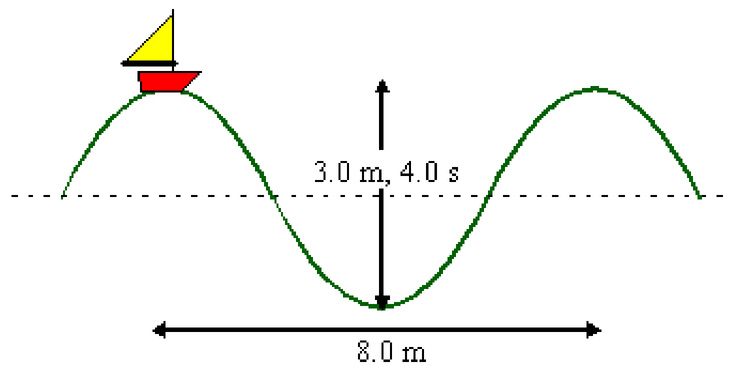
Physics 1100: Waves Solutions

1. A fisherman notices that his boat is moving up and down in a periodic way because of waves on the surface of the water. It takes 4.0 s for the boat to travel from its highest point to its lowest, a total distance of 3.0 m. The fisherman sees that the wave crests are spaced 8.0 m apart.

   (a) How fast are the waves moving?

   (b) What is the amplitude, frequency, wavelength, and period of the waves?



   The period, T, is the time that is takes the motion to repeat itself. The motion is peak down to trough and back up to the peak. Since it took 4.0 s to move from peak to trough, it should also take 4.0 s to move from trough to peak. The period is $T = 8.0$ s.

   The frequency is given by $f = 1/T = 1 / 8.0\ \text{s} = 0.125$ Hz.

   The wavelength, λ, is the distance that over which the wave repeats itself. The peaks are 8.0 m apart, so $\lambda = 8.0$ m.

   The amplitude, A, of a wave is the height of the wave above average height, or one-half the peak-to-trough distance. Hence $A = ½(3.0\ \text{m}) = 1.5$ m.

   The speed of a wave is $v = \lambda/T = 8.0\ \text{m} / 8.0\ \text{s} = 1.0$ m/s.

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2. Radio waves travel at the speed of light, 3.00×10^8 m/s. (a) AM radio waves have a frequency range of 530 kHz to 1600 kHz. What range of wavelengths does this correspond to? (b) FM waves have wavelengths of 2.77 m to 3.40 m. What range of frequencies does this correspond to?

Speed, wavelength, and frequency of a wave are related by $v = \lambda f$. We can rearrange this to get $\lambda = v/f$ or $f = v/\lambda$.

(a) For AM radio waves:

(i) $\lambda = 3.00 \times 10^8$ m/s / 530×10^3 s^{-1} = 566 m .

(ii) $\lambda = 3.00 \times 10^8$ m/s / 1600×10^3 s^{-1} = 188 m .

(b) For FM radio waves:

(i) $f = 3.00 \times 10^8$ m/s / 2.77 m = 1.08×10^8 Hz = 108 MHz.

(ii) $f = 3.00 \times 10^8$ m/s / 3.40 m = 8.82×10^7 Hz = 88.2 MHz.

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3. Provided that the amplitude is high enough, the human ears can detect sound in the range of frequencies from about 20.0 Hz to about 20,000 Hz. Compute the wavelengths corresponding to the frequencies

   (a) for waves in air (v = 343 m/s);

   (b) for waves in water (v = 1480 m/s).

   Speed, wavelength, and frequency of a wave are related by $v = \lambda f$. We can rearrange this to get $\lambda = v/f$ or $f = v/\lambda$.

   (a) In air

   (i) λ = 343 m/s / 20 s^{-1} = 17.2 m .

   (ii) λ= 343 m/s / 20×10^3 s^{-1} = 0.0172 m .

   (b) In water

   (i) λ = 1480 m/s / 20 s^{-1} = 74.0 m .

   (ii) λ = 1480 m/s / 20×10^3 s^{-1} = 0.74 m .

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4. With what tension must a rope of length 5.00 m and mass 0.160 kg be stretched for transverse waves of frequency 60.0 Hz to have a wavelength of 0.800 m?

The speed of a wave on a string is determined by the tension through the formula

$$v = \sqrt{F / M/L}\ .$$

The speed of a wave is also determined by the wavelength and frequency by $v = \lambda f$.

Letting these two expression equal yields

$$\sqrt{F / M/L} = \lambda f\ .$$

Solving for the tension F, we find

$$F = v^2 M / L = (48.0\ \text{m/s})^2 (0.160\ \text{kg})/(5.00\ \text{m}) = 73.7\ \text{N}\ .$$

The tension in the rope is thus 73.7 N

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5. Transverse waves travel with a speed of 20.0 m/s in a string that is under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s in the same string?

The speed of a wave on a string is determined by the tension through the formula $v = \sqrt{F / M/L}$.

For the first case we have $v_1 = \sqrt{F_1 / M/L}$ or $M/L = F_1 / v_1^2$.

In the second case, $v_2 = \sqrt{F_2 / M/L}$ or $M/L = F_2 / v_2^2$.

Now M/L is a constant since it is the same string. Thus we have $F_1 / v_1^2 = F_2 / v_2^2$, or

$$F_2 = (v_2/v_1)^2 F_1 = (30\ \text{m/s} / 30\ \text{m/s})^2 (6.00\ \text{N}) = 13.5\ \text{N}.$$

The tension is the string must be increased to 13.5 N.

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6. A wire of length 4.35 m and mass 137 g is under a tension of 125 N. What is the speed of a wave in this wire? If the tension is doubled, what is the speed? If the mass is doubled?

The speed of a wave on a string is given by the formula $v = \sqrt{F_{tension}/\mu}$, where is the linear density given by $\mu = M/L$. Thus the speed is

$$v = \sqrt{125N/(0.137kg/4.35m)} = 63.0\,m/s.$$

If we double the tension, v = 89.1 m/s.

If we double the mass, v = 44.5 m/s.

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7. The waves on a certain string move at 50.0 m/s. The string is 7.5 m long. If both ends are fixed, what are the first five resonance or standing wave frequencies? If one end is free, what are the first five frequencies? Is there any way of telling that a string is fixed or free at one end by looking at a sequence of resonance frequencies?

   (i) The standing wave of resonance frequencies of a string fixed at both ends is given by the formula $f_n = n\frac{v}{2L}$ where $n = 1,2,3,\ldots$ Since we are given v and L, we have

   $$f_n = n(50\ m/s)/(2\ 7.5\ m) = n(3.33\ Hz).$$

   The first five frequencies are thus: f_1 = 3.33 Hz, f_2 = 6.67 Hz, f_3 = 10.0 Hz, f_4 = 13.33 Hz, and f_5 = 16.67 Hz.

   (ii) The standing wave of resonance frequencies of a string fixed at only one end is given by the formula $f_n = n\frac{v}{4L}$ where $n = 1,3,5,\ldots$ Since we are given v and L, we have

   $$f_n = n(50\ m/s)/(4\ 7.5\ m) = n(1.67\ Hz).$$

   The first five frequencies are thus: f_1 = 1.67 Hz, f_3 = 5.00 Hz, f_5 = 8.33 Hz, f_7 = 11.67 Hz, and f_9 = 15.00 Hz.

   (iii) If we have a sequence of resonance frequencies, we can tell if the string is fixed at both ends or open at one end by looking at the ratio of the frequencies. To find the ratio divide through by the greatest common divisor. For a string fixed at both ends, the ratio will have both even and odd numbers. A string fixed at only one end will have only odd numbers. For example, from part (i) above, the ratio is 1:2:3:4:5, where the common factor 3.33 Hz is factored out. In part (ii), the ratio is 1:3:5:7:9, where the common factor 1.67 Hz is factored out. Notice that the greatest common factor is the fundamental frequency.

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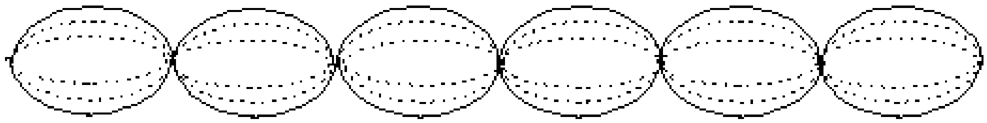
8. A wire of length 4.35 m and mass 137 g is under a tension of 125 N. A standing wave has formed which has seven nodes including the endpoints. Sketch the wave. What is the frequency of this wave? Which harmonic is it? What is the fundamental frequency?

First we sketch the standing wave.



The equation for a string fixed at both ends is given by $f_n = n\frac{v}{2L}$ and $\lambda_n = \frac{2L}{n}$. Examining the sketch , we see that n = #node - 1 = 6, so that this is the **sixth** harmonic. We are given L, so we need the speed of the wave v to determine f_n. The speed of the wave can be found from the formula $v = \sqrt{F_{tension}/\mu}$, where is the linear density given by $\mu = M/L$. Using the given data, the speed may be computed

$$v = \sqrt{\frac{125N}{(0.137kg/4.35m)}} = 63.0\,m/s$$

.

Hence, $f_6 = 6\frac{(63.0m/s)}{2(4.35m)} = 6 \times 7.24Hz = 43.4Hz.$

The fundamental, n = 1, frequency is f_1 = 7.24 Hz.

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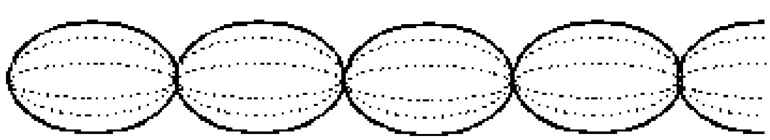
9. A string fixed at one end only is vibrating in its ninth harmonic mode. Sketch the wave. The speed of a wave on the string is v = 25.8 m/s and the string has a length of 8.25 m. What is the frequency of this wave? What is the wavelength of the wave? What is the fundamental frequency?

The resonance frequencies of a string fixed at only one end is given by the formula $f_n = n\frac{v}{4L}$ where n = *1,3,5,...* The first sentence tells us that n = 9 - the number of quarter wavelengths seen. We use this to sketch the standing wave.



Using the formula, since we have n, v and L,

$$f_9 = 9(25.8\ m/s)/(4\ 8.25\ m) = 7.04\ Hz.$$

The fundamental frequency occurs when $n = 1$, so

$$f_1 = 1(25.8\ m/s)/(4\ 8.25\ m) = 0.78\ Hz.$$

The wavelength of a string fixed at both ends is given by the formula $\lambda_n = \frac{4L}{n}$, where $n = 1,3,5,\ldots$ Thus we have

$$_9 = 4(8.25\ m)/9 = 3.67\ m.$$

Of course, we could just look at the sketch and see that one full wavelength is 4 quarters of the nine quarters displayed, so that the wavelength was just 4/9 times the length of the string.

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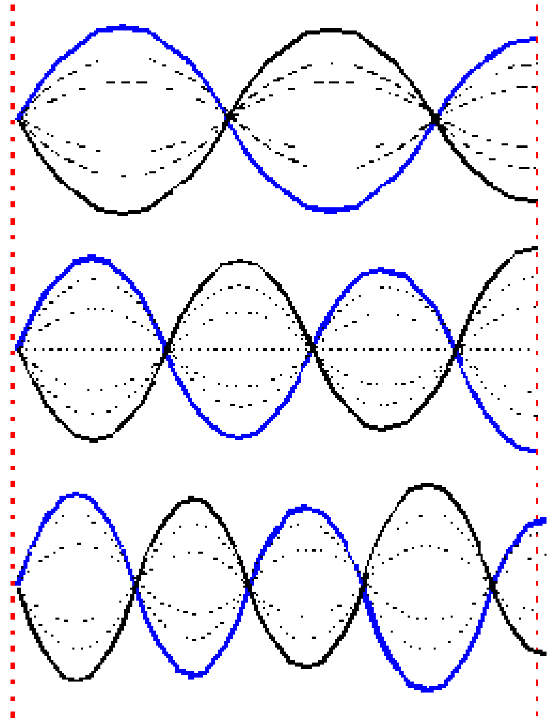
10. Three successive resonance frequencies for a certain string are 175, 245, and 315 Hz. (a) Find the ratio of these three modes. (b) How can you tell that this sting has an antinode at one end? (c) What is the fundamental frequency? (d) Which harmonics are these resonance frequencies? Sketch each wave. (e) If the speed of transverse waves on this string is 125 m/s, find the length of the string?

(a) The ratio of these three frequencies is 175:245:315, or 535:735:935, or 5:7:9.

(b) For a string fixed at both ends, the resonant frequencies are given by $f_n = n\frac{v}{2L}$, where $n = 1, 2, 3, 4, \ldots$ For a string fixed at only one end, the resonant frequencies are given by $f_n = n\frac{v}{4L}$, where $n = 1, 3, 5, 7, 9, \ldots$ Since the given sequence has only odd numbers, we may conclude that the string is fixed at only one end.

(c) The fundamental frequency is the greatest common factor of the sequence, so $f_1 = 35.0$ Hz.

(d) These are 5^{th}, 7^{th}, and 9^{th} harmonics.



(e) Since $f_1 = \frac{v}{4L}$, we may rearrange this equation to find the length,

$$L = \frac{v}{4f_1} = \frac{125 m/s}{4 \times 35 Hz} = 0.893 m.$$

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