Fraction Rules

Terminology:

Numerator – the number above the fraction line.

Denominator – the number below the fraction line.

$$\frac{3}{8}$$

Proper Fraction – a fraction in which the numerator is less than the denominator.

$$\frac{3}{4}$$

Improper Fraction – a fraction in which the numerator is greater than the denominator.

$$\frac{3}{2}$$

Mixed Number – a combination of both a whole number and a fraction.

$$4\frac{5}{8}$$

Reciprocal – when the numerator and denominator of a fraction are flipped.

Fraction`	Reciprocal Fraction
$\frac{7}{8}$	$\frac{8}{7}$
Whole Number	Reciprocal of Whole Number
8	$\frac{1}{8}$

Common Denominator – when the denominator of one fraction is equal to the denominator of another fraction.

Common Denominator	Non-common Denominator
$\frac{5}{8}, \frac{1}{8}$	$\frac{5}{8}, \frac{1}{3}$

Fraction Rules

Converting between mixed number/ improper fraction

Improper fraction ⟶ mixed number

$\frac{21}{8}$ Starting with an improper fraction.

$8\overline{)21} \atop $ $\begin{array}{r} 2 \\ 8\overline{)21} \\ -16 \\ \hline 5 \end{array}$ Divide the numerator by the denominator as usual but, keep the remainder. We cannot divide 8 into 5 so, 5 is the remainder.

Place the whole number to the left and the remainder over the original denominator.

Mixed number ⟶ improper fraction:

$2\frac{5}{8}$ Starting with a mixed number.

$\frac{2 \times 8 + 5}{8}$ Multiply the whole number by the denominator and add it to the numerator.

$\frac{21}{8}$

Example:

$\frac{22}{7} =$

$3\frac{1}{7} = \underline{\times+}$

How to reduce a fraction to lowest term

Some fractions can be reduced to smaller numbers on the top and bottom but still be the same value, this is called reducing a fraction.

$\frac{16}{32}$ To reduce a fraction, we need to check if the numerator and denominator are divisible by the same number. We call common factor.

$\frac{16 \div 2}{32 \div 2}$ In this case we can divide the top and bottom by 2 because they are both even.

$\frac{8}{16}$ And the fraction has been reduced to $\frac{8}{16}$

$\frac{8 \div 2}{16 \div 2}$ but we can see that it is still divisible by 2 on the top and bottom. So, we will do it again.

$\frac{4}{8}$ We don't always have to divide by 2, it can be any number. For example,

$\frac{4 \div 4}{8 \div 4}$ we can divide the numerator and denominator by 4.

$\frac{1}{2}$ And we can see that $\frac{16}{32}$ is actually $\frac{1}{2}$ we could have also divided the numerator and denominator by 16 in the first step to get the same result.

Example:

$\frac{15}{30} = \frac{\div}{\div} = \frac{\div}{\div} = \frac{\square}{\square}$

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Fraction Rules

Common Denominators

Finding a common denominator	Converting the fractions with the common denominator

A common denominator is required when adding or subtracting fractions.

$\frac{1}{12}, \frac{1}{16}$

To find the Lowest Common Denominator (LCD) for $\frac{1}{12}$ and $\frac{1}{16}$.

12 , 16

Place both denominators in the configuration shown.

2 | 12   16

Find a number that evenly divides into both denominators, in this case we will use 2. Place this number to the left.

2 | 12   16
  6 8

Divide both denominators by the chosen number and place the results below.

2 | 12   16
2 | 6 8
  3 4

Redraw the shape around the new numbers and pick a new number that evenly divides into both, we will use 2 again.

Now no number will evenly go into both 3 and 4, so we are done.

$2 \times 2 \times 3 \times 4 = 48$

To find the least common denominator, multiply all the numbers around the outside.

After finding the common denominator, we can convert both fractions, so they can be added or subtracted. Be aware not just change the bottom of a fraction but also multiple the top with the same factor, for example:

The original fraction is $\frac{1}{12} = 0.0833$, when you only change the denominator into 48 without changing the numerator, then the faction will be converted to $\frac{1}{12 \times 4} = \frac{1}{48} = 0.020833$. They are obviously not equal. You should multiple the numerator with the same factor as 4, which means $\frac{1 \times 4}{12 \times 4} = \frac{4}{48} = \frac{1}{12} = 0.0833$

2 | 12   16
2 | 6 8
  3 4

$2 \times 2 \times 3 \times 4 = 48$

LCD=48

$12 \times 4 = 48$
$16 \times 3 = 48$

You may also notice that after you finish using all the common factors, the denominator (12) on the left side multiplying the bottom factor on the right side (4) is equal to 48, the denominator on the right side multiplying the bottom factor on the left side is also equal to 48, which shows the easy way to get 48 (LCD) from 12 & 16.

Find the LCD of the following groups of fractions:

$\frac{4}{9}, \frac{5}{6}$ $\frac{1}{4}, \frac{3}{16}$ $\frac{1}{12}, \frac{1}{15}$ $\frac{1}{8}, \frac{5}{12}$

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Fraction Rules

Adding Fractions

Rule for Addition: When adding any type of fractions, a common denominator is needed. That is, the bottom number of both fractions must be the same.

Fraction + Fraction (common denominator)

$$\frac{5}{16} + \frac{7}{16}$$

Because these fractions already
have a common denominator we do not need to find one.

Just add the numbers on top.

$$\frac{5+7}{16}$$

$$\frac{12}{16}$$

$$\frac{12 \div 4}{16 \div 4}$$

Reduce the fraction if possible.

$$\frac{3}{4}$$

Fraction + Whole Number

$$\frac{5}{12} + 5$$

Simply combine the whole number with the fraction as a mixed number

$$5\frac{5}{12}$$

Fraction + Fraction

$$\frac{1}{12} + \frac{1}{16}$$

First find a lowest common denominator.

$$
\begin{array}{r|cc}
2 & 12 & 16 \\
2 & 6 & 8 \\
\hline
 & 3 & 4
\end{array}
$$

$$2 \times 2 \times 3 \times 4 = 48$$

$$\frac{48}{12} = 4, \qquad \frac{48}{16} = 3$$

The lowest common denominator is 48.

$$12 \times 4 = 48$$
$$16 \times 3 = 48$$

$$\frac{1 \times 4}{12 \times 4} + \frac{1 \times 3}{16 \times 3}$$

Convert each fraction.

$$\frac{4}{48} + \frac{3}{48}$$

Now just add the top numbers.

$$\frac{7}{48}$$

Mixed Number + Mixed Number

$$1\frac{7}{12} + 2\frac{7}{16}$$

Find a lowest common denominator.

$$
\begin{array}{r|cc}
2 & 12 & 16 \\
2 & 6 & 8 \\
\hline
 & 3 & 4
\end{array}
$$

$$2 \times 2 \times 3 \times 4 = 48$$

$$\frac{48}{12} = 4, \qquad \frac{48}{16} = 3$$

The lowest common denominator is 48.

$$1\frac{7 \times 4}{12 \times 4} + 2\frac{7 \times 3}{16 \times 3}$$

Convert each fraction.

$$1\frac{28}{48} + 2\frac{21}{48}$$

Now add the whole numbers and
add the fractions separately.

$$3\frac{49}{48}$$

$$3 + \frac{49}{48}$$

Convert the improper fraction portion to a mixed number and
add the whole number part to the existing whole number.

$$3 + 1\frac{1}{48}$$

$$4\frac{1}{48} \qquad \longleftarrow \qquad \frac{49}{48} = 1\frac{1}{48}$$

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Fraction Rules

Subtracting Fractions

Rule for Subtraction: When subtracting any type of fractions, a common denominator is always needed. That is, the bottom number on both fractions must be the same.

Fraction – Fraction (common denominator)

$$\frac{31}{32} - \frac{13}{32}$$

Because these fractions already have a common denominator we do not need to find one.

$$\frac{31 - 13}{32}$$

Just subtract the top numbers.

$$\frac{18}{32}$$

$$\frac{18 \div 2}{32 \div 2}$$

Reduce the fraction if possible.

$$\frac{9}{16}$$

Fraction – Fraction

$$\frac{5}{6} - \frac{3}{8}$$

Find the lowest common denominator.

$$2 \begin{array}{|cc} 6 & 8 \\ 3 & 4 \end{array}$$

$$2 \times 3 \times 4 = 24$$

The lowest common denominator is 24.

$$\frac{24}{6} = 4 \quad \frac{24}{8} = 3$$

$$\frac{5 \times 4}{6 \times 4} - \frac{3 \times 3}{8 \times 3}$$

Convert each fraction.

$$\frac{20}{24} - \frac{9}{24}$$

Now subtract the top numbers.

$$\frac{11}{24}$$

Whole number – Fraction

$$3 - \frac{5}{8}$$

Borrow 1 from the whole number and turn it into a fraction with the same denominator as the subtracting fraction.

$$2\frac{8}{8} - \frac{5}{8}$$

$$2\frac{8 - 5}{8}$$

Keep the whole number and make the fraction portions subtract.

$$2\frac{3}{8}$$

Mixed number – Mixed Number

$$3\frac{5}{12} - 2\frac{7}{16}$$

Find the lowest common denominator.

$$4 \begin{array}{|cc} 12 & 16 \\ 3 & 4 \end{array}$$

$$4 \times 3 \times 4 = 48$$

The lowest common denominator is 48.

$$\frac{48}{12} = 4 \quad \frac{48}{16} = 3$$

Convert each fraction.

$$3\frac{5 \times 4}{12 \times 4} - 2\frac{7 \times 3}{16 \times 3}$$

$$3\frac{20}{48} - 2\frac{21}{48}$$

Now 20-21 would give us a negative number so we must borrow 1 from the whole number 3.

$$2\frac{1 \times 48 + 20}{48} - 2\frac{21}{48}$$

When we borrow one from the whole number, we multiply the base by 1 and add it to the top.

$$2\frac{68}{48} - 2\frac{21}{48}$$

Now subtract the whole numbers and then subtract the tops.

$$\frac{47}{48}$$

Fraction Rules

Multiplying fractions

Rule for Multiplying: When doing multiplying, change all items (mixed numbers, numbers) into improper fraction forms first.

Fraction $\times$ Fraction		Fraction $\times$ Mixed Number	
$\frac{3}{8} \times \frac{17}{32}$	Multiply the numbers beside each other.	$\frac{4}{5} \times 2\frac{1}{3}$	Convert the mixed number to an improper fraction.
$\frac{3 \times 17}{8 \times 32}$		$\frac{4}{5} \times \frac{7}{3}$	Multiply the numbers beside each other.
$\frac{51}{256}$		$\frac{4 \times 7}{5 \times 3}$	
		$\frac{28}{15}$	Convert to mixed number.
		$1\frac{13}{15}$	

Fraction $\times$ Whole number		Mixed Number $\times$ Mixed Number	
$\frac{7}{16} \times 4$	Change the whole number to a fraction by placing it over 1.	$1\frac{3}{5} \times 3\frac{2}{8}$	Convert the mixed numbers to improper fractions.
$\frac{7}{16} \times \frac{4}{1}$	Multiply the numbers beside each other.	$1\frac{\overset{\div 8}{8}}{5} \times \frac{26}{\underset{1}{8} \div 8}$	Simplify the numbers if possible.
$\frac{7 \times 4}{16 \times 1}$			Multiply the numbers beside each other.
$\frac{28}{16}$		$\frac{26}{5}$	
$\frac{28 \div 4}{16 \div 4}$	Reduce the fraction.		
$\frac{7}{4}$		$5\frac{1}{5}$	Convert to a mixed number.
$1\frac{3}{4}$	Convert to a mixed number.		

Fraction Rules

Dividing fractions

Rule for Dividing: When doing dividing, change all items (mixed numbers, numbers) into improper fraction form, then flip the 2nd item and change the dividing sign into multiplying.

Fraction ÷ Fraction	Fraction ÷ Mixed Number
$\frac{3}{7} \div \frac{5}{8}$ Flip the 2nd fraction over (reciprocal) and Change the sign to multiply. $\frac{3}{7} \times \frac{8}{5}$	$\frac{5}{8} \div 1\frac{3}{16}$ Convert the mixed number to an improper fraction. $\frac{5}{8} \div \frac{19}{16}$
$\frac{3 \times 8}{7 \times 5}$ Multiply the numbers beside each other. $\frac{24}{35}$	$\frac{5}{8} \times \frac{16}{19}$ Flip the 2nd fraction over (reciprocal) and change the sign to multiply. $\frac{5 \times 16}{8 \times 19}$ Multiply the numbers beside each other. $\frac{80}{152}$ $\frac{80 \div 8}{152 \div 8}$ Reduce the fraction. $\frac{10}{19}$

Fraction ÷ Whole Number	Mixed Number ÷ Mixed Number
$\frac{5}{8} \div 3$ Change the whole number to a fraction by placing it over 1. $\frac{5}{8} \div \frac{3}{1}$ Flip the 2nd fraction over (reciprocal) and change the sign to multiply. $\frac{5}{8} \times \frac{1}{3}$ $\frac{5 \times 1}{8 \times 3}$ Multiply the numbers beside each other. $\frac{5}{24}$	$2\frac{5}{8} \div 1\frac{1}{6}$ Convert the mixed numbers to improper fractions. $\frac{21}{8} \div \frac{7}{6}$ Flip the 2nd fraction over (reciprocal) and change the sign to multiply. $\frac{\overset{3}{\cancel{21}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{7}}}$ Simplify the numbers if possible. $\frac{3 \times 3}{4 \times 1}$ Multiply the numbers beside each other. $\frac{9}{4}$ Check if the answer is reduced to the lowest term. $2\frac{1}{4}$ Convert to mixed number if required.

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