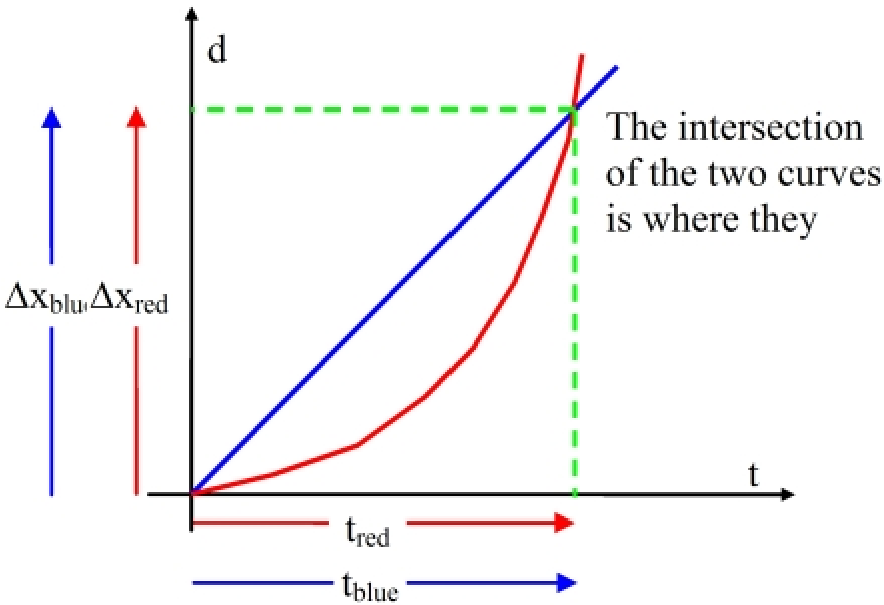
23. A red car is stopped at a red light. As the light turns green, it accelerates forward at 2.00 m/s^2. At the exact same instant, a blue car passes by traveling at 62.0 km/h. When and how far down the road will the cars again meet? Sketch the d versus t motion for each car on the same graph. What was the average velocity of the red car for this time interval? For the blue car? Compare the two and explain the result?

To solve this problem, we list the list the given information

Red Car	**Blue Car**
$v_{0\ red} = 0.0\ m/s$	$v_{0\ blue} = 62.0\ km/h = 17.222\ m/s$
$a_{red} = 2.00\ m/s^2$	$a_{blue} = 0\ m/s^2$ (constant velocity)
$\Delta x_{red} = ?$	$\Delta x_{blue} = ?$
$t_{red} = ?$	$t_{blue} = ?$

This is an example of a two–body constrained kinematics problem. Even if a sketch was not explicitly required, we would need one anyway to get the constraints. For the sketch, recall that on a d versus t curve an object moving forward with a uniform acceleration should be represented by a line curving upwards while an object with constant forward velocity is represented by a straight line with a positive slope.



Looking at the sketch, we see that our constraints are:

$$\Delta x_{red} = \Delta x_{blue} \quad (1), \text{ and}$$
$$t_{red} = t_{blue} \quad (2).$$

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and a, and the unknown quantities, Δx and t. Examining our equations we see that we can use $\Delta x = v_0 t + ½at^2$. We substitute this equation into both sides of equation (1). This yields,

$$v_{0\,red}t_{red} + ½a_{red}(t_{red})^2 = v_{0\,blue}t_{blue} + ½a_{blue}(t_{blue})^2.$$

We then use equation (2) to replace t_{red} and t_{blue} by t,

vΔ;0 red $t + ½a_{red}t^2 = v_{0\,blue}t + ½a_{blue}t^2$.

Plugging in the values of the given quantities yields,

$$½(2.00)t^2 = 17.2t.$$

The solution of this equaΔtion is t = 17.222 seconds. This is the time that elapses before the two cars meet again.

With a value for t, we can find how far down the road the red car has traveled;

$$\Delta x_{red} = v_{0\,red}t + ½a_{red}t^2 = ½(2.00)(17.2)^2 = 297 \text{ m}.$$

As a check, we can find how far down the road the blue car has traveled;

$$\Delta x_{blue} = v_{0\,blue}t + ½a_{blue}t^2 = (17.2)(17.2) = 297 \text{ m}.$$

So the cars meet 297 m down the road.

According to our definition of average velocity, $v_{average\ red} = \Delta x_{red}/t$ = (297 m)/(17.2 s) = 17.2 m/s. Since the blue car maintains a constant velocity, $v_{average\ blue} = v_{0\ blue}$ = 17.2 m/s. The two quantities are the same since the two cars have traveled the same distance in the same amount of time.

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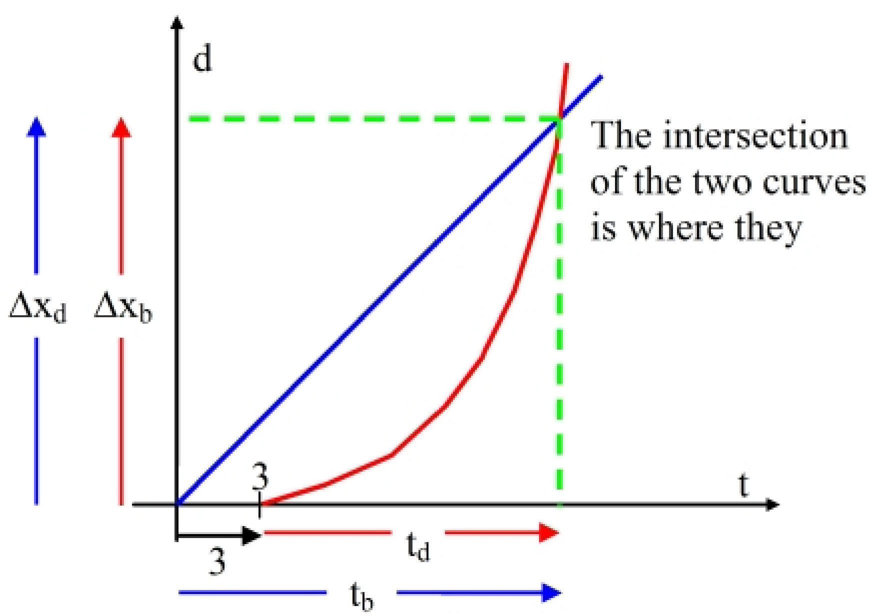
24. A bicyclist travelling at 8.0 m/s passes a napping dog. Startled, the dog barks for three seconds and then gives chase accelerating at 2.2 m/s^2. How far from her initial position does the dog catch up with the bicyclist? How long did this take? What is the dog's velocity when she catches the bicyclist? Her average velocity for the entire chase? Compare this to the bicyclist's average velocity.

To solve this problem, we list the given information

bicyclist		***dog***
Δx_b = ?		Δx_d = ?
v_{0b} = 8.0 m/s		v_{0d} = 0 m/s
v_{fb} = 8.0 m/s	(assuming constant velocity)	v_{fd} = ? m/s
a_b = 0	(assuming constant velocity)	a_d = 2.2 m/s^2
t_b = ?		t_d = ?

This is an example of a two–body constrained kinematics problem. Even if a sketch was not explicitly required, we would need one anyway to get the constraints. For the sketch, recall that on a d versus t curve an object moving forward with a uniform acceleration should be represented by a line curving upwards. An object with constant forward velocity is represented by a straight line with a positive slope.



Looking at the sketch, we see that our constraints are:

$$\Delta x_b = \Delta x_d \qquad (1),$$

and

$$t_b = t_d + 3 \qquad (2).$$

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and a, and the unknown quantities, Δ x and t. Examining our equations we see that we can use $\Delta x = v_0 t + ½at^2$. We substitute this equation into both sides of equation (1). This yields,

$$v_{0b}t_b + ½a_b(t_b)^2 = v_{0d}t_d + ½a_d(t_d)^2.$$

We then use equation (2) to replace t_d by t and t_b by t + 3,

$$v_{0b}(t+3) + ½a_b(t+3)^2 = v_{0d}t + ½a_d t^2.$$

Plugging in the values of the given quantities yields,

$$8(t+3) = ½(2.20)t^2 .$$

This gives a quadratic equation

$$1.10t^2 – 8t – 24 = 0 .$$

The solutions of this equation are t = 9.556 s and t = –2.283 seconds. The solution that we are looking for is the positive, future, value. So the boy has been travelling for

$$t_b = 9.556 \text{ s} + 3 \text{ s} = 12.556 \text{ s}$$

while the dog has run for

$$t_d = 9.556 \text{ s}$$

With a value for t, we can find how far down the road they have gone;

$$\Delta x_b = v_{0b}t_b + ½a_b(t_b)^2 = (8 \text{ m/s})(12.556) = 100.45 \text{ m}.$$

As a check, we can find how far down the road the dog has traveled;

$$\Delta x_d = v_{0d}t + ½a_d t^2 = ½(2.2 \text{ m/s}^2)(9.556)^2 = 100.45 \text{ m}.$$

So they meet 100.45 m down the road.

With the given and calculated information, we find the final velocity of the dog to be

$$v_{df} = v_{0d} + a_d t = 0 + (2.20 \text{ m/s})(9.556 \text{ s}) = 21.02 \text{ m/s} .$$

According to our definition of average velocity, $v_{\text{average d}} = \Delta x_d/t$ = (100.45 m)/(9.556 s) = 10.51 m/s. This is faster than the bicyclist’s constant speed since the dog has to overcome the bicyclist’s head start.

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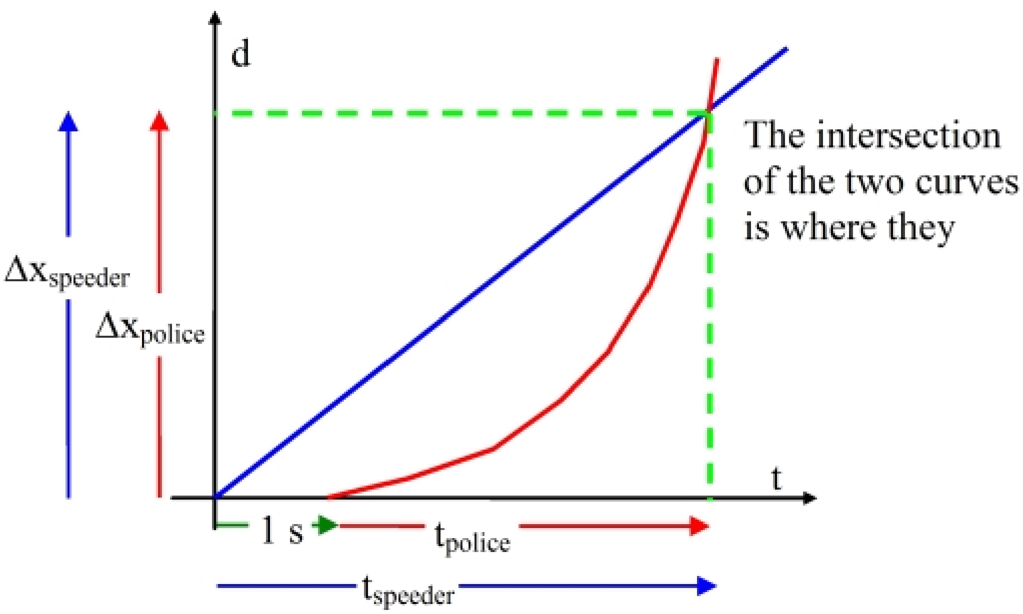
25. A speeding motorist traveling down a straight highway at 110 km/h passes a parked patrol car. It takes the police constable 1.0 s to take a radar reading and to start up his car. The police vehicle accelerates from rest at 2.1 m/s^2. When the constable catches up with the speeder, how far down the road are they and how much time has elapsed since the two cars passed one another?

$v_{0\,police} = 0.0$ m/s,	$v_{0\,speeder} = 110$ km/h $= 30.556$ m/s
$a_{police} = 2.00$ m/s^2	$a_{speeder} = 0$ m/s^2 (constant velocity)
$\Delta x_{police} = ?$	$\Delta x_{speeder} = ?$
$t_{police} = ?$	$t_{speeder} = ?$

This is an example of a two-body constrained kinematics problem. We need a sketch to get the constraints. For the sketch, recall that on a d versus t curve an object moving forward with a uniform acceleration should be represented by a line curving upwards while an object with constant forward velocity is represented by a straight line with a positive slope.



Looking at the sketch, we see that our constraints are:

$$\Delta x_{speeder} = \Delta x_{police} \qquad (1), \text{ and}$$

$$t_{speeder} = t_{police} + 1 \qquad (2)..$$

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and a, and the unknown quantities, Δx and t. Examining our equations we see that we can use $\Delta x = v_0 t + ½at^2$. We substitute this equation into both sides of equation (1). This yields,

$$v_{0\,speeder}t_{speeder} + ½a_{speeder}(t_{speeder})^2 = v_{0\,police}t_{\text{p\<p\>\<sup\>\<sub\>\<sub\>\<subolice}} + ½a_{police}(t_{police})^2.$$

We then use equation (2) to replace tspeeder by tpolice + 1,

$$v_{0\,speeder}(t_{police} + 1) + ½a_{speeder}(t_{police} + 1)^2 = v0\ policet_{police} + ½a_{police}(t_{police})^2.$$

Plugging in the values of the given quantities yields,

$$(30.556)(\ t_{police} + 1) = ½(2.1)(t_{police})^2.$$

This is a quadratic in t_{police}. Solving the quadratic yields, t_{police} = 30.07 seconds. It takes the police constable 30.1 s to catch up with the speeder. The speeder was traveling for 31.1 s.

With a value for t_{police}, we can find how far down the road the police car has traveled;

$$\Delta x_{police} = v_{0\,police}t_{police} + ½a_{police}(t_{police})^2 = ½(2.1)(30.07)^2 = 949\text{ m.}$$

As a check, we can find how far down the road the speeder's car has traveled;

$$\Delta x_{speeder} = v0\ speeder\ (t_{police} + 1) + ½a_{speeder}(t_{police} + 1)^2 = 30.556 \times 31.07 = 949\text{ m.}$$

So the cars meet 949 m down the road.

Top

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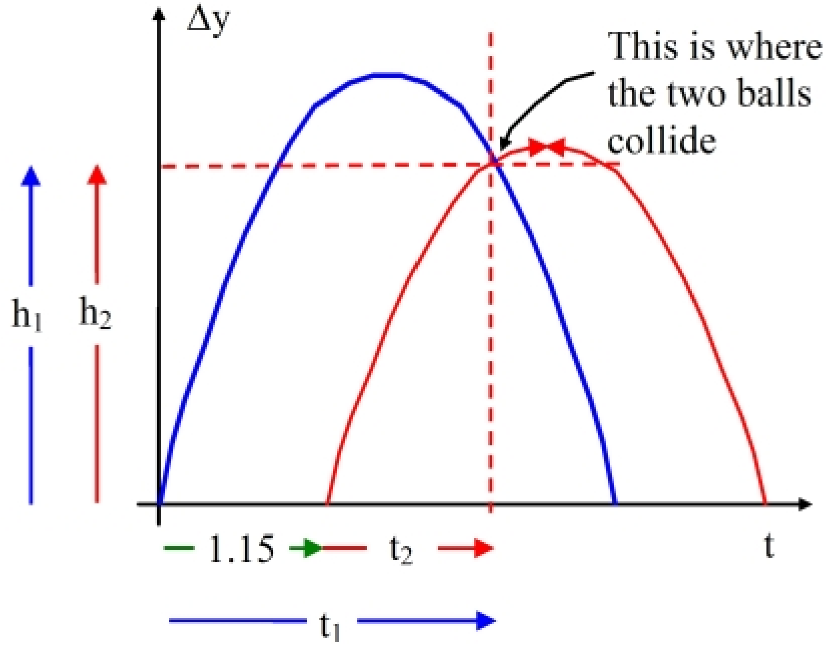
26. Two balls are thrown upwards from the same spot 1.15 seconds apart. The first ball had an initial velocity of 15.0 m/s and the second was 12.0 m/s. At what height do they collide?

To solve this problem, we list the list the given information

$v_{0\,1}$ = 15.0 m/s	$v_{0\,2}$ = 12.0 m/s
a_1 = -9.81 m/s2	a_2 = -9.81 m/s^2
Δy_1 = ?	Δy_2 = ?
t_1 = ?	t_2 = ?

This is an example of a two-body constrained kinematics problem. We need a sketch to get the constraints. For the sketch, recall the shape of the d versus t curve for an object thrown up into the air - a parabola.



Looking at the sketch, we see that our constraints are:

$\Delta y_1 = \Delta y_2$ (1), and<p><sup><sub><sub><sub

$t_1 = t_2 + 1.15$ (2).

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and $a = -g$, and the unknown quantities, Δy and t. Examining our equations we see that we can use $\Delta y = v_0 t - ½gt^2$. We substitute this equation into both sides of equation (1). This yields,

$$v_{01}t_1 - ½g(t_1)^2 = v_{02}t_2 - ½g(t_2)^2.$$

We then use equation (2) to replace t_1 by $t_2 + 1.15$,

$$v_{01}(t_2 + 1.15) - ½g(t_2 + 1.15)^2 = v_{02}t_2 - ½g(t_2)^2.$$

This reduces to

$$1.15v_{01} + v_{01}t_2 - ½g\,[t_2^2 + 2.30t_2 + 1.3225] = v_{02}t_2 - ½g\,t_2^2.$$

Upon rearrangement this becomes

$$(v_{01} - v_{02} - 1.15g)t^2 = -(1.15v_{01} - 0.66125g).$$

Thus $t_2 = 1.2997$ s, and $t_1 = 2.4497$ s. Now that we have the time that each ball is in the air, we can now find h

$$h = v_{01}t_1 - ½g(t_1)^2 = (15.0 \times 2.4497) - ½g(2.4497)^2 = 7.31\ \text{m},$$

and double-checking our result

$$h = v_{02}t_2 - ½g(t_2)^2 = (12.0 \times 1.2297) - ½g(1.2297)^2 = 7.31\ \text{m}.$$

So the balls collide when they are 7.31 m in the air.

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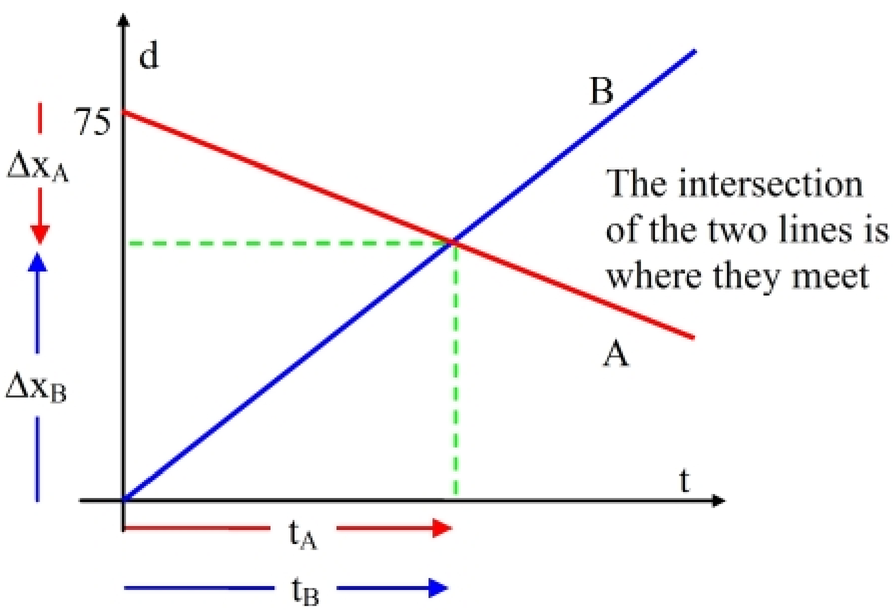
27. Two cars are separated by 75 km of straight highway. They both head toward each other at the same time. Car A travels at a constant 45 km/h and car B travels at a constant 65 km/h. How long after they start do they pass one another? How far from car A's starting point do they pass one another?

To solve this problem, we list the list the given information.

v_{0A} = –45 km/h ,	(down highway)	v_{0B} = +65.0 km/h
a_A = 0 velocity) ,	(constant	a_B = 0
Δx_A = ? ,		Δx_B = ?
t_A =? ,		t_B = ?

This is an example of a two-body constrained kinematics problem. We need a sketch to get the constraints. For the sketch, recall the shape of the d versus t curve for an object moving at constant velocity - a tilted straight line.

To solve this problem, we list the list the given information



Looking at the sketch, we see that our constraints are:

$$\Delta x_B - \Delta x_A = 75 \qquad (1), \text{ and}$$

$$t_A = t_B \qquad (2).$$

Note that Δx_A is pointing downwards meaning that the vector is negative.

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and a, and the unknown quantities, Δx and t. Examining our equations we see that we can use $\Delta x = v_0 t$. We substitute this equation into both sides of equation (1). This yields,

$$v_{0B}t_B - v_{0A}t_A = 75.$$

We then use equation (2) to replace t_A by t_B,

$$v_{0B}t_b - v_{0A}t_B = 75.$$

Plugging in the values of the given quantities yields,

$$65t_B - (-45)t_B = 75.$$

This yields t_B = (75 km)/(110 km/h) = 0.682 hours. Each vehicle has been travelling for about 41 minutes when they pass one another.

With a values for t_B and t_A, we can find how far down the road the vehicle A has travelled;

$$\Delta x_A = v_{0A}t_A = (-45 \text{ km/h}) \times (0.682 \text{ h}) = -30.7 \text{ km}.$$

Car A is 30.7 km from where it started.

As a check, we can find how far down the road car B has travelled;

$$\Delta x_B = v_{0B}t_B = (65 \text{ km/h}) \times (0.682 \text{ h}) = 44.3 \text{ km}.$$

The cars have travelled 75 km as required.

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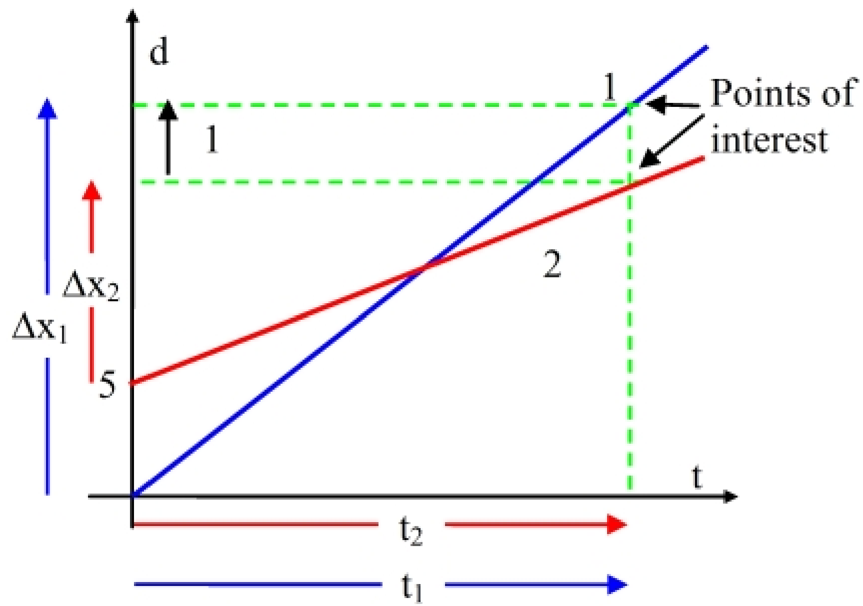
28. In short sprints, runners can be assumed to maintain constant speeds. In a practice run, runner 1 whose speed is 11.0 m/s, starts the race 5.0 m behind runner 2 whose speed is 10.5 m/s. Despite the headstart, runner 1 wins the race by a distance of 1.0 m.
(a) Provide a neat sketch of the position versus time graph for the runners.
(b) How long did the race last?
(c) How far did each runner travel?

To solve this problem, we list the list the given information

v_{01} = 11 m/s		v_{02} = 10.5 m/s
$a_1 = 0$	(constant velocity)	$a_2 = 0$
Δx_1 = ?		Δx_2 = ?
	t_1 =?	

This is an example of a two-body constrained kinematics problem. We would need a sketch to get the constraints even if it wasn't asked for. For the sketch, recall the shape of the d versus t curve for an object moving at constant velocity - a tilted straight line.



Looking at the sketch, we see that our constraints are:

$$\Delta x_1 = \Delta x_2 + 5\text{ m} + 1\text{ m} \qquad (1), \text{ and}$$

$$t_2 = t_1 \qquad (2).$$

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and a, and the unknown quantities, Δx and t. Examining our equations we see that we can use $\Delta x = v_0 t$. We substitute this equation into both sides of equation (1). This yields,

$$v_{01}t_1 = v_{02}t_2 + 6.$$

We then use equation (2) to replace t_2 by t_1,

$$v_{01}t_1 - v_{02}t_1 = 6.$$

Plugging in the values of the given quantities yields,

$$11t_1 - 10.5t_1 = 6.$$

This yields $t_1 = (6\text{ m})/(0.5\text{ m/s}) = 12.0$ s. Each runner has been travelling for 12 seconds.

With a value for t_1 and t_2, we can find how far down the road the runner 1 has travelled;

$$\Delta x_1 = v_{01}t_1 = (11\text{ m/s}) \times (12\text{ s}) = 132\text{ m}.$$

Runner 1 covered 132 m.

Runner 2 has travelled;

$$\Delta x_2 = v_{02}t_2 = (10.5\ \text{m/s}) \times (12\ \text{s}) = 126\ \text{m.}$$

Note runner 1 has travelled 6 m more than runner 2.

Questions?mike.coombes@kwantlen.ca

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