Physics 1100: Collision & Momentum Solutions

1. The diagrams below are graphs of Force in kiloNewtons versus time in milliseconds for the motion of a 5-kg block moving to the right at 4.0 m/s.
   (a) What is the magnitude and direction of the impulse acting on the block in each case?
   (b) What is the magnitude and direction of the average force acting on the block in each case?
   (c) What is the magnitude and direction of the final velocity of the block in each case?

(i)   (ii)   (iii)   (iv)
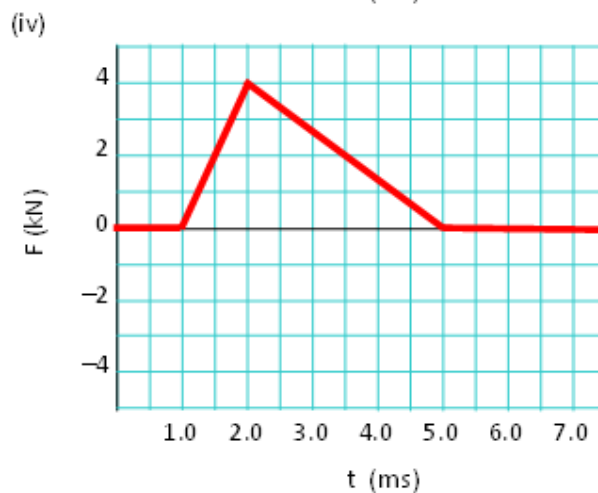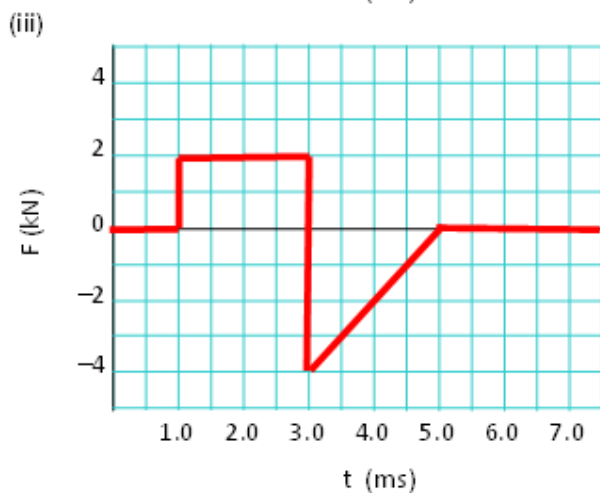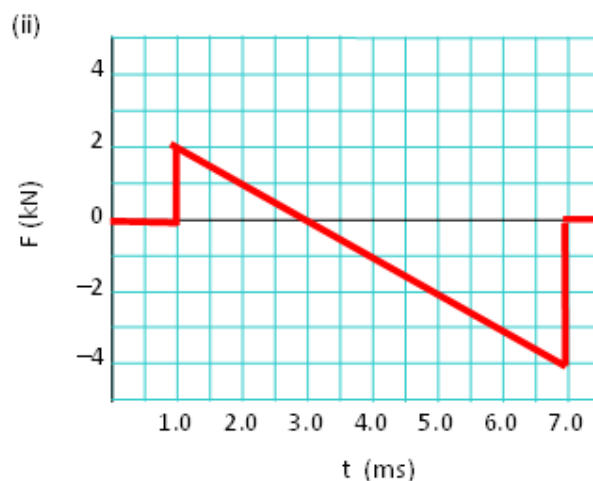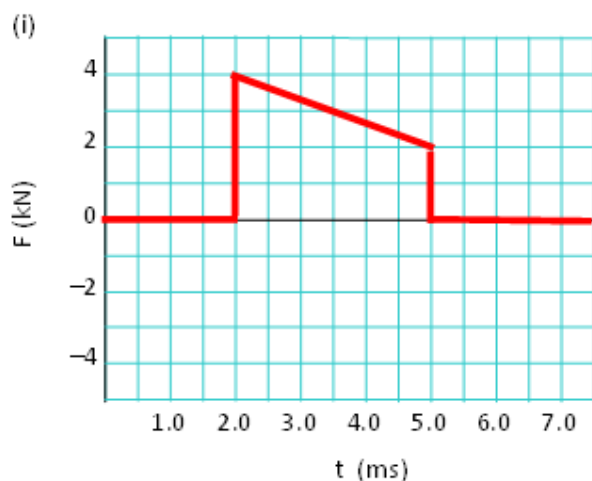
a. Impulse is given by the area under the F-t curves. Since we have simple shapes, it is easy to find the area. For rectangles area is height × base and for triangles area is half the height × base.

    i. $I = 3 \text{ kN} \times 3 \text{ ms} = 9$ N-s

    ii. $I = -1 \text{ kN} \times 6 \text{ ms} = -6$ N-s

    iii. $I = 2 \text{ kN} \times 2 \text{ ms} + \tfrac{1}{2}(-4 \text{ kN}) \times 2 \text{ ms} = 0$ N-s

    iv. $I = \tfrac{1}{2}(4 \text{ kN}) \times 4 \text{ ms} = 8$ N-s

If the impulse is positive, the net area was above the curve and it is directed to the right, if negative to the left.

b. We know $\mathbf{I} = \mathbf{F}_{ave}\Delta t$ where Δt is how long the collision lasts. We read Δt from the graphs, so $\mathbf{F}_{ave} = \mathbf{I}/\Delta t$.

   i. $F_{ave} = (9 \text{ N-s})/(3 \text{ ms}) = 3000 \text{ N}$

   ii. $F_{ave} = (-6 \text{ N-s})/(6 \text{ ms}) = -1000 \text{ N}$

   iii. $F_{ave} = (0 \text{ N-s})/(4 \text{ ms}) = 0 \text{ N}$

   iv. $F_{ave} = (8 \text{ N-s})/(4 \text{ ms}) = 2000 \text{ N}$

   If the average force is positive it is directed to the right, if negative to the left. The impulse and force have the same direction.

c. Impulse is also equal to the difference in momentum, $\mathbf{I} = m\mathbf{v}_f - m\mathbf{v}_i$. We can rearrange our equation for $\mathbf{v}_f$, $\mathbf{v}_f = \mathbf{I}/m + \mathbf{v}_i$.

   i. $v_f = (9 \text{ N-s})/(5.0 \text{ kg}) + 4 \text{ m/s} = 5.8 \text{ m/s}$

   ii. $v_f = (?6 \text{ N-s})/(5.0 \text{ kg}) + 4 \text{ m/s} = 2.8 \text{ m/s}$

   iii. $v_f = (0 \text{ N-s})/(5.0 \text{ kg}) + 4 \text{ m/s} = 4 \text{ m/s}$

   iv. $v_f = (8 \text{ N-s})/(5.0 \text{ kg}) + 4 \text{ m/s} = 5.6 \text{ m/s}$

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2. The diagrams below are the velocity versus time graphs for the collision of motion of a 4-kg block with a wall. The collision lasts for 20 milliseconds in each case.
(a) What is the magnitude and direction of the impulse acting on the block in each case?
(b) What is the magnitude and direction of the average force acting on the block in each case?
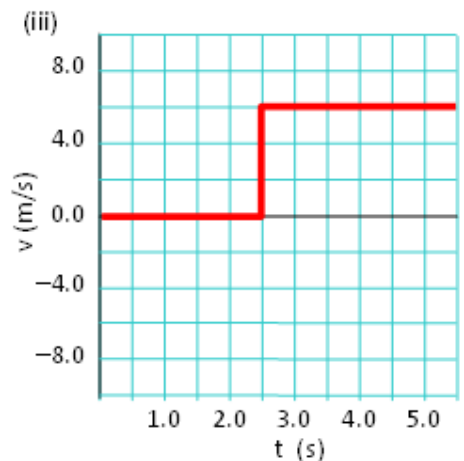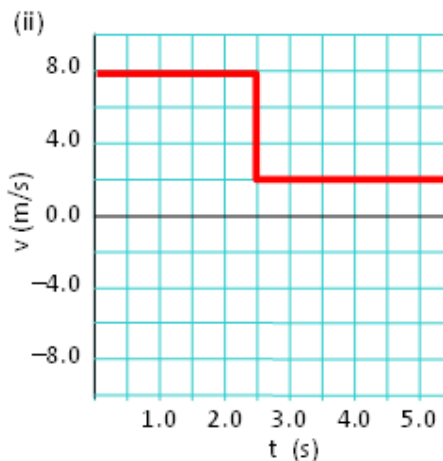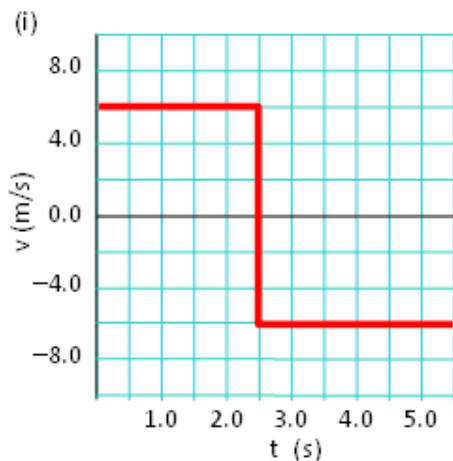
(i)   (ii)   (iii)

a. Impulse is also equal to the difference in momentum, $\mathbf{I} = m\mathbf{v}_f - m\mathbf{v}_i$. We have the mass, $m = 4$ kg.

i. $I = (4 \text{ kg}) \times (-6 \text{ m/s} - 6 \text{ m/s}) = -48 \text{ N-s}$

ii. $I = (4 \text{ kg}) \times (2 \text{ m/s} - 8 \text{ m/s}) = -24 \text{ N-s}$

iii. $I = (4 \text{ kg}) \times (6 \text{ m/s} - 0 \text{ m/s}) = +24 \text{ N-s}$

If the impulse is positive it is directed to the right, if negative to the left.

b. We know $\mathbf{I} = \mathbf{F}_{ave}\Delta t$ where Δt is how long the collision lasts. We have already calculated I and we are given $\Delta t =$ 20 ms, so $\mathbf{F}_{ave} = \mathbf{I}/\Delta t$.

i. $F_{ave} = (-48 \text{ N-s})/(20 \text{ ms}) = -2400 \text{ N}$

ii. $F_{ave} = (-24 \text{ N-s})/(20 \text{ ms}) = -1200 \text{ N}$

iii. $F_{ave} = (+24 \text{ N-s})/(20 \text{ ms}) = +1200 \text{ N}$

3. You've been rowdy and obnoxious in a bar and now are in the process of being thrown out by the bouncer by the scruff of the neck. The bouncer has hold of you for 5.0 s and you are given a final velocity of 2.75 m/s. If your mass is 70.0 kg, what was your final momentum? What impulse and average force did the bouncer exert on you? Assume all motion is in a straight line.

Momentum is defined by $p = mv$. Taking the direction of motion as positive, your initial momentum was zero and your final momentum is

$$p = (70.0 \text{ kg})(2.75 \text{ m/s}) = 192.5 \text{ kg-m/s} .$$

Impulse is defined as the change in momentum

$$I = p_f - p_i = 192.5 \text{ kg-m/s} .$$

Average force is related to impulse by $I = F_{average}t$, so

$$F_{average} = I / t = 192.5 \text{ kg-m/s} / 5 \text{ s} = 38.5 \text{ N}.$$

This is the average force exerted on you and is in the same direction as your motion.

4. A ball of mass 0.500 kg with speed 15.0 m/s collides with a wall and bounces back with a speed of 10.5 m/s. If the motion is in a straight line, calculate the initial and final momenta and the impulse. If the wall exerted a average force of 1000 N on the ball, how long did the collision last?

Momentum is defined by $p = mv$. Taking the right as positive, the initial momentum of the ball is

$$p_i = (0.5 \text{ kg})(-15 \text{ m/s}) = -7.5 \text{ kg-m/s} .$$

The final momentum is

$$p_f = (0.5 \text{ kg})(10.5 \text{ m/s}) = 5.25 \text{ kg-m/s} .$$

Impulse is defined as the change in momentum
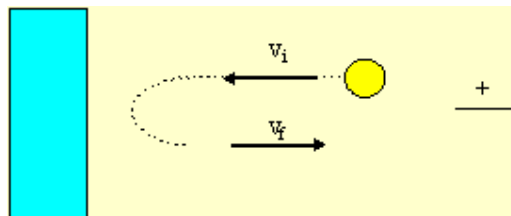
$$I = p_f - p_i = 12.75 \text{ kg-m/s} .$$

Average force is related to impulse by $I = F_{average}t$, and the wall would exert this force on the ball to the right. Therefore

$$t = I / F_{average} = 12.75 \text{ kg-m/s} / +1000 = 0.013 \text{ s}.$$

The ball is in contact with the wall for approximately 13 milliseconds.

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5. A ball of mass 0.25 kg glances of a wall as shown in the diagram. The ball approaches at 15 m/s at $\theta = 30°$ and leaves at 12 m/s at $\varphi = 20°$. The collision lasts for 15 milliseconds.
   (a) What are the components of the impulse experienced by the ball?
   (b) What are the components of the average force acting on the ball?
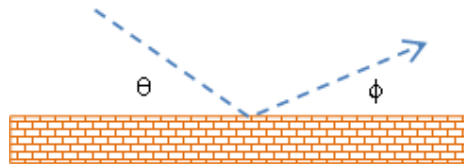
   a. We know Impulse is equal to the difference in momentum, $\mathbf{I} = m\mathbf{v}_f - \mathbf{v}_i$. This is a vector equation and to get components we consider the x and y components separately.

   $I_x = mv_{fx} - mv_{ix} = (0.25) \times (12\cos20° - 15\cos30°) = -0.4285$ N-s

   $I_y = mv_{fy} - mv_{iy} = (0.25) \times (12\sin20° - (-15\sin30°)) = +2.9011$ N-s

   or $\mathbf{I} = -i0.4285 + j2.9011$ N-s.

   b. We know $\mathbf{I} = \mathbf{F}_{ave}\Delta t$ where Δt is how long the collision lasts. We have already calculated I and we are given $\Delta t = 15$ ms, so $\mathbf{F}_{ave} = \mathbf{I}/\Delta t$

   so $\mathbf{F}_{ave} = (-i0.4285 + j2.9011$ N-s$) / (15 \text{ ms}) = -i28.6 + j193.4$ N.

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6. While chasing an armed suspect into and onto an ice rink, a police constable is shot. Fortunately, the constable is wearing a bullet-proof vest which absorbs the bullet. If the muzzle velocity of the bullet is 350 m/s and the its mass is 100 g. Find the final velocity of the constable and bullet if her mass is 69.5 kg. Assume all motion is in a straight line and ignore friction. Assume that the constable is at rest.

$v_{bi} = 350$ m/s
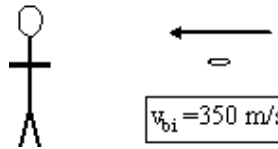
We have a totally inelastic collision, so momentum is conserved. For this particular problem

$$(m_{police} + m_{bullet})v_{pf} = m_{police}v_{pi} + m_{bullet}v_{bi}.$$

Since we are told $v_{pi} = 0$,

$$v_{pf} = m_{bullet}v_{bi}/(m_{police} + m_{bullet}) = (0.100 \text{ kg})(-350 \text{ m/s})/(69.5 \text{ kg} + 0.1 \text{ kg}) = -0.503 \text{ m/s} .$$

So the constable is knocked backwards at 0.50 m/s.

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7. A 70-kg man and a 55-kg woman are standing on a stationary sled which is on a frictionless surface. The man jumps horizontally off the sled with a velocity of 3.00 m/s at 25.0° west of north. The woman jumps off the sled horizontally with a speed of 3.25 m/s at 40.0° south of west. What is the magnitude and direction of the sled's final momentum? If the mass of the sled is 7.50 kg, what is the final velocity of the sled?

As is suggested by the word momentum in this question, this is an explosion in which momentum is conserved.

$$\mathbf{P}_{man} + \mathbf{P}_{woman} + \mathbf{P}_{sled} = \mathbf{0} . \qquad (1)$$

Momentum is a vector quantity so we will need to deal with the components. First we calculate the magnitude of the momentum of the man and the woman, using $p = mv$:

$$P_{man} = 70 \text{ kg } 3 \text{ m/s} = 210 \text{ kg-m/s} ,$$

$$P_{woman} = 55 \text{ kg } 3.25 \text{ m/s} = 178.75 \text{ kg-m/s} .$$

Examining equation (1), we see that $\mathbf{P}_{sled} = -(\mathbf{P}_{man} + \mathbf{P}_{woman})$, so we need to do a vector addition as shown in the diagram below.
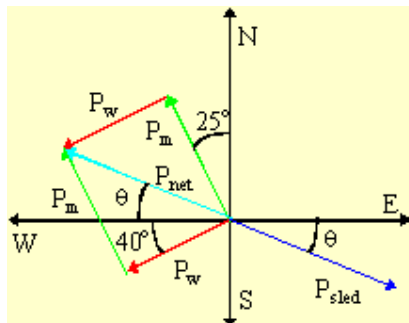
So we find $\mathbf{P}_{net}$ by components

i	j
$P_{man\ x} = -210sin(25°)$	$P_{man\ y} = 210cos(25°)$
$= -88.750$	$= 190.325$
$P_{woman\ x} = -178.75cos(40°)$	$P_{woman\ y} = -178.75sin(40°)$

$$= -136.930 \qquad\qquad = -114.898$$

$$P_{net\ x} = -225.68 \qquad\qquad P_{net\ y} = 75.426$$

$$P_{sled\ x} = +225.68 \qquad\qquad P_{sled\ y} = -75.426$$

Using the Pythagorean formula,

$$P_{sled} = [(P_{sled\ x})^2 + (P_{sled\ y})^2]^{\frac{1}{2}} = [(255.68)^2 + (-75.426)^2]^{\frac{1}{2}} = 237.951 \text{ kg-m/s} .$$

Using trigonometry,

$$\theta = arctan(P_{sled\ y} / P_{sled\ x}) = arctan(|-75.426/255.68|) = 18.48° .$$

So the final momentum of the sled is 238 kg-m/s at 18.5° south of east.

To find the final velocity of the sled recall that $\mathbf{p} = m\mathbf{v}$. This is a vector equation, so $\mathbf{p}$ and $\mathbf{v}$ must point in the same direction. The magnitude of the velocity of the sled is thus

$$v_{sled} = P_{sled} / m_{sled} = 237.95 \text{ kg-m/s} / 7.50 \text{ kg} = 31.7 \text{ m/s} .$$

So the velocity of the sled just after both people jump off the sled is 31.7 m/s at 18.5 south of east.

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8. A 50.0-kg skater is travelling due east at a speed of 3.00 m/s. A 70.0-kg skater is moving due south at a speed of 7.00 m/s. They collide and hold on to one another after the collision, managing to move off at an angle θ south of east with a speed v_f. Find (a) the angle θ and (b) the speed v_f, assuming that friction can be ignored.
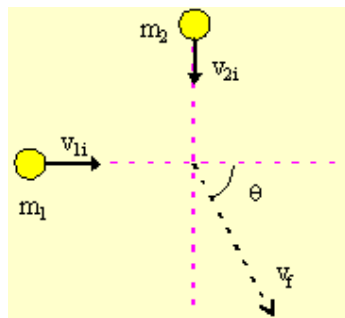
In any kind of collision, momentum is conserved so

$$(m_1 + m_2)\mathbf{v}_f = m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} . \qquad (1)$$

Now momentum and velocity are vector quantities and the i and j components must be handled separately

$$(m_1 + m_2)v_{fx} = m_1 v_{1ix} + m_2 v_{2ix} , \qquad (1a)$$

$$(m_1 + m_2)v_{fy} = m_1 v_{1iy} + m_2 v_{2iy} . \qquad (1b)$$

So we can rearrange these equations to find the components of the final velocity

$$v_{fx} = (m_1 v_{1ix} + m_2 v_{2ix}) / (m_1 + m_2) , \qquad (2a)$$

$$v_{fy} = (m_1 v_{1iy} + m_2 v_{2iy}) / (m_1 + m_2) . \qquad (2b)$$

Using the given values, we find

$$v_{fx} = [(50 \text{ kg})(3 \text{ m/s}) + (70 \text{ kg})(0)] / (50 \text{ kg} + 70\text{kg}) = 1.25 \text{ m/s}, \qquad (2a)$$

$$v_{fy} = [(50 \text{ kg})(0) + (70 \text{ kg})(-7 \text{ m/s}) / (50 \text{ kg} + 70 \text{ kg}) = 4.083 \text{ m/s}. \qquad (2b)$$

To find the magnitude and direction of the final velocity, we use the Pythagorean Theorem and trigonometry,

$$v_f = [(v_{fx})^2 + (v_{fx})^2]^{\frac{1}{2}} = 4.27 \text{ m/s, and}$$

$$\theta = arctan(|v_{fy}/v_{fx}|) = 72.98°.$$

The final velocity of the pair is 4.27 m/s at 73.0° south of east.

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9. Two opposing hockey players are racing up the ice for the puck when they collide at point A as shown in the diagram below. The first hockey player has mass 90 kg and a speed of 2.7 m/s while the other has mass 82 kg and speed 3.1 m/s. The angle in the diagram is $\theta = 32°$. After the collision, the players remain locked together (at least until the referee forces them apart). What is the magnitude and direction of the players' velocity just after they collide?
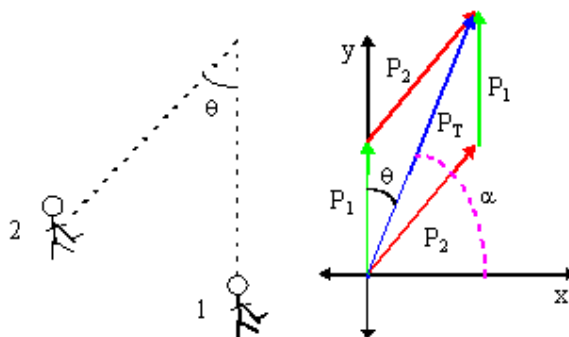
Since the collision is totally inelastic and in two dimensions, we find that we are dealing with a vector addition problem, $\mathbf{P_T} = \mathbf{P_1} + \mathbf{P_2}$. First we calculate the magnitude of each player's momentum using $p = mv$,

$$P_1 = m_1 v_1 = (90 \text{ kg})(2.7 \text{ m/s}) = 243 \text{ kg-m/s},$$

$$P_2 = m_2 v_2 = (82 \text{ kg})(3.1 \text{ m/s}) = 254.2 \text{ kg-m/s}.$$

Then we find P_T by the component method,

i	j
$P_{1x} = 0$	$P_{1y} = 243$
$P_{2x} = 254.2 sin(32°)$	$P_{2y} = 254.2 cos(32°)$
$= 134.705$	$= 215.574$
$P_{Tx} = 134.705$	$P_{Ty} = 458.574$

Using the Pythagorean formula we find,

$$P_T = [(P_{Tx})^2 + (P_{Ty})^2]^{\frac{1}{2}} = [(134.705)^2 + (458.574)^2]^{\frac{1}{2}} = 477.949 \text{ kg-m/s}.$$

Using trigonometry, we find the angle from

$$\theta = arctan(P_{Ty}/P_{Tx}) = arctan(458.574/134.705) = 73.63°.$$

So the total momentum of the two players is $\mathbf{P}_T = (478, 73.6°)$.

Now $\mathbf{P}_T = (m_1 + m_2)\mathbf{v}_f$, so the final velocity must be in the same direction as the total momentum. The magnitude of the velocity is

$$v_f = P_T/(m_1 + m_2) = 477.949 \text{ kg-m/s/ } (90 \text{ kg} + 82 \text{ kg}) = 2.78 \text{ m/s .}$$

So the final velocity of the two players just after the collision is $\mathbf{v}_f = (2.78 \text{ m/s, } 73.6°)$.

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10. In a curling match, a 6.0-kg rock with speed 3.50 m/s collides with another motionless 6.0-kg rock. What are the velocities of the rocks after the collision if it is (a) elastic or (b) totally inelastic? (c) How much energy was lost in the inelastic collision? Ignore friction and assume all motion is in a straight line.

(a) In an elastic collision, both momentum and kinetic energy is conserved. Thus we have the equations;

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i} \text{ , (1)}$$
$$v_{1f} - v_{2f} = -(v_{1i} - v_{2i}) \text{ . } \quad (2)$$

Since $v_{2i} = 0$, the two equations can be combine to yield

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} = 0$$

and

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} = \left(\frac{2 \times 6}{6 + 6}\right)(3.50\,m/s) = 3.50\,m/s$$

So after the collision, the first rock comes to a complete halt and the second rock takes off with the velocity of the first rock before the collision.

(b) In a totally inelastic collision, the two rocks stick together so that conservation of momentum becomes

$$(m_1 + m_2)v_f = m_1v_{1i} + m_2v_{2i} \text{ .}$$

since $v_{2i} = 0$,

$$v_f = m_1v_{1i}/(m_1 + m_2) = (6 \text{ kg } 3.5 \text{ m/s})/(6 \text{ kg} + 6 \text{ kg}) = 1.75 \text{ m/s .}$$

(c) We find the kinetic energy lost in the inelastic collision by examining the energies just before and after the collision:

$$K_i = \tfrac{1}{2}m_1(v_{1i})^2 + \tfrac{1}{2}m_2(v_{2i})^2 = \tfrac{1}{2}(6 \text{ kg})(3.5 \text{ m/s})^2 + 0 = 36.75 \text{ J ,}$$

$$K_f = \tfrac{1}{2}(m_1+m_2)(v_f)^2 = \tfrac{1}{2}(6 \text{ kg} + 6 \text{ kg})(1.75 \text{ m/s})^2 = 18.375 \text{ J .}$$

So the change in energy is $E = K_f - K_i = -18.4 \text{ J}$. Thus 18.4 J of energy was lost in the collision.

11. A 5.00-kg ball, moving to the right at a velocity of +2.00 m/s on a frictionless table, collides head-on with a stationary 7.50-kg ball. Find the final velocities of the balls if the collision is (a) elastic and (b) completely inelastic. (c) How much energy was lost in the inelastic collision?

(a) In an elastic collision, both momentum and kinetic energy is conserved. Thus we have the equations;

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} , \quad (1)$$
$$v_{1f} - v_{2f} = -(v_{1i} - v_{2i}) . \quad (2)$$

Since $v_{2i} = 0$, the two equations can be combine to yield

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{5 - 7.5}{5 + 7.5}\right)(2 \, m/s) = -0.40 \, m/s$$

and

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left(\frac{2 \times 5}{5 + 7.5}\right)(2 \, m/s) = 1.60 \, m/s$$

So after the collision, the first rock moves backward at 0.40 m/s and the second rock takes off with a forward velocity of 1.60 m/s.

(b) In a totally inelastic collision, the two rocks stick together so that conservation of momentum becomes

$$(m_1 + m_2)v_f = m_1 v_{1i} + m_2 v_{2i} .$$

since $v_{2i} = 0$,

$$v_f = m_1 v_{1i}/(m_1 + m_2) = (5 \text{ kg } 2 \text{ m/s})/(5 \text{ kg} + 7.5 \text{ kg}) = 0.80 \text{ m/s} .$$

(c) We find the kinetic energy lost in the inelastic collision by examining the energies just before and after the collision:

$$K_i = \tfrac{1}{2}m_1(v_{1i})^2 + \tfrac{1}{2}m_2(v_{2i})^2 = \tfrac{1}{2}(5 \text{ kg})(2 \text{ m/s})^2 + 0 = 10.0 \text{ J} ,$$

$$K_f = \tfrac{1}{2}(m_1 + m_2)(v_f)^2 = \tfrac{1}{2}(5 \text{ kg} + 7.5 \text{ kg})(0.80 \text{ m/s})^2 = 4.8 \text{ J} .$$

So the change in energy is $E = K_f - K_i = -5.2$ J. Thus 5.2 J of energy was lost in the collision.

12. A 60.0-kg person, running horizontally with a velocity of 3.80 m/s jumps on a 12.0-kg sled that is initially at rest. (a) Ignoring the effects of static friction, find the velocity of the sled and person as they move away. (b) The sled and person coast 30.0 m on level snow before coming to a rest. What is the coefficient of kinetic friction between the sled and the snow?
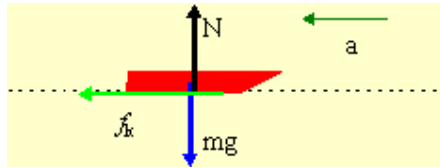
(a) We have a totally inelastic collision, so

$$(m_{person} + m_{sled})v_f = m_{person}v_{person} + m_{sled}v_{sled} .$$

since $v_{sled} = 0$,

$$v_f = m_{person}v_{person}/(m_{person} + m_{sled}) = (60 \text{ kg } 3.8 \text{ m/s})/(60 \text{ kg} + 12 \text{ kg}) = 3.17 \text{ m/s} .$$

(b) This portion of the question involves a force and a distance suggesting the use of Work-Energy methods.

Since there is friction, $W_{NC} = W_{friction}$ is not zero. We need a free body diagram to find f_k.

Using Newton's Second Law,

i	j
$F_x = ma_x$	$F_y = ma_y$
$-f_k = -ma$	$N - mg = 0$

The equation in the second column tells us that $N = mg$. Since $f_k = \mu_k N$, we have $f_k = \mu_k mg$. So the work done by friction is

$$W_{friction} = f_k \Delta x \, cos(180°) = -\mu_k \, mg\Delta x .$$

As well, we know that

$$W_{friction} = E_f - E_i = -\tfrac{1}{2}mv^2.$$

Combing these two results yields,

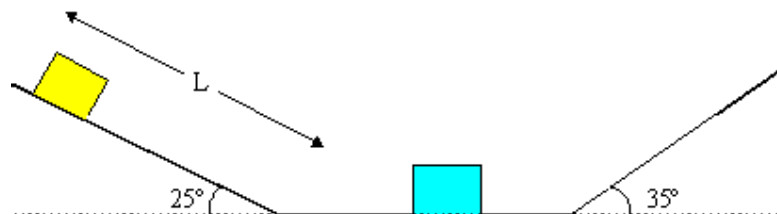$$-\mu_k mg\Delta x = -\tfrac{1}{2}mv^2.$$

Solving for μ_k

$$\mu_k = \tfrac{1}{2}v^2/g\Delta x = \tfrac{1}{2}(3.17 \text{ m/s})^2/(9.81 \text{ m/s}^2 \times 30 \text{ m}) = 0.017 .$$

The coefficient of kinetic friction was 0.017.

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13. A 5.0-kg block slides from rest down an L = 2.50 m long 25° incline. At the bottom it undergoes an elastic collision with a 10.0-kg block sending it towards a 35° incline. After the collision, how far along its incline does each block go? The surface is frictionless.

We have a change in height and speed in the first part of the problem, so that suggests that we have a Work-Energy problem. In the second part of the problem, there is a collision which suggest that we use conservation of momentum. In the final portion, there is a change in height and speed again, so this suggests that we have a Work-Energy problem.

(i) Since there is no friction, $W_{NC} = 0$. Hence $E_f = E_i$ or

$$\tfrac{1}{2}m_1 v^2 = m_1 gh.$$

The height h is related to L by $h = L\sin(25°)$. Substituting in this relation, and rearranging to get v by itself yields,

$$v = [2gL\sin(25°)]^{\frac{1}{2}} = [2(9.81)(2.5)\sin(25°)]^{\frac{1}{2}} = 4.553 \text{ m/s} .$$

This is the velocity of the first block just before the collision. This velocity will be the initial velocity for part (ii).

(ii) In an elastic collision, both momentum and kinetic energy is conserved. Thus we have the equations;

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} , \quad (1)$$
$$v_{1f} - v_{2f} = -(v_{1i} - v_{2i}) . \quad (2)$$

Since $v_{2i} = 0$, the two equations can be combine to yield

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{5 - 10}{5 + 10}\right)(4.553 \ m/s) = -1.518 m/s$$

and

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} = \left(\frac{2 \times 5}{5 + 10}\right)(4.553 \ m/s) = 3.035 \ m/s$$

So the first block will bounce backwards and return up the incline it came down. The second block will move up the incline on the right.

(iii) Applying conservation of energy for the first block

$$\tfrac{1}{2}m_1 v^2 = m_1 gL_1 \sin(25°) .$$

Solving for L_1, we find

$$L_1 = v^2/(2g\sin(25°)) = (-1.518)^2/(29.81\sin(25°)) = 0.278 \text{ m} .$$

Similarly, the second block moves up its incline a distance

$$L_2 = v^2/(2g\sin(35°)) = (3.035)^2/(29.81\sin(35°)) = 0.819 \text{ m} .$$

So the first block moves 0.28 m up the left incline after the collision while the second block moves 0.82 m up the right incline.

Questions? mike.coombes@kwantlen.ca

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