

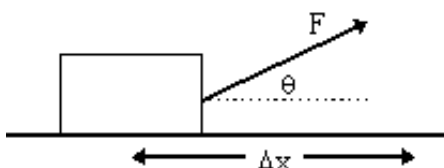
Questions: [1](#) [2](#) [3](#) [4](#) [5](#) [6](#) [7](#) [8](#) [9](#) [10](#) [11](#) [12](#) [13](#) [14](#) [15](#) [16](#) [17](#) [18](#) [19](#) [20](#) [21](#) [22](#) [23](#)

## Physics 1100: Work & Energy Solutions

### Definition of work: $W = F\Delta x \cos\theta$

1. In the diagram below, calculate the work done if

- (a)  $F = 15.0 \text{ N}$ ,  $\theta = 15^\circ$ , and  $\Delta x = 2.50 \text{ m}$ ,
- (b)  $F = 25.0 \text{ N}$ ,  $\theta = 75^\circ$ , and  $\Delta x = 12.0 \text{ m}$ ,
- (c)  $F = 10.0 \text{ N}$ ,  $\theta = 135^\circ$ , and  $\Delta x = 5.50 \text{ m}$ ,



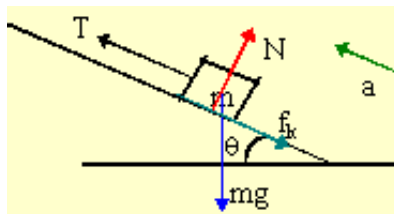
For constant forces, work is defined by  $W = F\Delta x \cos(\theta)$ .

- (a)  $W = 36.2 \text{ J}$
- (b)  $W = 77.6 \text{ J}$
- (c)  $W = -38.9 \text{ J}$

**Top**

2. In the diagram below, a rope with tension  $T = 150 \text{ N}$  pulls a  $15.0\text{-kg}$  block  $3.0 \text{ m}$  up an incline ( $\theta = 25.0^\circ$ ). The coefficient of kinetic friction is  $\mu_k = 0.20$ . Find the work done by each force acting on the block.

To find the work done by a force, we need to know the magnitude of the force and the angle it makes with the displacement. To find forces, we draw a FBD and use Newton's Second Law.



$$\begin{array}{cc}
 & \mathbf{i} & & \mathbf{j} \\
 F_x = ma_x & & & F_y = ma_y \\
 T - f_k - mg\sin(\theta) = ma & & & N - mg\cos(\theta) = 0
 \end{array}$$

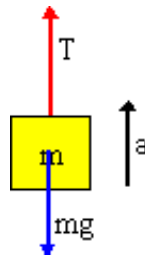
The second equation informs us that  $N = mg\cos(\theta)$ . We know  $f_k = \mu_k N = \mu_k mg\cos(\theta)$ .

<i>Force</i>	<i>Force (N)</i>	$\theta$	$W = F\Delta x\cos(\theta)(J)$
Tension	150	0	450
Weight	147.15	$\theta + \pi/2$	-187
Normal	133.36	$\pi/2$	0
Friction	26.67	$\pi$	-80

**Top**

3. A winch lifts a 150 kg crate 3.0 m upwards with an acceleration of  $0.50 \text{ m/s}^2$ . How much work is done by the winch? How much work is done by gravity?

To find the work done by a force, we need to know the magnitude of the force and the angle it makes with the displacement. To find forces, we draw a FBD and use Newton's Second Law.

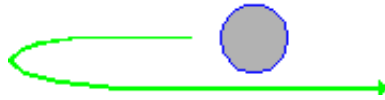


$$\begin{array}{c}
 \mathbf{j} \\
 F_y = ma_y \\
 T - mg = ma
 \end{array}$$

The work done by the winch is the work done by tension. The work done by gravity is the work done by the object's weight. Since we know  $m$  and  $g$ , we find  $T = mg + ma = 1546.5 \text{ N}$ . The work done by tension is  $W_{\text{tension}} = T\Delta y\cos(0) = 4.64 \times 10^3 \text{ J}$ . The work done by gravity is  $W_{\text{gravity}} = mg\Delta y\cos(\pi) = -4.41 \times 10^3 \text{ J}$ .

**Top**

4. What work does a baseball bat do on a baseball of mass  $0.325 \text{ kg}$  which has an initial speed forward of  $36 \text{ m/s}$  and a final speed of  $27 \text{ m/s}$  backwards. Assume motion is linear and horizontal. The work done by the bat is a non-conservative force.



Since we are asked for the work done and have a change in speed, we make use of the generalized Work-Energy Theorem. Since the height of the ball does not change, there is only a change in kinetic energy.

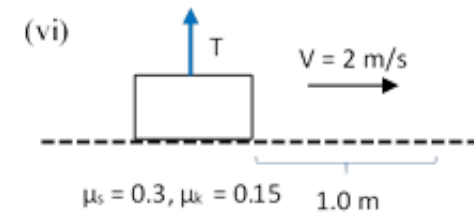
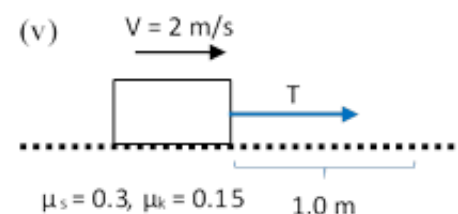
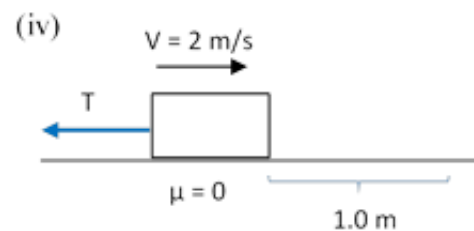
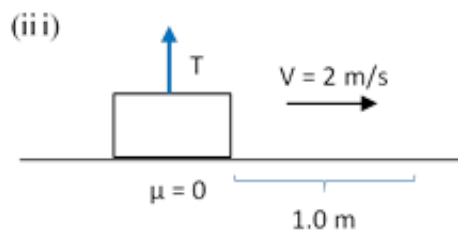
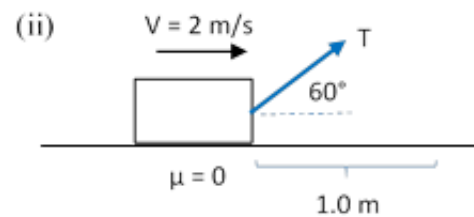
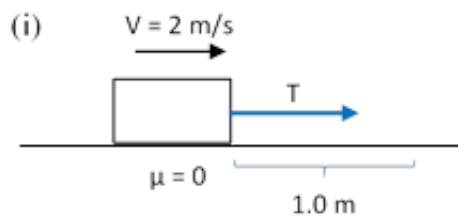
$$W_{\text{NC}} = E = K_f - K_i = \frac{1}{2}m[(v_f)^2 - (v_0)^2] = \frac{1}{2}(0.325\text{kg})[(-27 \text{ m/s})^2 - (36 \text{ m/s})^2] = -92.1 \text{ J} .$$

This is the work done on the ball by the bat. It's not a good hit as the ball slowed down. The batter decreased the energy of the ball. Perhaps he was trying for a bunt!

**Top**

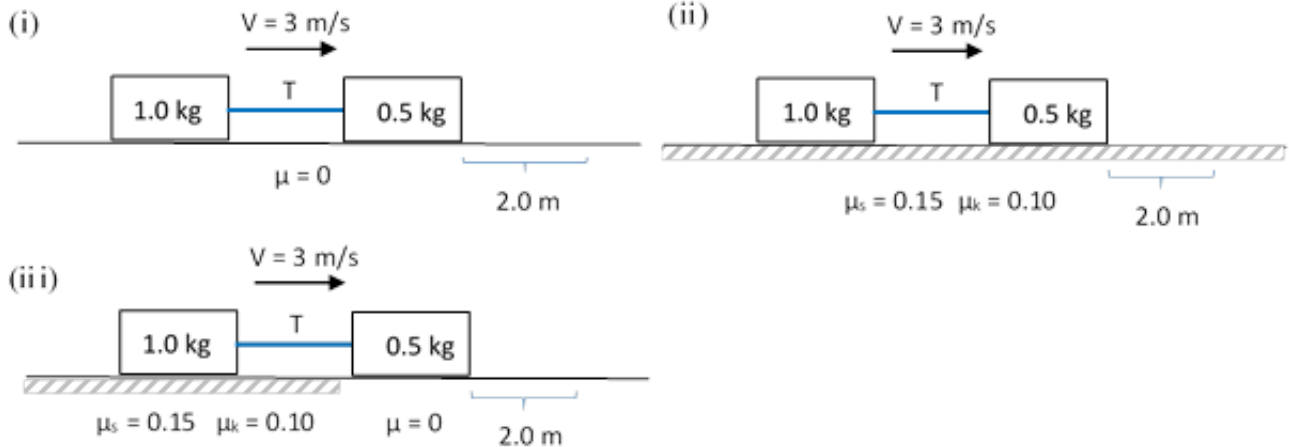
### Work and Energy: $W_{\text{external}} = E_{\text{final}} - E_{\text{initial}}$

5. Consider a 0.50-kg block travelling at 2.0 m/s on a horizontal surface. At the instant shown, there is a rope with a tension of 1.0 N attached while the block travels 1.0 m to the right. In each case:
- Determine what object(s) make(s) up the system.
  - Determine if the system is isolated or if there are external forces acting on the system.
  - Determine if there are internal forces to the system.
  - Find the final speed of the block.



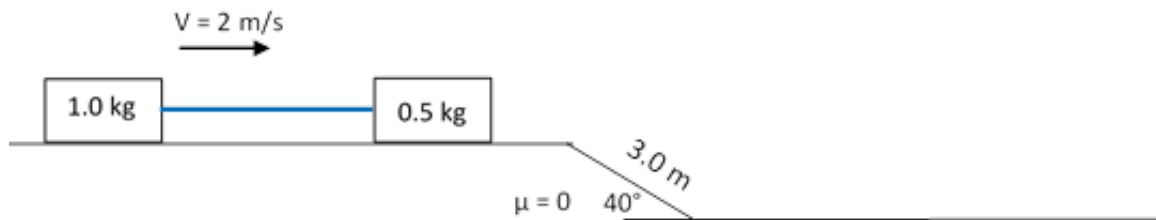
**Top**

6. Consider the blocks travelling at 5.0 m/s on a horizontal surface in the diagrams below. The blocks travel 2.0 m to the right. In each case:
- Determine what object(s) make(s) up the system.
  - Determine if the system is isolated or if there are external forces acting on the system.
  - Determine if there are internal forces to the system.
  - Find the final speed of the block(s).



**Top**

7. Consider the two blocks travelling at 5.0 m/s on a horizontal surface in the diagrams below. The blocks will one by one slide down the 3.0 m-long,  $40^\circ$  incline to the lower side. The string between the blocks is long enough so that the first block will be down on the lower level before the second block hits the incline. In each case:
- Determine what object(s) make(s) up the system.
  - Determine if the system is isolated or if there are external forces acting on the system.
  - Determine if there are internal forces to the system.
  - Find the final speed of the blocks when just the first block reaches the lower level.
  - Find the final speed of the blocks when both blocks reach the lower level.



**Top**

8. How much work must be done to stop a 2000-kg car travelling at 60 km/h in 15.0 m? What was the

average breaking force?

Since we are asked for the work done and have a change in speed, we make use of the generalized Work-Energy Theorem. Since the height of the car does not change, there is only a change in kinetic energy. First converting the initial velocity into SI

$$60 \text{ km/h} = 60 \text{ km/h} \times (1000 \text{ m})/(1 \text{ km}) \times (1 \text{ h})/(3600 \text{ s}) = 16.67 \text{ m/s} .$$

Therefore,

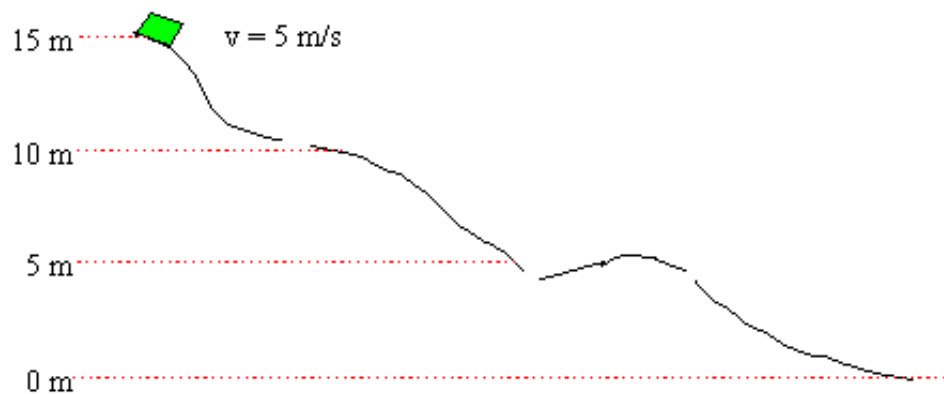
$$W_{\text{brake}} = E = K_f - K_i = \frac{1}{2}m[(v_f)^2 - (v_0)^2] = \frac{1}{2}(2000 \text{ kg})[(0)^2 - (16.67 \text{ m/s})^2] = 2.77810^5 \text{ J} .$$

Now the force doing this work,  $f_{\text{brake}}$ , is related to the work by  $W_{\text{brake}} = f_{\text{brake}} \cos(\theta)$ . Since the force is slowing the car down,  $\theta = 180^\circ$ ,  $\cos(180^\circ) = -1$ , and

$$f_{\text{brake}} = -W_{\text{brake}} / \Delta x = -(2.778 \times 10^5 \text{ J})/(15.0 \text{ m}) = 1.85 \times 10^4 \text{ N} .$$

**Top**

9. In the diagram below, determine the speed of the block at each point. Assume no friction. The mass of the block is 10.0 kg.



Since the problem involves a change in height and speed, we make use of the generalized Work-Energy Theorem,

$$W_{\text{NC}} = E = P_f - P_i + K_f - K_i = mg(h_f - h_i) + \frac{1}{2}m[(v_f)^2 - (v_i)^2] .$$

Since there is no mention of friction,  $W_{\text{NC}} = 0$ . Our equation therefore simplifies to

$$mg(h_f - h_i) + \frac{1}{2}m[(v_f)^2 - (v_i)^2] = 0 ,$$

or more simply

$$mgh_f + \frac{1}{2}m(v_f)^2 = mgh_i + \frac{1}{2}m(v_i)^2 .$$

We can divide through by  $m$ , and since we know  $h_f$ ,  $h_i$ , and  $v_i$ , we can rearrange the above to find  $v_f$

$$(v_f)^2 = (v_i)^2 + 2g(h_i - h_f) .$$

For the given values, we find

	$h_f$ (m)	$v_i$ (m/s)
1	15	5
2	10	11.1
3	5	14.9
4	0	17.9

**Top**

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10. A 2.0-kg rock is thrown with initial speed of 9.8 m/s at an unknown angle. The speed of the rock at the top of the parabola is 2.1 m/s. How high does it go? Assume no air resistance.

Since the problem involves a change in height and speed, we make use of the generalized Work-Energy Theorem,

$$W_{NC} = E = P_f - P_i + K_f - K_i = mg(h_f - h_i) + \frac{1}{2}m[(v_f)^2 - (v_i)^2] .$$

Since we are told that there is no air resistance,  $W_{NC} = 0$ . Our equation therefore simplifies to

$$mg(h_f - h_i) + \frac{1}{2}m[(v_f)^2 - (v_i)^2] = 0 ,$$

or more simply

$$mgh_f + \frac{1}{2}m(v_f)^2 = mgh_i + \frac{1}{2}m(v_i)^2 .$$

We can divide through by  $mg$ , and since we know  $h_i$ ,  $v_i$ , and  $v_f$ , we can rearrange the above to find  $h_f$

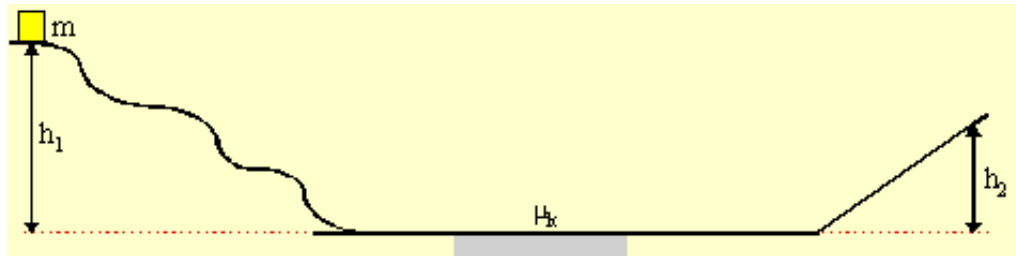
$$h_f = [(v_i)^2 - (v_f)^2]/2g = [(9.8 \text{ m/s})^2 - (2.1 \text{ m/s})^2]/(2 \cdot 9.81 \text{ m/s}^2) = 4.67 \text{ m} .$$

The rock reaches 4.67 m up into the air.

**Top**

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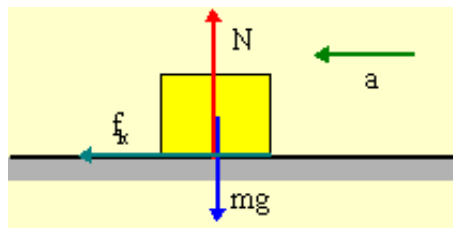
11. In the diagram below, a 5.00-kg block slides from rest at a height of  $h_1 = 1.75$  m down to a horizontal surface where it passes over a 2.00-m rough patch. The rough patch has a coefficient of kinetic friction  $\mu_k = 0.25$ . What height,  $h_2$ , does the block reach on the incline?



Since the problem involves a change of height and speed, we make use of the Generalized Work-Energy Theorem. Since the block's initial and final speeds are zero, we have

$$W_{NC} = E = U_f - U_i = mgh_2 - mgh_1 . \quad (1)$$

The nonconservative force in this problem is friction. To find the work done by friction, we need to know the friction. To find friction, a force, we draw a FBD at the rough surface and use Newton's Second Law.



$i$	$j$
$F_x = ma_x$	$F_y = ma_y$
$-f_k = -ma$	$N - mg = 0$

The second equation gives  $N = mg$  and we know  $f_k = \mu_k N$ , so  $f_k = \mu_k mg$ . Therefore, the work done by friction is  $W_{\text{friction}} = -f_k \Delta x = -\mu_k mg \Delta x$ . Putting this into equation (1) yields

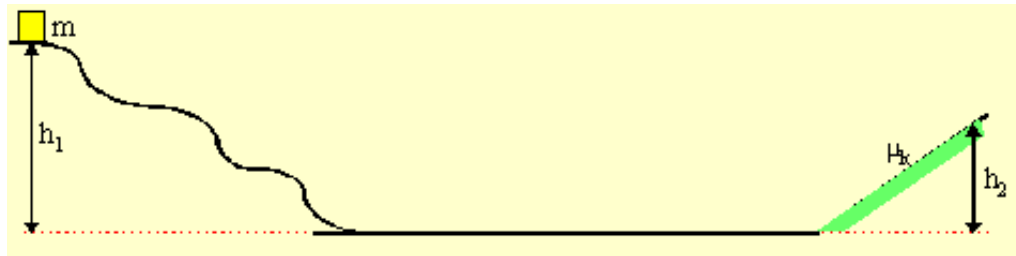
$$-\mu_k mg \Delta x = mgh_2 - mgh_1 .$$

Solving for  $h_2$ , we find

$$h_2 = h_1 - \mu_k \Delta x = 1.25 \text{ m} .$$

**Top**

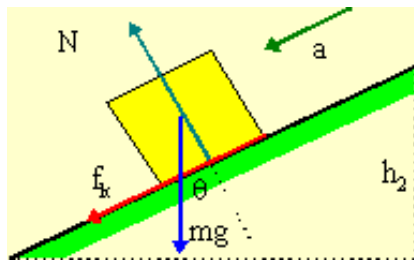
12. In the diagram below, a 5.00-kg block slides from rest at a height of  $h_1 = 1.75 \text{ m}$  down to a smooth horizontal surface until it encounters a rough incline. The incline has a coefficient of kinetic friction  $\mu_k = 0.25$ . What height,  $h_2$ , does the block reach on the  $\theta = 30.0^\circ$  incline?



Since the problem involves a change of height and speed, we make use of the Generalized Work-Energy Theorem. Since the block's initial and final speeds are zero, we have

$$W_{NC} = E = U_f - U_i = mgh_2 - mgh_1 . \quad (1)$$

The nonconservative force in this problem is friction. To find the work done by friction, we need to know the friction. To find friction, a force, we draw a FBD at the rough surface and use Newton's Second Law.



$$\begin{aligned} F_x &= ma_x & F_y &= ma_y \\ -f_k - mg\sin(\theta) &= -ma & N - mg\cos(\theta) &= 0 \end{aligned}$$

The second equation gives  $N = mg\cos(\theta)$  and we know  $f_k = \mu_k N$ , so  $f_k = \mu_k mg\cos(\theta)$ . Therefore, the work done by friction is  $W_{\text{friction}} = -f_k \Delta x = -\mu_k mg\cos(\theta) \Delta x$ . Putting this into equation (1) yields

$$-\mu_k mg\cos(\theta) \Delta x = mgh_2 - mgh_1 .$$

A little trigonometry shows that  $\Delta x$  is related to  $h_2$  by  $\Delta x = h_2 / \sin(\theta)$ . Putting this into the above equation yields

$$-\mu_k \cos(\theta) [ h_2 / \sin(\theta) ] = h_2 - h_1 .$$

Solving for  $h_2$ , we find

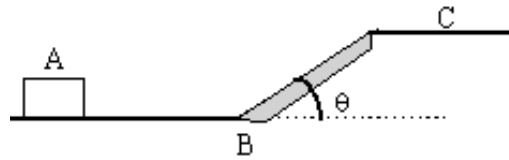
$$h_2 = h_1 / [1 + \mu_k / \tan(\theta)] = 1.22 \text{ m} .$$

**Top**

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13. In the figure below, a block of mass 5.0 kg starts at point A with a speed of 15.0 m/s on a flat frictionless surface. At point B, it encounters an incline with coefficient of kinetic friction  $\mu_k = 0.15$ . The block makes



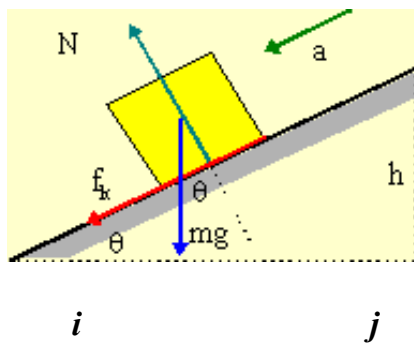
it up the incline to a second flat frictionless surface. What is the work done by friction? What is the velocity of the block at point C? The incline is 2.2 m long at an angle  $\theta = 15^\circ$ .



The problem involves a change in height and speed, so we apply the generalized Work-Energy Theorem.

$$W_{NC} = E = (K_f - K_i) + (U_f - U_i) = \frac{1}{2}m(v_C)^2 - \frac{1}{2}m(v_A)^2 + mgh. \quad (1)$$

Here the nonconservative force is friction, so  $W_{NC} = W_f$ . To find friction, a force, we draw a FBD and use Newton's Second Law.



$$\begin{aligned} F_x &= ma_x & F_y &= ma_y \\ -f_k - mg\sin(\theta) &= -ma & N - mg\cos(\theta) &= 0 \end{aligned}$$

The second equation gives  $N = mg\cos(\theta)$  and we know  $f_k = \mu_k N$ , so  $f_k = \mu_k mg\cos(\theta)$ . Therefore, the work done by friction is

$$W_f = -f_k \Delta x = -\mu_k mg\cos(\theta)\Delta x = -(0.15)(5 \text{ kg})(9.81 \text{ m/s}^2)\cos(15^\circ)(2.2 \text{ m}) = -15.635 \text{ J}.$$

Note from the diagram, that the height  $h$  is related to the length of the incline by  $h = \Delta x \sin(\theta)$ . Putting both results into equation (1) yields

$$W_f = \frac{1}{2}m(v_C)^2 - \frac{1}{2}m(v_A)^2 + mg[\Delta x \sin(\theta)].$$

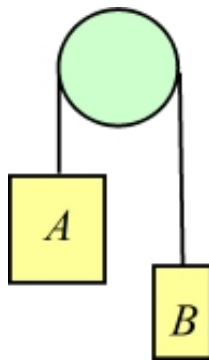
Solving for  $v_C$  yields

$$v_C = [2W_f/m - 2g\Delta x \sin(\theta) + (v_A)^2]^{1/2} = 14.4 \text{ m/s}.$$

**Top**

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14. Two blocks are connected by a string hung over a frictionless massless pulley. Block  $A$  has mass  $M_A$  and block  $B$  has mass  $M_B$ . Initially the blocks are held at rest before being allowed to move. How fast will

block  $B$  be moving when it has risen a distance  $h$ ?



Again we have a change in height and speed, so we apply the Work-Energy Theorem

$$W_{\text{NC}} = (K_f - K_i) + (U_f - U_i).$$

We are told that there is no friction so  $W_{\text{NC}} = 0$ .

The difference between this and earlier problems is that we are dealing with two objects. For each object there is an external force the tension  $T$  in the string. However the work done by the tension in each case is equal, since the distance each block moves is the same, but opposite. (Check this!) So for the system, energy is transferred from one block to the other. We solve the problem by applying the right hand side of the Work-Energy Theorem to each block in turn.

$$0 = [\frac{1}{2}M_B v_f^2 + M_B g h] + [\frac{1}{2}M_A v_f^2 - M_A g h]$$

Note that the two blocks are connected by a string so the final speed of each is the same. Also if block  $B$  moves up  $h$  block  $A$  drops  $h$ . Thus our equation becomes

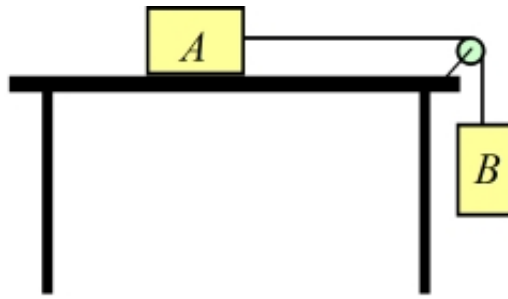
$$0 = \frac{1}{2} (M_A + M_B) v_f^2 - (M_A - M_B) g h.$$

When we solve this, we find

$$v_f = \sqrt{2gh \frac{M_A - M_B}{M_A + M_B}}$$

**Top**

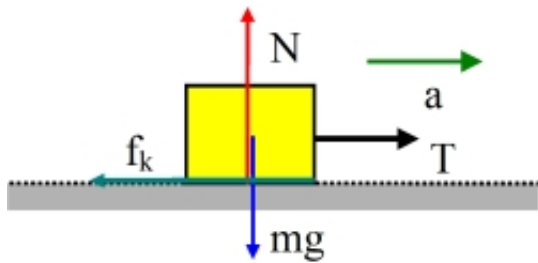
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15. Two blocks are connected by a string hung over a frictionless massless pulley. Block  $A$  has mass  $M_A$  and is on a table top. Block  $B$  has mass  $M_B$  and is hanging in the air. Initially the blocks are held at rest. The coefficients of friction between block  $A$  and the tabletop are  $\mu_S$  and  $\mu_K$ .
- (a)  $B$  is allowed to fall. How fast will block  $B$  be moving when it has fallen distance  $h$ ?
- (b) Block  $A$  is pulled to the left by a horizontal force  $F$  for a distance  $L$ . How fast will block  $B$  be moving?



(a) Again we have a change in height and speed, so we apply the Work-Energy Theorem

$$W_{NC} = (K_f - K_i) + (U_f - U_i).$$

We are told that there is friction so we need to determine  $W_{NC} = W_{\text{friction}}$ . Friction does negative work, takes energy out of the system, since it is opposite to the movement of block A. To find friction, a force, we draw a FBD of block A and use Newton's Second Law.



<i>i</i>	<i>j</i>
$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$T - f_k = M_A a$	$N - M_A g = 0$

The second equation gives  $N = M_A g$  and we know  $f_k = \mu_k N$ , so  $f_k = \mu_k M_A g$ . Therefore, the work done by friction is  $W_{\text{friction}} = -f_k \Delta x = -\mu_k M_A g h$  since block A will move as far as block B will drop.

For the pair of block, the tension T in the string, is internal and does not net work. So for the system, energy is transferred from one block to the other. We solve the problem by applying the right hand side of the Work-Energy Theorem to each block in turn.

$$-\mu_k M_A g h = \frac{1}{2} M_A v_f^2 + [\frac{1}{2} M_B v_f^2 - M_A g h]$$

Note that the two blocks are connected by a string so the final speed of each is the same. Thus our equation becomes

$$M_B g h - \mu_k M_A g h = \frac{1}{2} (M_A + M_B) v_f^2.$$

When we solve this, we find

$$v_f = \sqrt{2gh \frac{M_B - \mu_k M_A}{M_A + M_B}}$$

(b) Again we have a change in height and speed, so we apply the Work-Energy Theorem

$$W_{NC} = (K_f - K_i) + (U_f - U_i).$$

We are told that there is friction, and the work done by friction is still  $W_{friction} = f_k \Delta x = -\mu_k M_A g L$  since block A moves  $L$  not  $h$ . Because of the string block B rises  $L$  and both blocks will have the same speed. However there is an extra external force  $F$  which in the same direction as the motion of block A. It does positive work adding to the energy of the system.

We solve the problem by applying the right hand side of the Work-Energy Theorem to each block in turn.

$$FL - \mu_k M_A g h = \frac{1}{2} M_A v_f^2 + [\frac{1}{2} M_B v_f^2 + M_A g L]$$

Thus our equation becomes

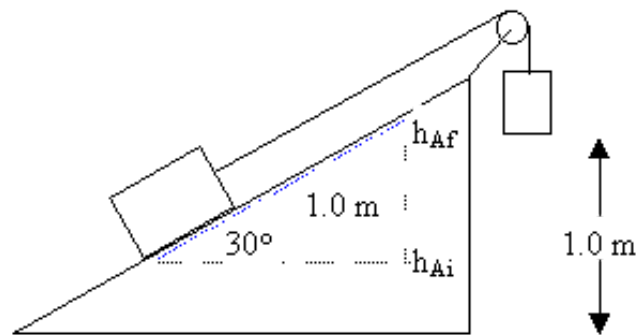
$$FL - M_B g h - \mu_k M_A g h = \frac{1}{2} (M_A + M_B) v_f^2 .$$

When we solve this, we find

$$v_f = \sqrt{2L \frac{F - (M_B + \mu_k M_A) g}{M_A + M_B}}$$

**Top**

16. Two blocks are connected by a string slung over a pulley as shown in the diagram below. The hanging block is allowed to drop. How fast will it be moving when it hits the ground? The block on the incline has mass  $M_A = 2.50$  kg. The hanging block has mass  $M_B = 1.50$  kg. The incline makes an angle  $\theta = 30^\circ$  with horizontal. Ignore friction.



Again we have a change in height and speed, so we apply the Work-Energy Theorem

$$W_{NC} = (K_f - K_i) + (U_f - U_i).$$

We are told to ignore friction so  $W_{NC} = 0$ .

The difference between this and earlier problems is that we are dealing with two objects. For each object there is an external force the tension  $T$  in the string. However the work done by the tension in each case is equal, since the distance each block moves is the same, but opposite. (Check this!) So for the system, energy is transferred from one block to the other. We solve the problem by applying the right hand side of the Work-Energy Theorem to each block in turn.

$$0 = (\frac{1}{2}M_B V_{Bf}^2 - 0) + (M_B g(0) - M_B g(1.0\text{m})) \\ + (\frac{1}{2}M_A V_{Af}^2 - 0) + (M_A g(h_{Af} - h_{Ai}))$$

Now the two blocks are connected by a string so the final speed of each is the same,  $V_{Bf} = V_{Af} = V_f$ . Next the block moves 1.0 m up the  $30^\circ$  degree incline, so  $h_{Af} - h_{Ai} = (1.0 \text{ m})\sin(30^\circ)$ . Thus our equation becomes

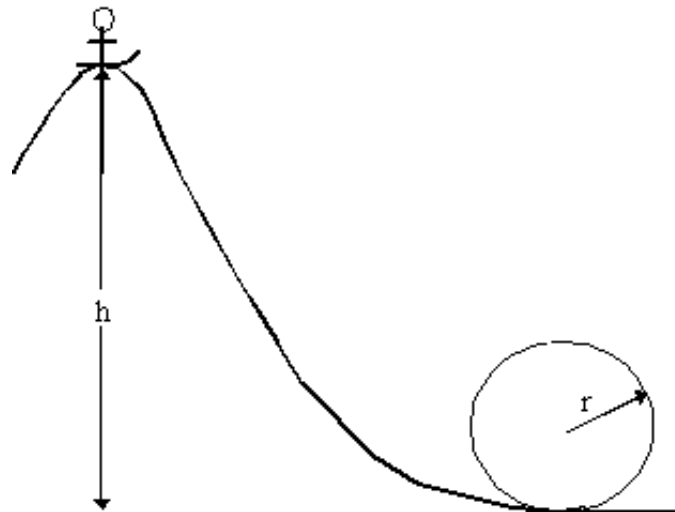
$$0 = \frac{1}{2}M_B V_f^2 + \frac{1}{2}M_A V_f^2 - M_B g(1.0\text{m}) + M_A g(1.0\text{m})\sin(30^\circ) .$$

When we solve this we find

$$V_f = \{2(9.81)[(1.50)(1.0\text{m}) - (2.50)(1.0\text{m})\sin(30^\circ)]/(1.50 + 2.50)\}^{1/2} = 1.338 \text{ m/s} .$$

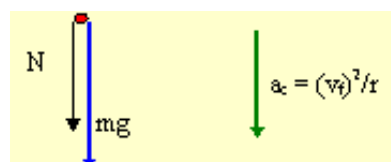
**Top**

17. In the diagram below, what is the minimum height that the skier must start from to successfully make it around the loop. Assume (a) no friction, and (b) that friction does  $-3.0 \times 10^3 \text{ J}$  of work on the skier. The radius is 5.00 m and the skier has mass 65.0 kg.



The problem involves a change of height and speed, so that suggests that we use the generalized Work-Energy Equation. However, the skier also travels in a circle, which suggests a centripetal acceleration problem. Centripetal acceleration problems are solved by drawing a free-body diagram (FBD) and applying Newton's Second Law. Let's do this first.

At the top of the inside of the loop, the centripetal acceleration acts straight down as does the normal force and the weight.



$$F_y = ma_y$$

$$-N - mg = -m(v_f)^2/r$$

The skier will lose contact with the inside of the loop when  $N$  goes to zero. This fact and our equation, let's us find a minimum value of  $v_f$

$$v_f = [gr]^{1/2} = [(9.81 \text{ m/s}^2)(5.00 \text{ m})]^{1/2} = 7.004 \text{ m/s} .$$

Now we consider the work energy portion of the problem.

The Work-Energy formula may be rewritten as

$$mgh_f + \frac{1}{2}m(v_f)^2 = mgh_i + \frac{1}{2}m(v_i)^2 .$$

We know  $v_i = 0$ , we see from the diagram that  $h_f = 2r$ , and  $v_f = [gr]^{1/2}$  from our earlier work, so we rearrange the above equation to find  $h_i$

$$h_i = h_f + \frac{1}{2}(v_f)^2/g = 2r + \frac{1}{2}r = (5/2)r = 12.5 \text{ m} .$$

If the trip is frictionless, the hill needs to be at least 12.5-m tall if the skier is to make it around the loop safely.

Since there are non-conservative forces, the generalized Work-Energy equation for this case is

$$W_{NC} = [mgh_f + \frac{1}{2}m(v_f)^2] - [mgh_i + \frac{1}{2}m(v_i)^2] .$$

We are told  $W_{NC} = -3000 \text{ J}$ , so we rearrange the equation to find that  $h_i$  is,

$$h_i = \{[mgh_f + \frac{1}{2}m(v_f)^2] - W_{NC}\}/mg = (5/2)r - W_{NC}/mg .$$

Using the given data,

$$h_i = 12.5 \text{ m} - (-3000 \text{ J})/(65.0 \text{ kg})(9.81 \text{ m/s}^2) = 17.2 \text{ m} .$$

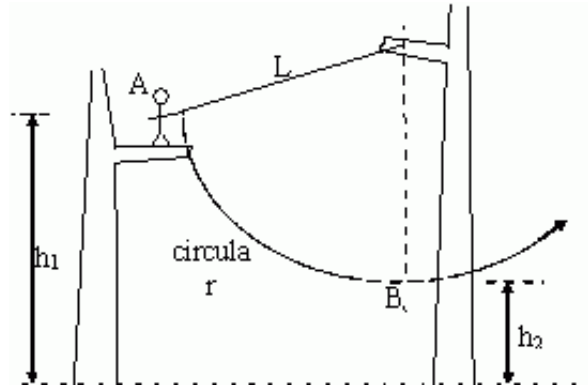
With this much friction, the hill needs to be at least 17.2-m tall if the skier is to make it around the loop safely.

**Top**

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18. Tarzan, Lord of Apes, is swinging through the jungle. In the diagram below, Tarzan is standing at point A on a tree branch  $h_1 = 22.0 \text{ m}$  above the floor of the jungle. Tarzan is holding one end of a vine which is attached to a branch on a second tree. The vine is  $L = 21.0 \text{ m}$  long. When Tarzan swings on the vine, his path is in an arc of a circle. At the bottom of his swing he is at point B,  $13.0 \text{ m}$  above the ground . Ignore

Tarzan's height. Tarzan has a mass of 90.0 kg. The vine does not stretch and has negligible mass.

- (a) Why does the tension in the vine do no work?
- (b) What will be his speed at point B?
- (c) What will be the tension in the rope at point B?



The problem involves a change in height and speed, so we apply the generalized Work-Energy Theorem.

$$W_{NC} = E = (K_f - K_i) + (U_f - U_i) = \frac{1}{2}m(v_B)^2 - mgh \quad (1)$$

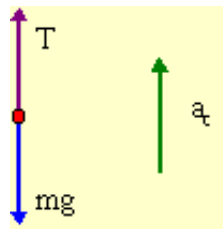
(a) Here the only possible nonconservative force is friction, so  $W_{NC} = W_T$ . The definition of work is  $W = Fx\cos$ , but in this problem the tension is along a radius and is thus always at 90 to the displacement. As a result,  $W_T = 0$ . Thus we have

$$0 = \frac{1}{2}m(v_B)^2 - mgh$$

(b) To find the speed at point B, we need to know  $h$ , the distance Tarzan dropped. Examining the question, we see that  $h = h_1 - h_2 = 22.0 \text{ m} - 13.0 \text{ m} = 9.0 \text{ m}$ . Rearranging our equation, we find

$$v_B = [2gh]^{1/2} = [2(9.81 \text{ m/s}^2)(9 \text{ m})]^{1/2} = 13.29 \text{ m/s}$$

(c) Tension is a force. To find a force we need to draw a FBD and apply Newton's Second Law. Since Tarzan is swinging in a circle, we are dealing with centripetal acceleration.



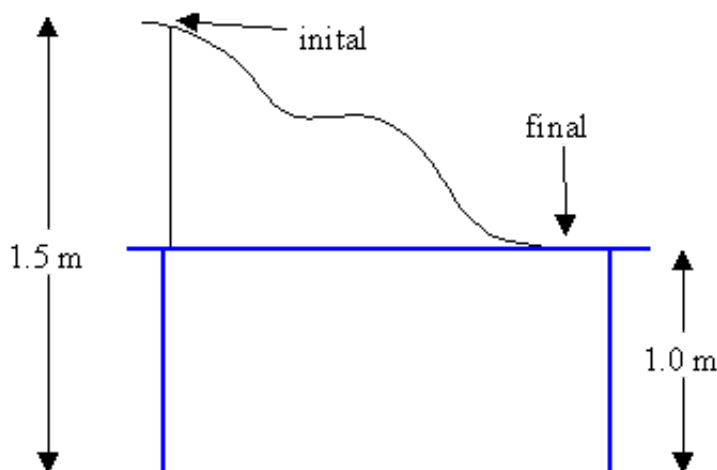
$$F_y = ma_y$$

$$T - mg = mv^2/L$$

Solving for T,

$$T = mg + mv^2/L = (90.0 \text{ kg})[ 9.81 \text{ m/s}^2 + (13.288 \text{ m/s})^2/(21.0 \text{ m})] = 1.64 \cdot 10^3 \text{ N}$$

19. A 0.200-kg block slides down the track and horizontally off a table as shown in the diagram below.
- (a) Assuming that friction is negligible, how far from the table does the block land?
- (b) The block only land 1.20 m away. How much work was done by friction and other non-conservative forces?



The first part of the problem involves a change in height and speed, so we can use the Work-Energy Theorem there. When the block leaves the surface it becomes a projectile.

- (a) Applying the Work-Energy Theorem and assuming that the initial velocity of the block is zero.

$$W_{NC} = \Delta E = (K_f - K_i) + (U_f - U_i) = \frac{1}{2}m(v_B)^2 - 0 + mg(h_f - h_i) .$$

The mass  $m$  cancels out and we find

$$v_B = [-2g(h_f - h_i)]^{1/2} = [-2(9.81)(1.0 - 1.5)]^{1/2} = 3.3121 \text{ m/s} .$$

Now this velocity is the initial velocity for the projectile.

$i$	$j$
$v_{0x} = 3.3121 \text{ m/s}$	$v_{0y} = 0 \text{ m/s}$ (horizontal flight)
$a_x = 0 \text{ m/s}^2$	$a_y = -9.81 \text{ m/s}^2$
$\Delta x = ?$	$\Delta y = -1.0 \text{ m}$
----- $t$ (common) -----	

From the  $j$  information we can find the time that the block is in the air using  $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ . This becomes  $-1.0 \text{ m} = \frac{1}{2}(-9.81 \text{ m/s}^2)t^2$  or  $t = \pm 0.4515 \text{ s}$ . We need the positive, forward in time, solution. We then find  $\Delta x$  using



$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2 = (3.3121 \text{ m/s})(0.4515 \text{ s}) = 1.4955 \text{ m}.$$

The block lands 1.50 m from the edge of the table.

(b) If the block only lands 1.20 m away, then its velocity must have been  $v_{0x} = (1.20 \text{ m}) / (0.4515 \text{ s}) = 2.6578 \text{ m/s}$ .

This is also the velocity at the bottom of the slide. To find  $W_{\text{NC}}$  we again use the Work-Energy Theorem.

$$W_{\text{NC}} = (K_f - K_i) + (U_f - U_i) = \frac{1}{2}m(v_{0x})^2 - 0 + mg(h_f - h_i).$$

So the work done by non-conservative forces is

$$W_{\text{NC}} = \frac{1}{2}(0.200)(2.6578)^2 + (0.200)g(1.0 - 1.5) = -0.2746 \text{ J}.$$

**Top**

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20. A 50-hp engine is used to lift heavy loads at a worksite. It is used to lift a load of bricks weighing 2000 N to the top of a new building 35.0 m above ground. How long does it take for the load to get to the top?

We are given the power of the engine

$$P = 50 \text{ hp} \times (740 \text{ W/hp}) = 37,000 \text{ W}.$$

Power is defined as work done per given time,  $P = W/t$ . The time  $t$  is what we are asked for. Work done is force times distance, here  $W = 2000 \text{ N} \times 35 \text{ m} = 70,000 \text{ J}$ .

So the time needed is

$$t = W/P = 70,000 \text{ J} / 37,000 \text{ W} = 1.89 \text{ seconds}.$$

Note however that rated power is seldom the same as the actual power that does useful work.

**Top**

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21. A 5.0 MW generating station is situated at a 22-m high dam. The energy used to generate the electricity comes from the loss in potential energy of the water as it falls the height of the dam. What is the minimum amount of water going through the dam every day?

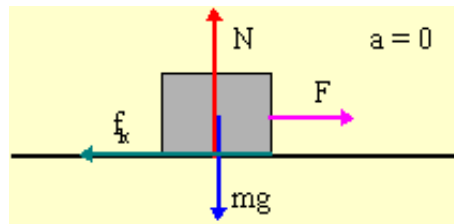
We are given the power (50,000,000 W) and power is defined as work done per given time,  $P = W/t$ . The time  $t$  we are given is one day. We are told that the work done equals the loss in potential energy of the water falling from the top of the dam, so  $W = mgh$  where  $h = 22.0 \text{ m}$ . Thus the amount of water, i.e. its mass, is found from  $P = mgh / t$  or

$$m = Pt/gh = (5 \times 10^6)(1 \text{ d} \times 24 \text{ h/d} \times 3600 \text{ s/h}) / (9.81)(22.0) = 2.00 \times 10^9 \text{ kg}.$$

Top

22. What power is required to pull a 5.0-kg block at a steady speed of 1.25 m/s? The coefficient of friction is 0.30.

The power required to move the block at constant speed is  $P = Fv$ . We are given  $v$ , the speed of the block. To get  $F$ , a force, we draw a FBD and apply Newton's Second Law,



$$\begin{array}{cc} i & j \\ F_x = ma_x & F_y = ma_y \\ F - f_k = 0 & N - mg = 0 \end{array}$$

The second equation gives  $N = mg$  and we know  $f_k = \mu_k N$ , so  $f_k = \mu_k mg$ . Therefore, the applied force is  $F = \mu_k mg$ . Thus the power is

$$P = \mu_k mgv = (0.3)(5 \text{ kg})(9.81 \text{ m/s}^2)(1.25 \text{ m/s}) = 18.4 \text{ Watts.}$$

Top

23. A 7000-W engine is propelling a speedboat at 30 km/h. What force is the engine exerting on the speedboat? What force and how much power is water resistance exerting on the speedboat?

First we convert the velocity to SI units,

$$30 \text{ km/h} \times (1000 \text{ m})/\text{km} \times (1 \text{ h})/(3600 \text{ s}) = 8.333 \text{ m/s} .$$

We know  $P = Fv$ , so

$$F = P/v = 7000 \text{ W} / 8.333 \text{ m/s} = 840 \text{ N} .$$

By Newton's Third Law, the water is exerting 840 N in the reverse direction. It is also removing 7000 W of power which is going into increasing the kinetic energy of the water.

Top

**Physics**

**Coombes**

**Handouts**

**Problems**

**Solutions**

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